Hamiltonian Approach for Electromagnetic Field in One-dimensional Photonic Crystal

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Abstract - *We present a novel method of determination of classical electromagnetic field distribution in a one-dimensional structured medium using microscopic approach for interaction between the field and the medium. Using the Hamiltonian constructed from the microscopic approach, the quantum electric field operator for the one-dimensional photonic crystal is obtained and the electric field distribution inside the photonic crystal is determined as the expectation value of this operator with coherent states taken as the field states.*

Key Words: Hamiltonian, Photonic Crystals, Metamaterials, Coherent States, Electric Field Operator, Structured Media.

1. INTRODUCTION

The structured media like the photonic crystals and the metamaterials are attracting lot of interest among the researchers in recent times [1,2,3,4,5,6]. These materials show the properties that are not readily available in the natural materials. Photonic band gap and negative refractive index are among the main properties shown by these materials. Because of their unusual properties, they show lot of promise in making novel optical and microwave devices. If the structure size at submicron level they may quantum nature and to study their properties fully quantum treatment of field and medium is necessary. In this short letter we present a novel method of studying the classical and quantum nature of electromagnetic field in structured media. Crenshaw [7,8] obtained the macroscopic Maxwell's equations in media starting from the microscopic quantum electrodynamic principles. In this approach both the field and the medium are considered as quantized harmonic oscillators and the interaction between the field and the medium is given by dipole interaction. Transition from microscopic regime to macroscopic regime is made by suitable approximations and as a consequence of this a macroscopic Hamiltonian in terms of averaged quantum field operators and material susceptibilities is obtained. We take this Hamiltonian as the starting point to construct the Hamiltonian for a structured medium and equations of motion for the averaged field operators are obtained from Heisenberg's equation of motion. Quantum electric field operator in the structured medium can be constructed from the field operators. The coherent states of electromagnetic fields represent quantum states which are very close in properties to the classical fields. Taking the coherent states of the fields, we obtain the expectation value of electric field operator. This represents the classical electric field distribution in the structured medium. In this letter, we determine the electric field distribution in one dimensional photonic crystal in the form of alternating layers of two different dielectrics. The same procedure can be straightforwardly extended to two and three dimensional structured media.

2. MACROSCOPIC HAMILTONIAN FROM MICROSCOPIC HAMILTONIAN

The Hamiltonian for the electromagnetic field in dielectric medium is formed from quantum electrodynamic principles. Here the electromagnetic field is considered as a collection of harmonic oscillators with creation and destruction field operators $\hat{a}^\dagger$ and $\hat{a}$ respectively. The medium is modeled as a field of discrete, quantized harmonic oscillators with the interaction between the field and medium given by electric and magnetic diploe interactions. The interaction of the electric field of the radiation with the medium is given by electric diploe interaction with the electric diploe operator $\hat{\mu}_e$. The creation and destruction operators for the medium oscillators are given as $\hat{b}^\dagger$ and $\hat{b}$ respectively. Similarly the interaction of magnetic field of radiation with medium is given by magnetic dipole moment operator $\hat{\mu}_m$ and corresponding creation and destruction operators for the medium oscillators are $\hat{c}^\dagger$ and $\hat{c}$ respectively. The Hamiltonian is [7]

$$
\begin{aligned}
H = & \sum_{l\lambda} \hbar\omega_l \hat{a}_l^\dagger \hat{a}_l + \sum_n \hbar\omega_b \hat{b}_n^\dagger \hat{b}_n + \sum_m \hbar\omega_m \hat{c}_m^\dagger \hat{c}_m \\
& - i\hbar \sum_{nl\lambda} (h_l \hat{a}_l \hat{b}_n^\dagger e^{i\mathbf{k}_l\cdot\mathbf{r}_n} - h_l^* \hat{a}_l^\dagger \hat{b}_n e^{-i\mathbf{k}_l\cdot\mathbf{r}_n}) \\
& - i\hbar \sum_{ml\lambda} (f_l \hat{a}_l \hat{c}_m^\dagger e^{i\mathbf{k}_l\cdot\mathbf{r}_m} - f_l^* \hat{a}_l^\dagger \hat{c}_m e^{-i\mathbf{k}_l\cdot\mathbf{r}_m})
\end{aligned} \quad \text{... (1)}
$$

where ω_l the frequency of radiation in the mode l and ω_b, ω_c are the resonance frequencies associated with the electric and magnetic dipoles respectively. The indices n and m enumerate the locations $\mathbf{r}_n$ and $\mathbf{r}_m$ of atoms or molecules of the medium and λ indicate the summation over polarization states of the radiation. The coupling between the fields and the medium are given by the terms

$$h_l = (2\pi\omega_l / \hbar V)^{1/2} \hat{\mu}_e \cdot \hat{\mathbf{e}}_{\mathbf{k}_l\lambda} \text{ and}$$

$$f_l = (2\pi\omega_l / \hbar V)^{1/2} \hat{\mu}_m \cdot (\hat{\mathbf{k}} \times \hat{\mathbf{e}}_{\mathbf{k}_l\lambda}) .$$

