

On the Selection of Optimum Blend of WPO – Combinatorial Mathematics Based Approach

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Abstract - This paper presents the methodology of selection of optimum blend using combinatorial mathematics based approach from blends of waste plastic oil designated as WPO10, WPO20 and WPO30 based on the performance parameters like brake power, specific fuel consumption, mechanical efficiency, brake thermal and indicated thermal efficiency. It is found that WPO20 at 10kg load forms the optimum blend out of all the test fuels.

Key Words: Waste plastic oil, Combinatorial mathematics based approach, CMBA, Blend

1. INTRODUCTION

With the rapid depletion of fossil fuels at a faster rate, an alternative fuel is needed to fulfill the needs of mankind. Out of the various alternative fuels, Waste Plastic Oil (WPO) is

acclaimed to be a suitable alternative fuel as the properties of WPO are very close to diesel [1-3]. Researchers have put a lot of effort in production and characterization of WPO and its use as fuel on engines in neat and blend form [4-6].

The present study deals with the selection of optimum blend of WPO-Diesel. The statistical data for the present study if taken from the experimental results of Ankit et al. [7]. The blends of WPO are designated as WPO10 (10%WPO+90%Diesel), WPO20 (20%WPO+80%Diesel) and WPO30 (30%WPO+70%Diesel). The load on the engine is varied as 2kg, 4kg, 6kg, 8kg and 10kg. the results of experimental investigation are shown in Table1.

Table - 1: Experimental results

| Exp No. | Blend | Load (kg) | BP (kW) | SFC Kg/kWh) | Mech Efficiency (%) | BTE (%) | ITE (%) |
|---------|--------|-----------|---------|-------------|---------------------|---------|---------|
| 1 | WPO 10 | 2 | 0.43 | 0.71 | 17.21 | 12 | 69.72 |
| 2 | WPO 10 | 4 | 0.87 | 0.43 | 29.36 | 20.13 | 68.55 |
| 3 | WPO 10 | 6 | 1.3 | 0.34 | 38.4 | 24.99 | 65.08 |
| 4 | WPO 10 | 8 | 1.74 | 0.27 | 45.39 | 32.07 | 70.66 |
| 5 | WPO 10 | 10 | 2.17 | 0.24 | 50.96 | 35.03 | 68.75 |
| 6 | WPO 20 | 2 | 0.43 | 0.71 | 18.77 | 12.16 | 64.78 |
| 7 | WPO 20 | 4 | 0.87 | 0.41 | 31.6 | 20.94 | 66.25 |
| 8 | WPO 20 | 6 | 1.3 | 0.33 | 40.94 | 26.29 | 64.23 |
| 9 | WPO 20 | 8 | 1.74 | 0.27 | 48.03 | 31.74 | 66.08 |
| 10 | WPO 20 | 10 | 2.17 | 0.24 | 53.6 | 36.41 | 67.93 |
| 11 | WPO 30 | 2 | 0.43 | 0.71 | 16.74 | 12.12 | 72.42 |
| 12 | WPO 30 | 4 | 0.87 | 0.42 | 26.68 | 20.21 | 70.46 |
| 13 | WPO 30 | 6 | 1.3 | 0.32 | 37.63 | 27.06 | 71.93 |
| 14 | WPO 30 | 8 | 1.74 | 0.27 | 44.58 | 31.57 | 70.83 |
| 15 | WPO 30 | 10 | 2.17 | 0.24 | 50.13 | 36.08 | 71.98 |

2. COMBINATORIAL MATHEMATICS BASED APPROACH

Combinatorial mathematics based approach (CMBA) is an integration of combinatorial mathematics matrix function and analytic hierarchy process and a [8]. The step wise procedure of CMBA is shown below:

Step 1: Decision matrix

Decision matrix is a collection of data for each experiment and is same as shown in Table1.

