An analytical approach for one-dimensional advection-diffusion equation with temporally dependent variable coefficients of hyperbolic function in semi-infinite porous domain

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Abstract - *An analytical solution is obtained for one-dimensional advection-diffusion equation with time dependent variable coefficients in semi-infinite porous medium. We consider the solute dispersion parameter and seepage velocity is hyperbolically decreasing function of time. The dispersion parameter is proportional to velocity of the flow. The first order decay and zero-order production terms are also considered into account which is time dependent and reducing in nature. The nature of pollutant and porous medium are considered chemically non-reactive. Initially porous domain is considered solute free and uniform continuous. A new time and space variables are introduced to get the analytical solution. The governing transport equation is solved analytically by employing Laplace transformation technique. The effect of various physical parameters on the contaminant concentration is illustrated graphically. Numerical solution obtained by PDEPE Method is compared with analytical solution.*

Key Words: *Advection, Diffusion, Continuous Input, First-order Decay, Zero-order Production, PDEPE Method, Homogeneous Medium.*

1. INTRODUCTION

Solute transport phenomenon is always an important issue for groundwater contamination which depends on the various mixing processes caused by dispersion. The solute transport in groundwater is affected by large number of physical and chemical processes. If groundwater becomes polluted, it is very difficult to rehabilitate. The slow rate of groundwater flow and low microbiological activities limit any self-purification processes which takes place in days or weeks, in surface water systems can take decades. Mathematical modeling of contaminant behaviour in porous media is considered to be a powerful tool for a wide range of pollution problems related with groundwater quality rehabilitation. Development of an analytical solutions for groundwater pollution problems are major interesting area for civil engineers, hydrologist, chemical engineer and mathematicians, because these solutions are very much helpful to understand the mechanism of contaminant transport, predict the movement of contaminant plumes, measure the field parameters related to solute transport, and verify the numerical results. Most theories of solute transport in porous media include limited assumptions, such as spatially stationary groundwater flow and the absence or negligibility of boundary influences.

