

Review on Viscoelastic Materials used in Viscoelastic Dampers

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Abstract - Visco-elastic dampers utilize high damping from viscoelastic materials to dissipate energy through shear deformation. Viscoelastic materials are highly influenced by parameters like temperature, frequency, dynamic strain rate, time effects such as creep and relaxation, aging, and other irreversible effects. Hence selecting a proper viscoelastic material is the key. This paper presents an overview of literature related to the viscoelastic materials used in visco elastic dampers. The review includes different materials like asphalt, rubber, polymer and glassy substances. There have been few investigations on these materials, its advantages and disadvantages are discussed and detailed review is carried out.

Key Words: Viscoelastic damper, viscoelastic materials, energy dissipation,

1.INTRODUCTION

The repetitive motion like earthquake waves causes fatigue and reduction of the performance of the structure. The energy released can cause high amount of damage to all components of the structure. Thus it is the need to reduce vibrations or maintaining the performance of the structure for life safety and economic loss.

The current trend toward buildings of ever increasing heights and the use of lightweight, high strength materials, and advanced construction techniques have led to increasingly flexible and lightly damped structures. Understandably, these structures are very sensitive to environmental excitations such as wind, ocean waves and earthquakes. This causes unwanted vibrations inducing possible structural failure, occupant discomfort, and malfunction of equipment. Hence it has become important to search for practical and effective devices for suppression of these vibrations. This has opened up a new area of research in the last decade, aptly titled structural control. [1]

These dampers act like the hydraulic shock absorbers in cars. Much of the sudden jerks are absorbed in the hydraulic fluids and only little is transmitted above to the chassis of the car. When seismic energy is transmitted through them, dampers absorb part of it, and thus damp the motion of the building. Dampers were used since 1960s to protect tall buildings against wind effects. However, it was only since 1990s, that they were used to protect buildings against earthquake effects. [2] These seismic control devices for civil structures can be divided into four classes: passive, active,

semi active and hybrid. Passive devices are those that have fixed properties and require no energy to function. In contrast, the controllable forces generated by active devices are induced directly by energy (electrical or otherwise) put into the device. Between passive and active are semi active devices that are passive devices with properties that are controllable by application of a small amount of energy. Hybrid devices are combinations of the other three classes. Commonly used types of seismic dampers include viscous dampers (energy is absorbed by silicone-based fluid passing between piston-cylinder arrangement), friction dampers (energy is absorbed by surfaces with friction between them rubbing against each other), and yielding dampers (energy is absorbed by metallic components that yield). [3]

Of all the available seismic response control devices, Viscoelastic dampers are considered to have an upper hand over others because of its effectiveness against not only seismic forces but also wind forces. When wind blows against the building, vortices are created. When vortex shredding frequency reaches near to the natural frequency, resonance occurs. When this vortex shredding takes place along the large side of the building height, it results in large forces and amplitudes. Fluctuations in the upstream wind cause the building to vibrate, resulting in accelerations. The accelerations can negatively influence the inhabitants of tall structures. Visco Elastic dampers are observed to reduce these accelerations.

1.1 Viscoelastic Damper

Viscoelastic damper is a passive energy dissipation device. This damper converts a portion of mechanical energy into heat energy. The medium in which the transfer of energy takes place is viscoelastic material. Viscoelastic dampers are known to get activated at lower displacements, hence optimum against governing wind forces too. Visco elastic dampers are known for providing restoring forces. As visco elastic damper follows a linear behavior, it makes the modeling simpler. Viscoelastic damper does not require any external source for providing energy. There are basically three ways of employing viscoelastic material as a damper medium.

1. Single Layer Viscoelastic damper: Here the damping is achieved by extensional deformation of viscoelastic layer.
2. Double Layer Viscoelastic damper: Here the viscoelastic material is placed between two constraints

layers. Damping achieved is mostly due to shear deformation rather than extensional deformation.