Step 2: Normalization

The size of the matrix is equal to the number of attributes considered. Normalization is to set the attribute data on same scale so that, comparisons can be made easier [9]. Let x_{ij} is the normalized value of y_{ij} for attribute i , then,

$$x_{ij} = \frac{y_{ij}}{\max_j (y_{ij})} ; \text{ if } j\text{th attribute is beneficial}$$

$$x_{ij} = \frac{\min_j (y_{ij})}{y_{ij}} ; \text{ if } j\text{th attribute is non-beneficial}$$

The normalized values of attributes are shown in Table 2.

Table – 2: Normalized values of attributes

| Exp No. | Blend | Load (kg) | BP (kW) | SFC Kg/kWh) | Mech Efficiency (%) | BTE (%) | ITE (%) |
|---------|--------|-----------|---------|-------------|---------------------|---------|---------|
| 1 | WPO 10 | 2 | 0.198 | 1.000 | 0.321 | 0.330 | 0.963 |
| 2 | WPO 10 | 4 | 0.401 | 0.606 | 0.548 | 0.553 | 0.947 |
| 3 | WPO 10 | 6 | 0.599 | 0.479 | 0.716 | 0.686 | 0.899 |
| 4 | WPO 10 | 8 | 0.802 | 0.380 | 0.847 | 0.881 | 0.976 |
| 5 | WPO 10 | 10 | 1.000 | 0.338 | 0.951 | 0.962 | 0.949 |
| 6 | WPO 20 | 2 | 0.198 | 1.000 | 0.350 | 0.334 | 0.895 |
| 7 | WPO 20 | 4 | 0.401 | 0.577 | 0.590 | 0.575 | 0.915 |
| 8 | WPO 20 | 6 | 0.599 | 0.465 | 0.764 | 0.722 | 0.887 |
| 9 | WPO 20 | 8 | 0.802 | 0.380 | 0.896 | 0.872 | 0.912 |
| 10 | WPO 20 | 10 | 1.000 | 0.338 | 1.000 | 1.000 | 0.938 |
| 11 | WPO 30 | 2 | 0.198 | 1.000 | 0.312 | 0.333 | 1.000 |
| 12 | WPO 30 | 4 | 0.401 | 0.592 | 0.498 | 0.555 | 0.973 |
| 13 | WPO 30 | 6 | 0.599 | 0.451 | 0.702 | 0.743 | 0.993 |
| 14 | WPO 30 | 8 | 0.802 | 0.380 | 0.832 | 0.867 | 0.978 |
| 15 | WPO 30 | 10 | 1.000 | 0.338 | 0.935 | 0.991 | 0.994 |

Step 3: Relative Importance

After analyzing the attributes, the relative importance of attributes is assigned. Table 3 shows the scale for pairwise comparison [10].

The geometric mean approach of AHP is used to determine the relative normalized weights of the attributes and the consistency check is carried out. It is required that the consistency ratio value of the relative importance of attributes should be less than 0.10 [11]. The consistency ratio in the present study is found to be 0.068. The consistency evaluation is shown in Table 4.

Step 4: Formation of alternate selection attribute matrix

The alternative selection matrix is formed by keeping the normalized values for attributes for the alternative as diagonal elements. The matrix is represented by, B.

$$B = \begin{bmatrix} R_1 & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & R_2 & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & R_3 & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & R_4 & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & R_5 \end{bmatrix} \quad (1)$$

Step 5: Permanent function

The permanent function used in Combinatorial mathematics characterizes the configuration of a system [12]. The characteristic permanent function for the standard matrix is shown in Eq. 2.