Mathematical models describing groundwater flow and solute transport in homogeneous and heterogeneous porous media have been developed in the past and are available in literature. Yule and Gardener [35] obtained analytical solutions to describe solute transport from line source neglecting longitudinal dispersion and van Genuchten [29] derived analytical solutions for chemical transport with simultaneous adsorption zero-order production and first-order decay. Davis [6] was derived the analytical solution of a diffusion-convection equation over a finite domain by using Laplace transform technique. Barry and Sposito [1] are obtained analytical solution of a convection-dispersion model with time-dependent transport coefficients while Yadav et al. [31] presented a solution of one-dimensional dispersion in unsteady flow in an adsorbing medium. Yates ([33], [34]) obtained the analytical solutions for one-dimensional advection dispersion equation considering linear and exponential increasing dispersion coefficient. Leij et al. [18] obtained analytical solutions in an infinite or semi-infinite medium assume a uniform initial concentration. Basha et al. [2] was examined an analytical solution of the one dimensional time dependent transport equation in the porous domain. Fry et al. [8] derived analytical solution for the advection–dispersion equations in a one-dimensional homogeneous isotropic porous medium with rate-limited desorption and first-order decay. Logan [19] developed an analytical solution to the one dimensional advection dispersion equations for solute transport in a heterogeneous porous medium considering sorption and first-order decay under time-varying boundary conditions. Hunt [10] discussed one, two and three-dimensional advection dispersion equation with scale dependent dispersion coefficients for unsteady flow and Marinoschi et al. [20] obtained analytical solutions of three dimensional convection-dispersion equation with time Dependent Coefficients in porous domain. Pang and Hunt [22] obtained analytical solutions for a one-dimensional advection dispersion equation with scale-dependent dispersion and linear equilibrium sorption and first-order degradation while Kumar and Kumar [17] discussed a model of Horizontal solute dispersion in unsteady flow through homogeneous finite aquifer with time dependent variable coefficients. Singh et al. [27] obtained analytical solution for conservative solute transport in one-dimensional homogeneous porous formation with time-dependent velocity. Kumar et al. [15] obtained analytical solutions for temporally and spatially dependent solute dispersion in a one-dimensional, semi-infinite, porous medium. One-dimensional temporally dependent advection-dispersion equation in finite homogeneous porous media was examined by Jaiswal et al. [12] while Singh and Kumari [25] derived an analytical solution of advection diffusion equation for contaminant transport in two-dimensional homogeneous semi-infinite domain. Yadav et al. [32] presented analytical solution of horizontal solute transport from a pulse type source along temporally and spatially dependent flow in porous domain while Kumar et al. [16] obtained analytical solutions of one-dimensional advection-diffusion equation with temporally dependent variable coefficients in longitudinal semi-infinite homogeneous porous medium for uniform flow. The first order decay and zero-order production terms are also considered. Guerrero et al. [9] obtained analytical solutions of the one-dimensional advection–dispersion solute transport equation subject to time- dependent boundary conditions and Generalized analytical formulas are established which relate the exact solutions to corresponding time-independent auxiliary solutions. Jaiswal and Yadav [11] obtained analytical solutions of one-dimensional advection-diffusion equation with temporally dependent coefficient of second order space derivative in a finite domain for two sets of pulse type input boundary conditions. Kumar and Yadav [14] developed an analytical solution is developed for conservative solute transport in a one-dimensional heterogeneous porous medium while Singh et al. [26] derived a analytical solution of a two-dimensional advection diffusion equation with time dependent coefficients by using Laplace Transformation Technique. Khaled et al. [13] explained analytical solution of advection diffusion equation by different methods. Das et al. [5] resents mathematical modeling of groundwater contamination with varying velocity field. Singh and Chatterjee [24] present a solution of three dimensional advection dispersion equation with non-point source of contamination in semi-infinite aquifer with specified concentration along an arbitrary plane using Laplace transform technique. Sanskrityayn et al. [23] presents an analytical solution of advection dispersion equation with spatiotemporal dependence of dispersion coefficient and velocity using green's function method. Moghaddam et al. [21] developed a numerical model for one dimensional solute transport in rivers.

In the present study, Advection–dispersion equation is solved analytically using Laplace Transform Technique (LTT) in a semi-infinite homogeneous porous domain. The hydrodynamic dispersion, seepage velocity is considered hyperbolically decreasing function of time and directly proportional to each other. First order decay and zero-order production terms are also considered which is taken as time dependent. The medium is considered semi-infinite homogeneous in longitudinal direction. Solution is obtained for uniform input point source condition. Initially porous domain is considered solute free that is there is no concentration in the medium. The input condition is introduced at the origin of the domain and second condition is considered at the end of the domain. Concentration gradient at infinity is considered zero. New time and space variables are introduced at the different stage through different transformations. It helps to reduce the variable coefficients into constant coefficients. So a much simpler but more viable Laplace transformation technique is used to get the analytical solution. Solution has been demonstrated using a set of hypothetical input data taken from the previous published works related to immiscible pollutant particle transport down the groundwater flow. Ground water velocity ranges from 2 m/year to 2 m/day (Todd, [28]).

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The advection-dispersion equation in one-dimension is written as (Freeze and Cherry, [7] and Bear, [3]).

$$\frac{\partial C}{\partial t}=\frac{\partial}{\partial x}\left(D(x,t)\frac{\partial C}{\partial x}-u(x,t)C\right)-\gamma C+\mu \qquad (1)$$

where $C[ML^{-3}]$ is the solute concentration in the liquid phase. $x[L]$, $t[T]$ are space and time variable respectively. $D[L^2T^{-1}]$ is the solute dispersion parameter. $u[LT^{-1}]$ is the velocity of the flow. $\gamma[T^{-1}]$ is first order decay rate coefficient and $\mu[ML^{-3}T^{-1}]$ zero-order production which represents internal/external production of the solute. In equation (1), D and u may be constants or functions of time or space. Velocity at the scale of pores causes a solute particle to spread from initial position. This spreading phenomenon is described at the Darcy scale through dispersion coefficient.