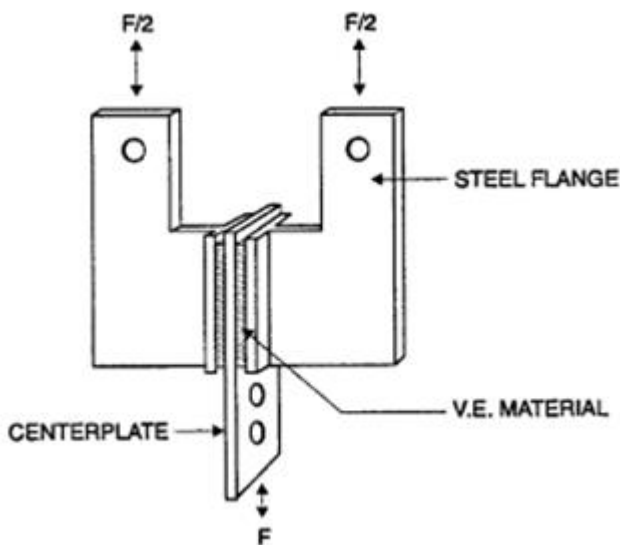


Fig-1 Sketch of Viscoelastic damper

3. Sandwiched viscoelastic damper: Here all the observed deformation is due to shear. This type of damper is more efficient when large amount of energies are to be damped.

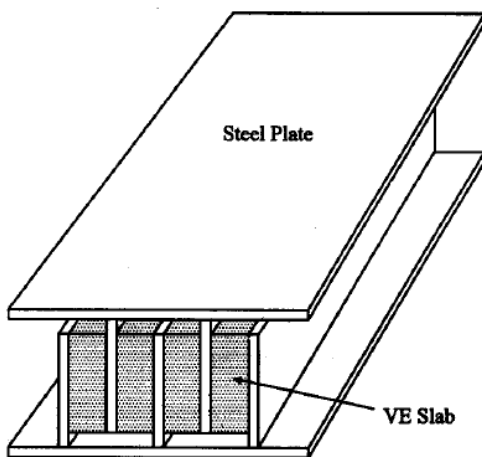


Fig-2 Sketch of Sandwiched viscoelastic damper [20]

1.2 Viscoelastic Materials

A viscoelastic material is characterized by possessing both viscous and elastic behavior. A purely elastic material is one in which all the energy stored in the sample during loading is returned when the load is removed. As a result, the stress and strain curves for elastic materials move completely in phase. For elastic materials, Hooke's Law applies where the stress is proportional to the strain, and the modulus is defined at the ratio of stress to strain. A complete opposite to

an elastic material is a purely viscous material, also shown in figure. This type of material does not return any of the energy stored during loading. All the energy is lost as "pure damping" once the load is removed. In this case, the stress is proportional to the rate of the strain, and the ratio of stress to strain rate is known as viscosity, μ . These materials have no stiffness component, only damping. While in case of viscoelastic materials, Some of the energy stored in a viscoelastic system is recovered upon removal of the load and the remainder is dissipated in the form of heat. Most important of these include temperature and frequency effects. These materials exist in various unique states or "phases" over the broad temperature and frequency ranges in which they are used. These regions are typically referred to as the Glassy, Transition, Rubbery, and Flow Regions. [4]

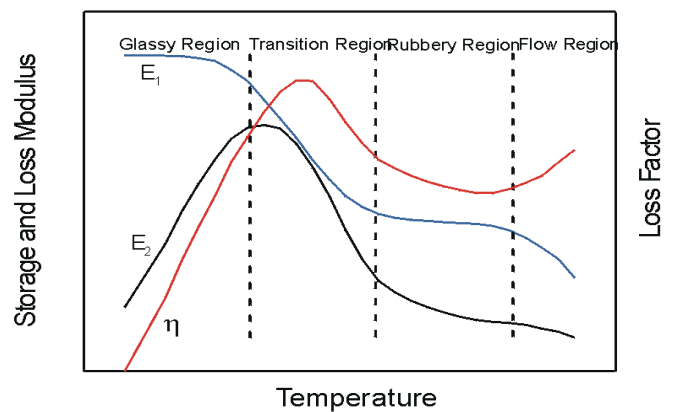


Fig 3: Phases of Viscoelastic material. [4]

2. LITERATURE REVIEW

The literature on viscoelastic dampers is reviewed briefly in the following paragraphs while giving special attention to ViscoElastic materials used in these viscoelastic dampers.

