Echo Function

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Abstract - *This Paper develop a function called Echo Function (Echo Function is called as Echo Function because of its periodic nature of output) , Which work similar to a Mathematical function . If x is a real number and Echo Function is applied on x than output of Echo Function will be some real number say y , now if again Echo function is applied on y output we get output as x and if we repeat this process again and again we get only two value x and y from Echo function for a real number x which is echoing periodically .*

Key Words: Hyperbolic function .

1. INTRODUCTION

In Mathematics we have many function along with its inverse function . I have come across a mathematical equation which is derived from a basic mathematical equation , Which act like a function and its inverse function too , named as Echo function. Result produced by Echo function is periodically oscillating between two value for a real number when applied again and again as shown in this paper.

1.1 BASIC DEFINITION

The purpose of this section is to recall some results on the proposed method. From basics of hyperbolic function we know that

$$\tanh(x) = (e^{2x} - 1)/(e^{2x} + 1) \qquad \_\_[1]$$

where x is a real number and Domain ($-\infty,\infty$) , Range (-1,1)

$$\tanh^{-1}(z) = (1/2)\ln[(1+z)/(1-z)] \qquad \_\_[2]$$

where z is a real number , The domain is the open interval (−1, 1).

2. ECHO FUNCTION

Let us consider a real number x such that $-\infty < x < \infty$. So hyperbolic tangent of x will be

$$\tanh(x) = (e^{2x} - 1)/(e^{2x} + 1)$$

As per eq^n [1]. Modifying above equation we get ,

$$-\tanh(x) = (1 - e^{2x})/(1 + e^{2x}) \qquad \_\_(A)$$

Using eq^n [2] we have ,

$$\tanh^{-1}(z) = (1/2)\ln[(1+z)/(1-z)]$$

where z is a real number with domain open interval (−1, 1). Put $z = e^{2x}$ in above equation , we get

$$\tanh^{-1}(e^{2x}) = (1/2)\ln[(1+e^{2x})/(1-e^{2x})]$$

Since we have replace $z = e^{2x}$ in above equation due to which it working range get modified as [x < 0] . Therefore we have

$$\tanh^{-1}(e^{2x}) = (1/2)\ln[(1+e^{2x})/(1-e^{2x})] \qquad \_\_[\text{ for } 0 > x]$$

Modify above equation we get ,

$$\tanh^{-1}(e^{2x}) = -(1/2)\ln[(1-e^{2x})/(1+e^{2x})] \qquad \_\_(B)$$

Solving eq^n (A) and (B), we get

$$\tanh^{-1}(e^{2x}) = -(1/2)\ln(-\tanh(x))$$

$$e^{2x} = \tanh[-(1/2)\ln(-\tanh(x))]$$

$$x = (1/2)\ln\{\tanh[-(1/2)\ln(-\tanh(x))]\}$$

We know that $\tanh(-x) = -\tanh(x)$, modified above eq^n will be ,

$$x = (1/2)\ln\{-\tanh[(1/2)\ln(-\tanh(x))]\} \qquad \_\_(C)$$

Which is valid if x is a real number and $0 > x$. looking carefully at eq^n(c) we can see that on right hand side , x is bounded with a function { $(1/2)\ln(-\tanh(x))$ } which is again bounded with the same function so as to obtain x on left hand side . It looks like value x is getting echo backed to x through a function applied twice . To make it understandable let us define that new function as Echo Function (ε), which is defined as

$$\varepsilon(x) = (1/2)\ln(-\tanh(x))$$

where x is a real number and $0 > x$

So eqn (C) becomes ,

$$x = \varepsilon(\varepsilon(x))$$

In above equation take Echo Function on both side ,we get

$$\varepsilon(x) = \varepsilon(\ \varepsilon(\ \varepsilon(x)\ )\ )$$

Again take Echo Function on both side of above eqn , we get

$$x = \varepsilon(\ \varepsilon(x)\ ) = \varepsilon(\ \varepsilon(\ \varepsilon(\ \varepsilon(x)\ )\ )\ )$$

So if repeat this process we get x for even repetition and ε(x) for odd repetition . Also on odd repetition it act as a function and on even repetition it act its own inverse.