The Heisenberg's equations of motion give the evolution equations for field operators and the medium operators. Elimination of medium degree of freedom in equations of motion and adiabatic-following and continuum approximations were made to move from microscopic regime to macroscopic regime. The effective Hamiltonian corresponding to the macroscopic regime is given as [7]

$$H = \sum_{l\lambda} (1 - 2\pi\chi_l^e - 2\pi\chi_l^m)(\hbar\omega_l \bar{a}_l^\dagger \bar{a}_l + C) \quad (2)$$

where χ_l^e and χ_l^m denote the electric and the magnetic susceptibilities of the medium and the term C corresponds to zero-point energy which can be neglected when we are primarily interested in classical electromagnetic fields. Here $\bar{a}_l$ indicate the spatially averaged field operator of the mode l. This averaging process results in smoothing of field fluctuations that occur in-between the oscillators.

We can use the Hamiltonian in Eq.(2) for determining electromagnetic field distribution in structured media. The Hamiltonian for the structured media neglecting the zero-point energy is

$$H = [1 - 2\pi\chi_e(\mathbf{r}) - 2\pi\chi_m(\mathbf{r})]\bar{a}^\dagger \bar{a} \quad (3)$$

Here the electric and magnetic susceptibilities $\chi_e(\mathbf{r})$ and $\chi_m(\mathbf{r})$ respectively are functions of position vectors and the subscripts l and λ are dropped because we are interested in fundamental mode of a plane polarized wave. The terms $2\pi\chi_e \bar{a}^\dagger \bar{a}$ and $2\pi\chi_m \bar{a}^\dagger \bar{a}$ in Eq. (3) represent the part of the energy that is used up to polarize and also to maintain the polarization of the medium. The Heisenberg's equation of motion for the operator $\bar{a}$ with the above Hamiltonian results in the evolution equation [9,10]

$$\bar{a}(t) = \bar{a}(0) e^{-i[1-2\pi\chi_e(\mathbf{r})-2\pi\chi_m(\mathbf{r})]\omega t} \quad (4)$$

3. ELECTRIC FIELD OPERATOR AND ELECTRIC FIELD DISTRIBUTION

In quantum optics when interaction at microscopic scales are considered the dimensions of atoms are negligible compared to the wavelength of radiation resulting in the approximation [9,10] $e^{\pm \mathbf{k}\cdot\mathbf{r}} \approx 1$. With this approximation the electric field operator for the structured medium can constructed using Eq.(1.4) as

$$\hat{E}(\mathbf{r},t) = i\left(\frac{\hbar\omega}{2\varepsilon_0 V}\right)^{1/2} \left[\bar{a}(0) e^{-i[1-2\pi\chi_e(\mathbf{r})-2\pi\chi_m(\mathbf{r})]} - \bar{a}^\dagger(0) e^{i[1-2\pi\chi_e(\mathbf{r})-2\pi\chi_m(\mathbf{r})]}\right] \quad (5)$$

where V denotes the effective volume of the medium used for electromagnetic field quantization. The electric field distribution in structured media can be obtained as the expectation value of electric field operator with coherent states of radiation. Coherent states show properties very close to the classical fields. For quantized electromagnetic fields if $\hat{a}$ and $\hat{a}^\dagger$ are replaced by continuous variables results in classical fields. One way of doing this through the eigenstates of $\hat{a}$ which is given as $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$, where $|\alpha\rangle = \exp(-\tfrac{1}{2}|\alpha|^2)\sum_{n=0}^{\infty}\frac{\alpha^2}{\sqrt{n!}}|n\rangle$ are coherent states given in terms of number states $|n\rangle$ and $\alpha = |\alpha| e^{i\theta}$ a complex number [9,10]. The electric field distribution in structured media for the fundamental mode is

$$\langle \hat{E} \rangle = \langle \alpha | \hat{E} | \alpha \rangle = 2|\alpha| \left(\frac{\hbar\omega}{2\varepsilon_0 V}\right) \sin\{[1 - 2\pi\chi_e(\mathbf{r}) - 2\pi\chi_m(\mathbf{r})]\omega t - \theta\} \quad \dots (6)$$

The key feature of above expression is that arriving at this is relatively simpler than solving the wave equation for a structured media. Once electric filed is known the magnetic field configuration can be found by standard procedure.