2.1 Researches done regarding Viscoelastic dampers

Tsai (1993), In this paper, the features of energy-absorbing capacities of the viscoelastic damper and its effect on the structure during earthquakes are investigated. To clarify the behavior of the structure with added viscoelastic dampers, a new analytical model for the viscoelastic damper are been developed considering the earthquake like loading and the temperature effect, in good agreement with experimental results, and an advanced finite element formulation are developed. The proposed method could be implemented easily in the finite element program. In this study the behavior of a 10-story building equipped with viscoelastic dampers is examined while it is subjected to earthquake ground motions. Both analytical and experimental results show that the energy-absorbing capacity of the viscoelastic damper decreases with increasing the ambient temperature.

Numerical examples show that the responses, including displacements and stresses, of the high-rise building to earthquake loadings are significantly reduced while viscoelastic dampers are added to it.

K. L. Shen. (1995) This paper presents an analytical model for predicting the hysteretic behavior of viscoelastic dampers based on the Boltzmann's superposition principle and the method of reduced variables. Test procedure and determination of model parameters are also discussed. The test in the time domain conducted for the present study is the stress relaxation test, in which a step strain is imposed on the damper and then kept constant, and the output of stress versus time is recorded. The test in the frequency domain is conducted using a sinusoidal strain as input and measuring the stress output. Both tests were conducted on MTS hydraulic actuators. Finally, the presented model is compared with the ones based on the concept of fractional derivatives. A significant simplification is presented for the frequency and temperature dependent properties of the storage and loss moduli, which is very useful for the damper design and structural applications. With this model, more precise and rational analysis of VE-damped structures subjected to dynamic loads can be developed and carried out.

Michael Montgomery (2014) In this paper new damping systems - the Viscoelastic Coupling Damper (VCD) and Viscoelastic-Plastic Coupling Damper have been studied to improve the performance of tall, reinforced concrete (RC) buildings subject to both wind and earthquake loads. VCDs are introduced in lieu of RC coupling beams to take advantage of differential shear deformations between adjacent walls during lateral loading of the structure. The VCDs utilize multiple viscoelastic material layers that are bonded to alternating steel plates with each consecutive steel layer extending out to the opposite side and anchored into the walls using a number of alternate connection details. When the building is subject to frequent or design level wind storms or low level earthquakes, the damper exhibits both a displacement-dependent elastic restoring force providing coupling to the walls and a velocity-dependent viscous force, providing supplemental damping to the building. In regions of severe seismicity, a ductile "fuse" element can also be included in the damper to enhance its performance. The "fuse" is capacity designed such that if predefined load levels are reached in the damper during extreme seismic loading, connection elements act as force-limiting members and prevent damage from occurring in adjacent structural elements. The response during severe earthquakes is viscoelastic at small amplitudes and becomes plastic once the connections start yielding, resulting in what is termed a Viscoelastic-Plastic hysteresis. Replaceable connections are utilized to allow for repair or replacement after an earthquake. This paper describes the Viscoelastic and Viscoelastic-Plastic response of this new damping system

and provides some examples of the design concept and applications of this technology.

Michael Montgomery (2015) Here a new damping system, the viscoelastic coupling damper (VCD), has been developed and studied to enhance the wind and seismic performance of coupled shear wall high-rise buildings by adding high damping elements in place of reinforced concrete coupling beams. VCDs replace structural members, such as outriggers or coupling beams, and therefore do not occupy any usable architectural space. When they are properly configured in high-rise buildings, they provide supplemental viscous damping to all lateral modes of vibration, which mitigates building tenant vibration perception problems and reduces both the wind and earthquake response. Experimental results from tests on five small-scale viscoelastic (VE) damper specimens of 5- and 10-mm thicknesses are first presented, followed by the results from six full-scale VCDs representing two alternative configurations. The first was designed for areas where moderate seismic ductility is required, and the second was designed with built-in ductile structural fuses for areas where high seismic ductility is required. The VE material tests exhibited stable hysteretic behavior under the loading conditions that are expected in high-rise buildings under wind and earthquake loading. The full-scale tests validated the overall system performance within a realistic coupled wall configuration, and confirmed the performance of the wall anchorages and all connecting elements as well as the VE material behavior. The full-scale test results also demonstrated the targeted viscoelastic response during wind and low level earthquake loading and the targeted viscoelastic-plastic response for extreme earthquakes, where the response is a combination of the VE response and the nonlinear behavior of the structural fuses.