$$\begin{aligned}
 Per(B) = & \prod_{i=1}^M D_i + \sum_{i=1}^{M-1} \sum_{j=i+1}^M \dots \sum_{M=i+1}^M (d_{ij}d_{ji})D_iD_jD_nD_o\dots D_iD_m \\
 & + \sum_{i=1}^{M-2} \sum_{j=i+1}^{M-1} \sum_{k=j+1}^M \dots \sum_{M=i+1}^M (d_{ij}d_{jk}d_{ki} + d_{ik}d_{kj}d_{ji})D_iD_jD_kD_o\dots D_iD_m \\
 & + (\sum_{i=1}^{M-3} \sum_{j=i+1}^M \sum_{k=i+1}^{M-1} \sum_{l=j+1}^M \dots \sum_{M=i+1}^M (d_{ij}d_{jk}d_{kl}d_{li} + d_{il}d_{lk}d_{kj}d_{ji})D_iD_jD_kD_lD_o\dots D_iD_m) + \\
 & (\sum_{i=1}^{M-3} \sum_{j=i+1}^M \sum_{k=i+1}^M \sum_{l=j+1}^M \dots \sum_{M=i+1}^M (d_{ij}d_{jk}d_{kl}d_{li} + d_{il}d_{lk}d_{kj}d_{ji})D_iD_jD_kD_lD_o\dots D_iD_m) \\
 & + (\sum_{i=1}^{M-4} \sum_{j=i+1}^M \sum_{k=i+1}^M \sum_{l=i+1}^M \sum_{m=j+1}^M \dots \sum_{M=i+1}^M (d_{ij}d_{jk}d_{kl}d_{lm}d_{mi} + d_{im}d_{ml}d_{lk}d_{kj}d_{ji})D_iD_jD_kD_lD_mD_o\dots D_iD_m) \\
 & + (\sum_{i=1}^{M-4} \sum_{j=i+1}^M \sum_{k=i+1}^M \sum_{l=j+1}^M \sum_{m=i+1}^M \dots \sum_{M=i+1}^M (d_{ij}d_{jk}d_{kl}d_{lm}d_{mi} + d_{im}d_{ml}d_{lk}d_{kj}d_{ji})D_iD_jD_kD_lD_mD_o\dots D_iD_m) \\
 & + (\sum_{i=1}^{M-5} \sum_{j=i+1}^M \sum_{k=i+1}^M \sum_{l=i+1}^M \sum_{m=i+1}^M \sum_{n=m+1}^M \dots \sum_{M=i+1}^M (d_{ij}d_{jk}d_{kl}d_{lm}d_{mn}d_{ni} + d_{in}d_{nm}d_{ml}d_{lk}d_{kj}d_{ji})D_iD_jD_kD_lD_mD_nD_o\dots D_iD_m) \\
 & + (\sum_{i=1}^{M-5} \sum_{j=i+1}^M \sum_{k=i+1}^M \sum_{l=i+1}^M \sum_{m=i+1}^M \sum_{n=k+2}^M \dots \sum_{M=i+1}^M (d_{ij}d_{jk}d_{kl}d_{lm}d_{mn}d_{ni} + d_{in}d_{nm}d_{ml}d_{lk}d_{kj}d_{ji})D_iD_jD_kD_lD_mD_nD_o\dots D_iD_m) \\
 & + (\sum_{i=1}^{M-5} \sum_{j=i+1}^M \sum_{k=i+1}^M \sum_{l=i+1}^M \sum_{m=i+1}^M \sum_{n=j+1}^M \dots \sum_{M=i+1}^M (d_{ij}d_{jk}d_{kl}d_{lm}d_{mn}d_{ni} + d_{in}d_{nm}d_{ml}d_{lk}d_{kj}d_{ji})D_iD_jD_kD_lD_mD_nD_o\dots D_iD_m) \\
 & + \dots
 \end{aligned}$$

(2)

Table - 3: Scale for pairwise comparison

| Degree of importance | Definition |
|--|--|
| 1 | Equal |
| 2 | Intermediate between 1 and 3 |
| 3 | Moderately preferable |
| 4 | Intermediate between 3 and 5 |
| 5 | Strongly preferable |
| 6 | Intermediate between 5 and 7 |
| 7 | Very strongly preferable |
| 8 | Intermediate between 7 and 9 |
| 9 | Extremely strongly preferable |
| 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9 | Reciprocals of 2, 3, 4, 5, 6, 7, 8 and 9 |