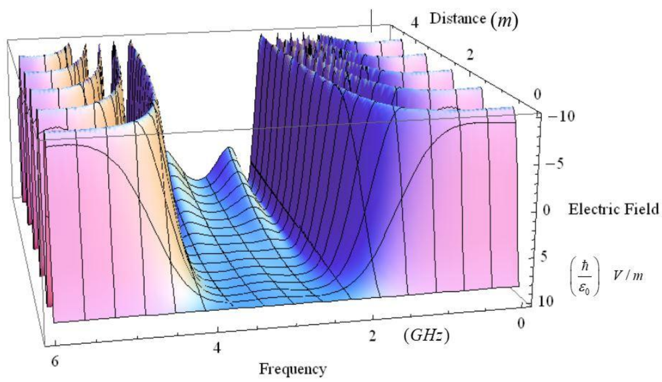

Fig -1: Electric Field Distribution in Periodic Dielectric

As an example a one-dimensional periodically structured dielectric medium like one-dimensional photonic crystal is considered. The lattice constant is $d = 1$ unit and the layers have relative dielectric constant alternating between $\varepsilon_{r1} = 1$ and $\varepsilon_{r2} = 13$. The electric field distribution along the length of the photonic crystal for varying frequencies is shown in the graph Fig.1. The range of frequencies for which the expectation value of the electric field operator is almost zero corresponds to the forbidden frequency band which is the photonic band gap. The Hamiltonian in Eq. (3) can be used to determine the electric field distribution inside the nanostructured dielectric medium where the structure scale is small enough for the quantum effects to manifest.

3. FIRST ORDER QUANTUM COHERENCE FUNCTION IN PERIODICALLY STRUCTURED DIELECTRIC MEDIUM

The Quantum Coherence Function gives the measure of much the coherence of the radiation is degraded in a medium. It is also a measure of how much of the quantum nature of the radiation is lost due to the fluctuation in field which occur due the discreetness of the matter at the microscopic level The first order quantum coherence function for the electric fields at two different places in the one-dimensional structured medium can be given as [9,11,12]

$$G^{(1)}(x_1,x_2)=\langle\alpha|\hat{E}^{(-)}(x_1)\cdot\hat{E}^{(+)}(x_2)|\alpha\rangle \qquad (7)$$

where $\hat{E}^{(+)}(\mathbf{r},t)=i\left(\frac{\hbar\omega_l}{2\varepsilon_0 V}\right)^{1/2}\bar{a}_l(0)e^{-i[(1-2\pi\chi_l(\mathbf{r}))\omega_l t-\mathbf{k}_p\cdot\mathbf{r}]}$ and $\hat{E}^{(-)}=\left[\hat{E}^{(+)}\right]^{\dagger}$. Here the function points $x_1=(z_1,t)$ and $x_2=(z_2,t)$ are two positions at which the electric fields are measured at the same time t. For a perfect periodic structure in which there is no random variation in the distance between the lattice points, the magnitude of coherence function is unity, $|G^{(1)}|=1$. For a one-dimensional periodic structure which is not perfect, there is some random variation in the thickness of each layer. The random variation in the thickness of the dielectric layers can be accounted by introducing a random step function, $\psi(z)$ in the susceptibility function $\chi(z)$ as

$$e^{i2\pi\chi_a(z)}=e^{i2\pi\chi(z)}e^{i\psi(z)} \qquad (8)$$

where $\chi_a(z)$ and $\chi(z)$ denote the susceptibility functions of the structure with a random variation in the width of a layer and of an ideal structure with perfect periodicity respectively. The random step function is defined as

$$\psi(z+z_0)-\psi(z)=random\ number\ between$$
$$0\ and\ p\ for\ z<p.$$
$$=0\ for\ z\geq p.$$
$$\ldots(9)$$

The magnitude of the first order coherence is given as

$$|G^{(1)}(z_1,z_2)|=G^{(1)}(z_1,z_2)\cdot G^{(1)}(z_1,z_2)^*$$
$$|G^{(1)}(z_1,z_2)|=e^{-\frac{2|z|}{p}} \qquad (10)$$

For perfect periodic structure the fluctuations in electric field is

$$\langle E^2(\mathbf{r},t)\rangle=\{4|\alpha|^2\sin^2[(1-2\pi\chi_l(\mathbf{r}))\omega_l t-\mathbf{k}_l\cdot\mathbf{r}-\theta]+1\} \qquad \ldots(11)$$

$$\Delta E=\left\langle\left(\Delta\hat{E}\right)^2\right\rangle^{1/2}=\left(\frac{\hbar\omega_l}{2\varepsilon_0 V}\right)^{1/2} \qquad (12)$$

which means that fluctuations are same as in free space.