Rajneesh Kakar (2016) An analytical and numerical approach was used to study the propagation of Love waves in an inhomogeneous viscoelastic layer overlying a gravitational half-space when the upper boundary plane was assumed to be free. The dispersion relationship for the Love wave in closed form was obtained with Whitaker's function. The effect of various nondimensional inhomogeneity factors, the gravity factor, and internal friction on the nondimensional Love wave velocity was shown graphically in this study. The researcher observed that the dispersion curve of the Love wave increases as the inhomogeneity factor increases. An increment in gravity, inhomogeneity, and internal friction decreases the damping phase velocity of Love waves, but it is more prominent with internal friction. The surface plot of Love waves revealed that the velocity ratio increases with the increase of a nondimensional phase velocity and a nondimensional wave number, and these results may interest seismologists and geologists.

Yutaka Nakamura (2016), In this study, a viscoelastic damper is developed by using a viscoelastic material with low temperature dependence, and a performance-based

placement-design procedure of viscoelastic dampers is developed for finding the story-wise distribution of viscoelastic dampers in a building such that each peak interstory drift coincides with the prescribed value. The necessary number of viscoelastic dampers at each story is obtained by dividing the obtained area of the viscoelastic material by the area of the VE material of the employed single wall-type VED and rounding off the result to a whole number.

2.2 Researches done regarding Viscoelastic materials

Lee and Kim (1998) studied the nonlinear viscoelastic behavior of asphalt mixture with the nonlinear elastic-viscoelastic correspondence principle, and introduced an initial pseudo secant stiffness to reflect the scatter of test specimens. The pseudo stiffness was assumed to be between 0.9 and 1.1 without statistical analysis, so it cannot accurately describe the probability distribution of the random material characteristics. Damage accumulation under uniaxial tensile cyclic loading without rest periods was modeled using the elastic-viscoelastic correspondence principle and the time-dependent damage parameter. The pseudo variables greatly simplified the task of separately evaluating viscoelastic behavior and damage growth of asphalt concrete. The damage parameter successfully eliminated stress-strain-level dependence of asphalt concrete on fatigue behavior. The constitutive equation developed from the controlled-strain fatigue tests satisfactorily predicted the stress strain behavior of asphalt concrete all the way up to failure under the controlled-stress mode as well as under the monotonic loading with varying strain rates.

Kim (1999) Test method and data analysis procedures are presented in order to develop a constitutive model which can describe the behavior of visco-elasto-plastic materials due to the external load. A series of creep tests were performed on asphalt concrete. It is shown that the elastic, plastic, viscoelastic and viscoplastic strain components are incorporated and present simultaneously in the total creep strain during the loading process. Elastic and plastic strains of the tested material are linearly proportional to the stress level. The Burgers model and the Bingham's model are employed to represent the viscoelastic and viscoplastic response, respectively. Model parameters are extracted from a series of uniaxial compression creep/recovery tests. A good agreement was observed between the verification test result and the prediction from the model.

Based upon the study presented in this paper, the following conclusions are made for tested material within a framework: 1. In a uniaxial creep response the total creep strain can be separated into four components (elastic, plastic, viscoelastic, and viscoplastic strain), and the recoverable strain can be separated into two components (elastic and

viscoelastic strain). 2. A set of creep tests with various times to unloading are required to separate these components. 3. Elastic and plastic strains are proportional to stress, and the yield stress for plastic strain may be zero. 4. The viscoelastic response can be expressed with a Burger's model. 5. The viscoplastic response can be expressed with a Bingham's model. In this study, the yield stress of the viscoplasticity for the tested material is zero.

Renata G. Faisca(2001) Experimental procedures for the determination of physical properties of viscoelastic materials are discussed in this paper. Two well-known methods, a standard ASTM method and the other based on indirect measures are compared to the method proposed here that uses direct measures of forces and response in terms of displacements or accelerations. Two kinds of materials were tested using these three methods. Through the experimental results used for the characterization of viscoelastic materials in the three analyzed methods, the results obtained by the ASTM method were the most reliable (ASTM 1993). It must be emphasized that the use of thin viscoelastic materials may cause small imperfections in the final assembly. However, the problems observed in the experimental tests should be minimized in real situations, as thicker materials are necessary for practical use.