2.1 GRAPH

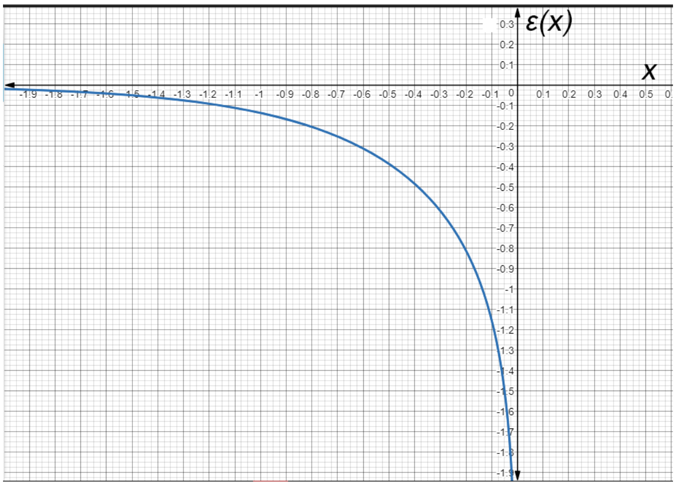

Fig (2.1.1) Graph of ε(x)

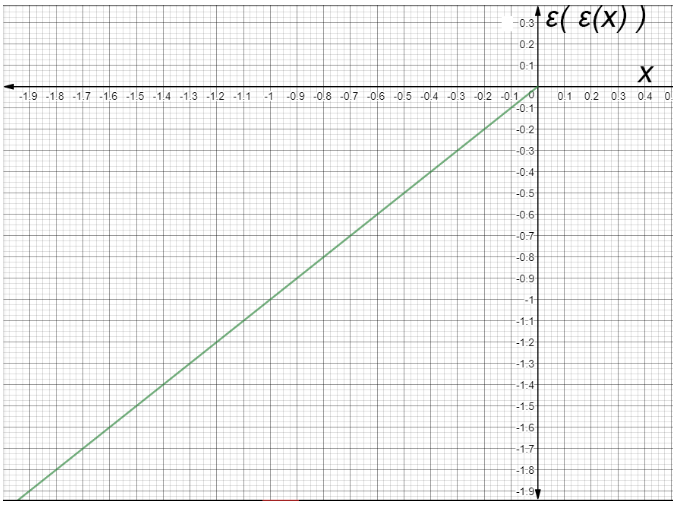

Fig (2.1.2) Graph of ε(ε(x))

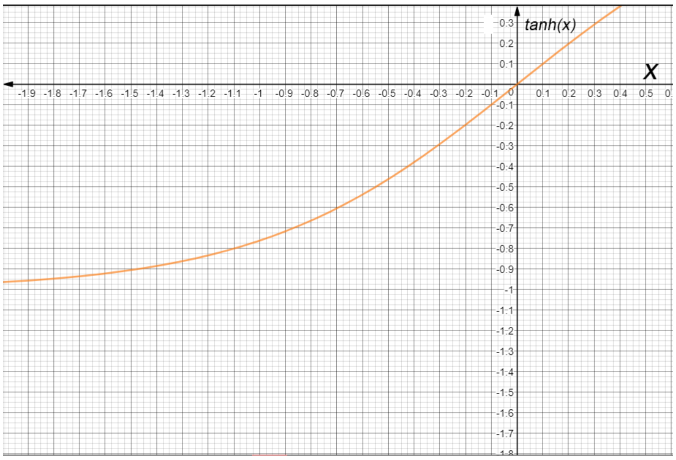

Fig (2.1.3) Graph of tanh(x)

3. CONCLUSIONS

This paper shows that Echo Function produce two different value one when applied even times on given value and another when applied odd times on given value. Also it act as a function and its own inverse function too.

REFERENCES

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