Now considering a one-dimensional dielectric structure in which the lattice distance is normalized to unity, $p=1$ and a random variation in the thickness of the each layer up to the one layer thickness, the first order quantum coherence in Eq. (10) gives the measure of the quantum coherence degradation with distance of the one-dimensional structure as shown in the Fig. 2.

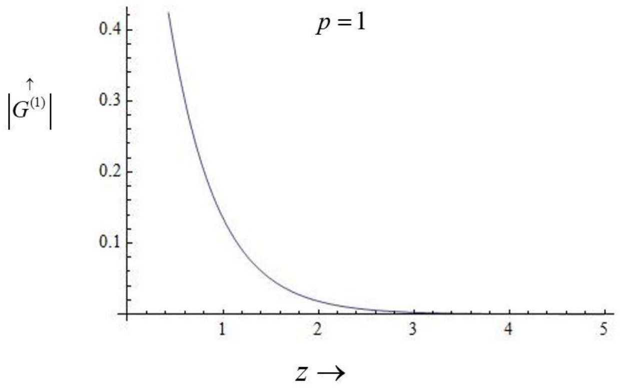

Fig -2: Magnitude of First Order Coherence Imperfect Periodic Structure

When the random variation in the thickness of the layers vary up to one layer thickness, then the quantum coherence property degrades exponentially and almost completely disappears at three layers thickness as seen in Fig. 2. This shows that the random variation in the periodic structure leads to fluctuations in the electric field which destroys the quantum nature of the field at a rate exponentially to the distance in the periodic structure.

3. CONCLUSIONS

Microscopic approach for determining the electric field distribution in structured medium is presented starting from the Hamiltonian for electromagnetic field in medium. The Hamiltonian is expressed in terms of macroscopic material parameters like susceptibility and spatially averaged quantum field operators. Evolution equations are determined from which electric field operator is constructed. Finally electric field in structured medium is found as the expectation value of the electric field operator in the structured medium. This formalism could be easily used to study the quantum nature of electromagnetic fields in structured media because it offers the convenience of using macroscopic material parameters of interest. The loss of the quantum nature of the electric field along the one-dimensional periodic structure is determined from the first order quantum coherence function. This shows that the quantum nature of the field decays exponentially with the distance in a structured medium.

REFERENCES

[1] Joannopoulos, J. The almost-magical world of photonic crystals. Braz. J. Phys. 26, 58–67, (1996)

[2] Pendry, J. Photonic band structures. J. Mod. Opt. 41, 209–229 (1994).

[3] Sihvola, A., "Metamaterials in electromagnetics," Metamaterials, Vol. 1, No. 1, 2 - 11, 2007.

[4] Shalaev, V. M., "Optical negative-index metamaterials", Nature Photonics.

[5] M.E. Crenshaw, "Quantum electrodynamic foundations of continuum electrodynamics," Phys.

[6] M.E. Crenshaw, "Microscopic foundations of macroscopic quantum optics," Phys. Rev. A 67

(2003) 033805.

[7] Christopher C. Gerry and Peter L. Knight, Introductory Quantum Optics, Cambridge

University Press, Cambridge, 2005.

[8] Rodney Loudon, The Quantum Theory of Light, 3rd ed., Oxford University Press, 2000.

[9] J.D. Joannopoulos, S.G. Johnson, J.N. Winn, and R.D. Meade, Photonic Crystals: Molding the Flow of Light, Second ed., Princeton University Press, (2008).

[10] J.D. Joannopoulos, Pierre R. Villeneuve and Shanhui Fan, "Photonic Crystals: Putting a New Twist on Light", Nature 386 (1997) 143-149.

[11] Mark Fox, "Quantum Optics: An Introduction", Oxford University Press, 2006.

[12] Marlan O. Scully and M. Suhail Zubairy, "Quantum Optics", Cambridge, University Press, 1997.