2.1 Uniform Input Point Source Condition

The model simulates concentration along one-dimensional time dependent flow through homogeneous semi infinite porous medium. Groundwater flow is considered along the *x*-axis, means the direction of the flow of water is from $x=0$ to $x\to\infty$. Initially the porous domain is supposed to be solute free, means before solute injection in the domain there is no concentration present in the domain. A continuous mass injection of solute is introduced into the aquifer at $x=0$.

To proceed further, the initial and boundary conditions are defined as

$$C(x,t)=0 \quad ; \; t=0 \; , \quad x>0 \tag{2}$$

$$C(x,t)=C_0 \quad ; \; t\geq 0 \; , \; x=0 \tag{3}$$

$$\frac{\partial C(x,t)}{\partial x}=0 \quad ; \; t\geq 0 \; , \quad x\to\infty \tag{4}$$

2. 2 Analytical solution

Let us write $\quad u(x,t)=\dfrac{u_0}{1+\sinh(mt)}$

$$D(x,t)\propto u \quad \Rightarrow \quad D(x,t)=\frac{D_0}{1+\sinh(mt)}$$

$$\gamma(x,t)=\frac{\gamma_0}{1+\sinh(mt)} \quad \text{and} \quad \mu(x,t)=\frac{\mu_0}{1+\sinh(mt)}$$

where u_0, D_0, γ_0 and μ_0 are initial constants of velocity of flow, dispersion parameter, first order decay and zero order production respectively.

Substituting these values in Eq. (1) we have

$$\frac{\partial C}{\partial t}=\frac{1}{1+\sinh(mt)}\left(D_0\frac{\partial^2 C}{\partial x^2}-u_0\frac{\partial C}{\partial x}-\gamma_0 C+\mu_0\right) \tag{5}$$

Let us introduce a new time variable using the following transformation (Crank, [4])

$$T=\int_0^t\left\{\frac{1}{1+\sinh(mt)}\right\}dt \tag{6}$$

$$\frac{dT}{dt}=\{1+\sinh(mt)\}^{-1}$$

$$T=\frac{1}{\sqrt{2}\,m}\Big[\log\{2+\sqrt{2}+(4+3\sqrt{2})\tanh(mt/2)\} -\log\{2+\sqrt{2}-\sqrt{2}\tanh(mt/2)\}\Big] \tag{7}$$

where $m[T^{-1}]$ is a resistive coefficient whose dimension is inverse of that of time variable t. Here m is chosen such that for $m=0$ or $t=0$, Gives $T=0$. The new time variable obtained from Eq. (6) satisfies the conditions $T=0$ for $t=0$ and $T=t$ for $m=0$. The first condition ensures that the nature of the initial condition does not change in the new time variable domain. The second condition refers to the temporally independent dispersion and Eq. (5) reduces to one with constant coefficients.

As a result of this transformation Eq. (6), Eq.(5) reduces in to advection dispersion equation with constant coefficients as

$$\frac{\partial C}{\partial T}=D_0\frac{\partial^2 C}{\partial x^2}-u_0\frac{\partial C}{\partial x}-\gamma_0 C+\mu_0 \tag{8}$$

These initial and boundary conditions are written in the terms of new time variable T is as

$$C(x,T)=0 \quad ; \quad T=0 \; , x>0 \tag{9}$$

$$C(x,T)=C_0 \quad ; \quad T\geq 0 \; , x=0 \tag{10}$$

$$\frac{\partial C(x,T)}{\partial x}=0 \quad ; \; T\geq 0 \; , \; x\to\infty \tag{11}$$