Table - 4: Consistency evaluation

| | BP (kW) | SFC (Kg/kWh) | Mech Efficiency (%) | BTE (%) | ITE (%) | Total | Average | Consistency Measure |
|---------------------|---------|--------------|---------------------|---------|---------|-------|---------|---------------------|
| BP (kW) | 0.353 | 0.444 | 0.343 | 0.235 | 0.222 | 1.598 | 0.320 | 5.259 |
| SFC (Kg/kWh) | 0.118 | 0.222 | 0.343 | 0.235 | 0.222 | 1.140 | 0.228 | 5.435 |
| Mech Efficiency (%) | 0.176 | 0.111 | 0.171 | 0.353 | 0.222 | 1.034 | 0.207 | 5.360 |
| BTE (%) | 0.176 | 0.111 | 0.057 | 0.118 | 0.222 | 0.685 | 0.137 | 5.090 |
| ITE (%) | 0.176 | 0.111 | 0.086 | 0.059 | 0.111 | 0.543 | 0.109 | 5.102 |
| | | | | | | | CI | 0.062 |
| | | | | | | | RI | 0.91 |
| | | | | | | | CR | 0.068 |

Step 6: Rank of alternatives

The rank of alternatives is based on the permanent function value of the alternative selection matrix, also called as Index score. The alternative for which the value of Index score is highest is the best choice for the considered decision making problem. The Index score for all alternatives is sorted and ranked as shown in Table 5.

Table – 5: Index score for alternatives

| Exp No. | Blend | Load (kg) | Index score | Rank |
|---------|--------|-----------|-------------|------|
| 10 | WPO 20 | 10 | 141.4409 | 1 |
| 15 | WPO 30 | 10 | 141.2131 | 2 |
| 5 | WPO 10 | 10 | 139.798 | 3 |
| 4 | WPO 10 | 8 | 132.5319 | 4 |
| 14 | WPO 30 | 8 | 131.9812 | 5 |
| 9 | WPO 20 | 8 | 131.8858 | 6 |
| 13 | WPO 30 | 6 | 123.9184 | 7 |
| 8 | WPO 20 | 6 | 122.7776 | 8 |
| 3 | WPO 10 | 6 | 121.7662 | 9 |
| 2 | WPO 10 | 4 | 115.2242 | 10 |
| 7 | WPO 20 | 4 | 115.138 | 11 |
| 12 | WPO 30 | 4 | 114.5299 | 12 |
| 11 | WPO 30 | 2 | 111.311 | 13 |
| 1 | WPO 10 | 2 | 110.7055 | 14 |
| 6 | WPO 20 | 2 | 110.0223 | 15 |

3. CONCLUSION

The proposed CMBA is adapted to select optimum blend of waste plastic oil. The computation used is comparatively simple compared to other multi attribute decision making methods. The measures of the attributes and their relative importance are used together to rank the alternatives. Hence, it provides a better evaluation. The use of permanent concept characterizes the considered approach as it contains all possible structural components of the attributes and their relative importance hence, no information is lost. This method can deal with problems considering both qualitative and quantitative attributes. The uniqueness of CMBA is that it offers a general procedure that can be applicable to diverse selection problems that incorporates vagueness and a number of selection attributes. The approach is logical, simple and convenient to implement.

NOMENCLATURE

- BP: Brake power
- SFC: Specific fuel consumption
- BTE: Brake thermal efficiency
- ITE: Indicated thermal efficiency

CMBA: Combinatorial mathematics based approach

WPO: Waste plastic oil

WPO10: 10%WPO+90%Diesel

WPO20: 20%WPO+80%Diesel

WPO30: 30%WPO+70%Diesel

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