To eliminate the convection term from advection-diffusion equation Eq. (8) we use the following transformation (12) in terms of new dependent variable K(x, T) defined as

$$C(x,T)=K(x,T)\exp\left[\frac{u_0}{2D_0}x-\left(\frac{u_0^{\,2}}{4D_0}+\gamma_0\right)T\right]+\frac{\mu_0}{\gamma_0} \tag{12}$$

The initial and boundary value problem in terms of new dependent variable $K(x,T)$ are as follows

$$\frac{\partial K}{\partial T}=D_0\frac{\partial^2 K}{\partial x^2} \tag{13}$$

$$K(x,T)=-\frac{\mu_0}{\gamma_0}\exp\left(-\frac{u_0}{2D_0}x\right) \quad ; T=0 \; , \; x>0 \tag{14}$$

$$K(x,T)=\left(C_0-\frac{\mu_0}{\gamma_0}\right)\exp\left(\alpha^2 T\right) \quad ; T\geq 0 \; , \; x=0 \tag{15}$$

$$\frac{\partial K(x,T)}{\partial x}+\frac{u_0}{2D_0}K=0 \quad ; \; T\geq 0 \; , x\to\infty \tag{16}$$

where $\quad \alpha^2=\left(\dfrac{u_0^{\,2}}{4D_0}+\gamma_0\right)$

Applying the Laplace transformation in the set of above initial and boundary value problem which reduce to it in an ordinary differential equation of second order boundary value problem, which comprises of following three equations

$$\frac{d^2\overline{K}}{dx^2}-\frac{p}{D_0}\overline{K}=\frac{\mu_0}{\gamma_0 D_0}\exp\left(-\frac{u_0}{2D_0}x\right) \tag{17}$$

where $\overline{K}=\int_0^\infty K(x,T)e^{-pT}\,dT$

$$\overline{K}(x,p)=\left(C_0-\frac{\mu_0}{\gamma_0}\right)\frac{1}{p-\alpha^2} \quad ; \; x=0 \tag{18}$$

$$\frac{d\overline{K}}{dx}+\frac{u_0}{2D_0}\overline{K}=0 \quad ; \quad x\to\infty \tag{19}$$

Where p is Laplace parameter.

The particular solution of this boundary value problem in Laplace domain may be obtained as

$$\overline{K}(x,p)=\left\{\left(C_0-\frac{\mu_0}{\gamma_0}\right)\frac{1}{(p-\alpha^2)}+\frac{\mu_0}{\gamma_0}\frac{1}{(p-\beta^2)}\right\}\exp\left(-x\sqrt{\frac{p}{D_0}}\right)-\frac{\mu_0}{\gamma_0}\exp\left(-\frac{u_0}{2D_0}x\right)\frac{1}{(p-\beta^2)} \quad (20)$$

where $\beta^2=\frac{u_0^2}{4D_0}$

Applying inverse Laplace transformation on it, using the appropriate tables (van Genuchten and Alves [30]) and using the necessary transformations defined earlier in the text, backwards, we may get the desired analytical solution as

$$C(x,T)=\frac{\mu_0}{\gamma_0}+\left(C_0-\frac{\mu_0}{\gamma_0}\right)F(x,T)-\frac{\mu_0}{\gamma_0}G(x,T) \quad (21)$$

where

$$F(x,T)=\frac{1}{2}\exp\left(\frac{\{u_0-(u_0^2+4\gamma_0 D_0)^{1/2}\}x}{2D_0}\right)\operatorname{erfc}\left(\frac{x-(u_0^2+4\gamma_0 D_0)^{1/2}T}{2\sqrt{D_0 T}}\right)+\frac{1}{2}\exp\left(\frac{\{u_0+(u_0^2+4\gamma_0 D_0)^{1/2}\}x}{2D_0}\right)\operatorname{erfc}\left(\frac{x+(u_0^2+4\gamma_0 D_0)^{1/2}T}{2\sqrt{D_0 T}}\right)$$

$$G(x,T)=\exp(-\gamma_0 T)\left\{1-\frac{1}{2}\operatorname{erfc}\left(\frac{x-u_0 T}{2\sqrt{D_0 T}}\right)-\frac{1}{2}\exp\left(\frac{u_0 x}{D_0}\right)\operatorname{erfc}\left(\frac{x+u_0 T}{2\sqrt{D_0 T}}\right)\right\}$$

and T is defined in Eq. (7).

2.3 Result and discussion

Solution obtained in Equation (21) describes the solute concentration distribution for uniform input point source through a homogeneous medium with temporally dependent coefficients. Concentration attenuation with position and time is studied in a finite domain $0 \le x(\text{meter}) \le 10$ at different values of parameters. All concentration values are evaluated from Eq. (21). The common input data are chosen as $C_0=1.0$, $t=5\,(\text{day})$ initial seepage velocity $u_0=0.25\,(\text{meter/day})$, initial dispersion coefficient $D_0=0.45\,(\text{meter}^2/\text{day})$, unsteady parameter $m=0.1\,(\text{day})^{-1}$, initial first order decay $\gamma_0=0.02(\text{day})^{-1}$, and initial zero order production $\mu_0=0.001\,(\text{kg/meter}^3\,\text{day})$. The solute concentration distribution behaviour in the domain at various parameters is represented graphically.

Figure 1. Illustrate the concentration distribution behaviour for different time *t*(day)= 5, 10 and 15 in the domain. It is observed that contaminant concentration decreases with distance travelled in presence of source contaminants and at a particular position increases with increase in time. This decreasing tendency of contaminant concentration with time and distance travelled may help to rehabilitate the contaminated aquifer. It is also observed that at particular position the concentration level is lower for smaller time and higher for larger time. Figure 2. Shows the concentration profile on various flow resistance for m(day)$^{-1}$ = 0.1, 0.2 and 0.3 at particular time t(day)=5. It is observed that the contaminant concentration is faster for the higher flow resistance and slower for lower flow resistance with distance travelled in presence of source contaminant at particular time. Figure 3. Represents the concentration distribution for various velocity of flow u_0(meter/day)= 0.05, 0.15, 0.25 at particular time t (day) = 5.0 in the domain. It is observed that the contaminant concentration is faster for lower velocity and slower for higher velocity with distance travelled in presence of source contaminant at particular time and dispersion parameter D_0(meter^2/day)= 0.45. Figure 4. Depict the effect of various dispersion parameter D_0(meter^2/day)= 0.45, 0.65, 0.85 on concentration distribution at particular time *t*(day)=5.0 and seepage velocity u_0(meter/day)= 0.25. It is observed that the contaminant concentration is faster for the smaller and slower for higher dispersion parameter with distance travelled in presence of source contaminant at particular time and velocity of flow u_0(meter/day)= 0.25. Figure 5. shows comparison of analytical and numerical solutions. Numerical solution is obtained by numerical method PDEPE of Matlab9.0. An analytical and numerical solution shows nearly same pattern as figure-5 at particular time and other parameters. Figure 6. Represents the concentration distribution in surface graph between dimensionless solute concentration, time and space for numerical solution. It is shows that solute concentration is of reducing with increase in time and space. The Time-dependent behaviour of solute in subsurface is of interest for many practical problems where the concentration is observed or needs to be predicted at particular time. These solutions have practical application for many field problems. Some examples of transport phenomena are smoke coming out from a chimney of a factory, particulate particles coming out of a volcano, the sewage outlet of a municipal area or effluent outlet of a factory or industry in a surface water medium, infiltrations of wastes from garbage disposal sites, septic tanks, mines, discharge from surface water bodies polluted due to industrial and municipal influents and reaching the ground water level, particularly with rainwater.

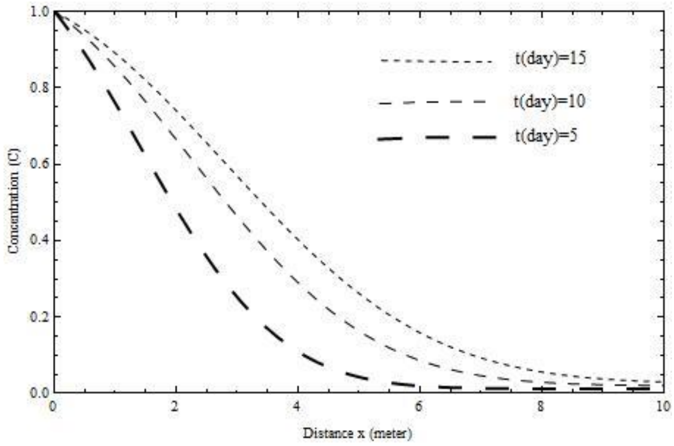

Figure 1. Comparison of dimensionless solute concentration distributions for obtained solution as Eq. (21) at various time.

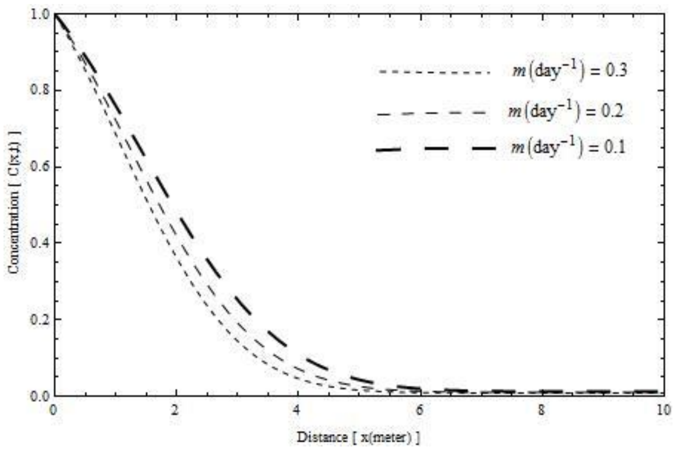

Figure 2. Comparison of dimensionless solute concentration distributions for obtained solution as Eq. (21) for various flow resistance at particular time.

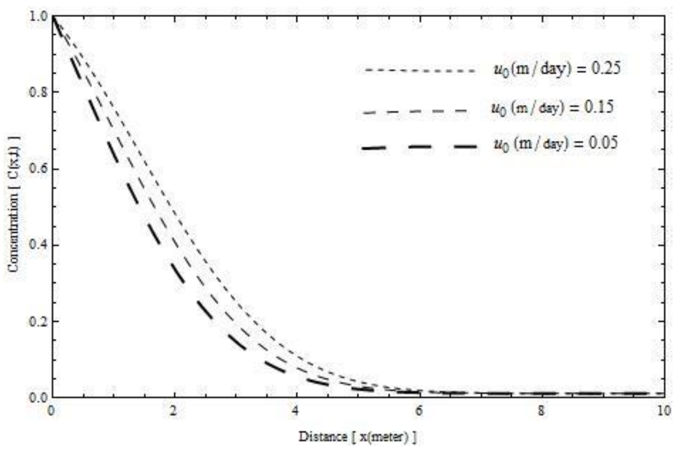

Figure 3. Comparison of dimensionless solute concentration distributions for obtained solution as Eq. (21) for various seepage velocity at particular time and dispersion parameter.

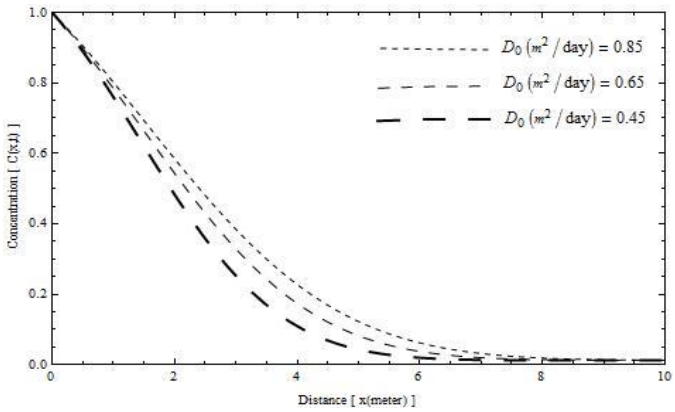

Figure 4. Comparison of dimensionless solute concentration distributions for Eq. (21) for various dispersion parameter at particular time and seepage velocity.

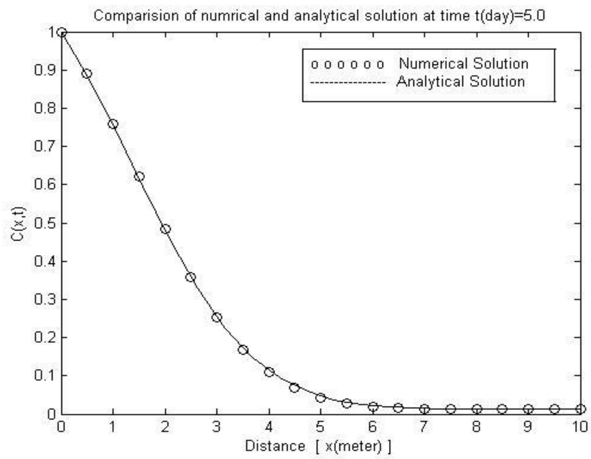

Figure 5. Comparison of dimensionless solute concentration distributions obtained analytically in Eq. (21) and numerical solution obtained by numerical method PDEPE of Matlab-9.0 at particular time t(day)= 5.0

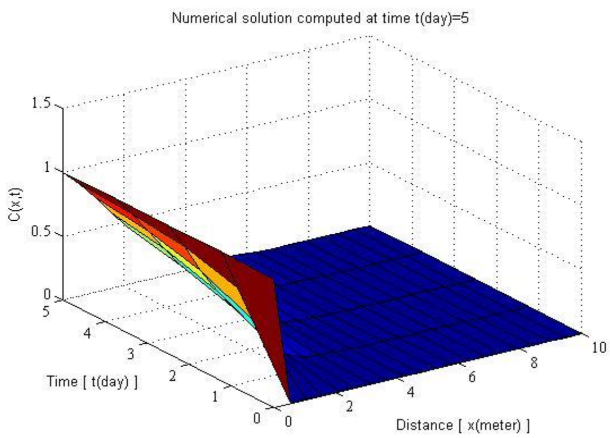

Figure 6. Surface graph drawn from numerical solution obtained by numerical method PDEPE of Matlab-9.0

3. CONCLUSIONS

An analytical solution of advection dispersion equation for non reactive constant and continuous point source is obtained in one dimensional semi-infinite homogenous porous medium using Laplace transform technique. Certain transformations are helped us to obtain the analytical solution. LTT is simpler, more viable and commonly used in assessing the stability of numerical solutions in more realistic dispersion problems, to understand and manage the pollution distributions along ground water, surface water and air flow domains. The analytical solution is obtained in the present work for uniform input conditions which may be useful in examining the degradation levels of the surface. Such solution will be very useful in validating a numerical solution of a more general dispersion problem by infinite element technique. In this work, advective dispersive term, transient velocity, first order decay and zero order production term are considered temporally dependent hyperbolically decreasing function. The analytical solutions of the advection-diffusion equation are useful to assess the time and position at which the concentration level of the pollutants will start affecting polluted water and eco-system. The solution in all possible combination for decreasing temporally dependent coefficients is compared with each other with the help of graphs.

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