David W. Dinehart (2001) Twelve viscoelastic dampers were tested under static loading to failure. Half of the dampers were comprised of VE material and A36 steel, typical of most VE dampers, while the other half used wood in lieu of steel. Two wood and steel specimens were tested for each of the three different thicknesses of VE material used. Comparison of the replicate dampers showed that there was no difference between the performance of the steel and wood dampers. The damper tests indicated that VE material could be applied directly to wood. The initial fullscale results showed that the application of viscoelastic sheet material between the sheathing and wall frame could be an effective method for improving the energy dissipation characteristics of wood-frame shear walls. Furthermore, the installation of the sheet material was such that there was no impact to the dimensions of the wall. This innovative application of VE material could be used in new design or the retrofitting of wood-frame structures. However, it must be emphasized that only limited testing, on the dampers and wall, was conducted and that further testing is necessary to verify these initial results, and investigate such matters as environmental effects, optimal of material thickness and type, and repeatability.

Haruhiko Kurino et al (2003) A simple semi active oil damper developed for an actual application. This device can dissipate twice as much energy imposed by wind or earthquake forces as an ordinary passive damper by switching the opening of the on/off valve according to a signal from a controller. Because the system employs a decentralized control algorithm that uses only a built-in

sensors' information, each device can be equipped with all the necessary control equipment such as sensors and a controller. Four large earthquake records were obtained from a high-rise building equipped with semi-active oil dampers in the transverse direction and conventional passive oil dampers in the longitudinal direction. Estimated damping verified the high damping property of the building with oil dampers. Equivalent damping ratios were from 6.4% to 6.7% in the transverse direction and from 4.3% to 4.8% in the longitudinal direction. The effect of the dampers was also discussed with reference to a seismic design model. By comparing the damping ratio added by the dampers estimated from the earthquake records with that estimated from the seismic design model with passive oil dampers, it was verified that the passive oil dampers showed almost the same results as expected, and the semi-active oil dampers showed high performance in adding damping, which was about 1.7 times as large as that expected by a passive oil damper.

Hajime Kishi , Manabu Kuwata (2004) In this paper, The temperature and frequency dependencies of the viscoelastic properties (storage modulus and mechanical loss tangent) of polymer films were evaluated by the dynamic mechanical analysis (DMA) method in shear mode. Samples were tested over a wide temperature range between 100 and 150 C with a heating rate of 0.5 C/min. Eight frequencies (0.1, 0.2, 0.5, 1, 2, 5, 10 and 20 Hz) were used to probe the dynamic response of the samples. Master curves on G' and t and of each polymer were made from the results of the DMA experiments by shifting in accordance with time-temperature superposition. The base temperature of the master curve for time-temperature superposition was set at 25 C. The polymer films were surely located between fiber-reinforced plies with keeping the original thickness. Good adhesion between the films and epoxy resins was achieved in all laminates. Thermoplastic polyurethane had the highest damping effect among them. It was also seen that the damping effect became higher with being higher the resonant frequency.

This indicates that the damping effect depends not only on the viscoelastic properties of the interlaminar polymer films but also on the arrangements of the reinforcing carbon fibers in the composite laminates. The arrangements of the reinforcing carbon fibers control the stiffness of the intralaminar zone and would have considerable influence on the amount of local strain of the interlaminar films.

Interleaving of fiber-reinforced composites had a significant effect on the damping properties. In this study, several types of thermoplastic-elastomer films were used as interleaf materials. Laminate stacking sequence (layup arrangements of carbon-fiber prepreg, lay-up number and so on) determined the resonant frequencies of the laminates. The viscoelastic properties of interleaved polymer films at the

temperatures where the laminates were evaluated reflected the damping properties of the interleaved laminates.

Lan Wang (2009) Crumb rubber modified asphalt (CRMA) mixture is a typical viscoelastic material. In order to study the rheological characteristics of CRMA mixture, a series static creep testes for the material has been performed at -15 °C, 0°C, 15 °C, 30 °C and 45 °C respectively. The experimental results indicate that the Burgers model is good in simulating the viscoelastic characteristics of CRMA mixtures. Viscoelastic parameters and time dependence of creep stiffness modulus can be obtained from the simulation of the experimental data. The result shows that creep stiffness modulus decreases rapidly at the initial stage of loading, then the rate of change decreases, and finally creep stiffness modulus approaches to a stable value at the end of loading.

Balch, Lakes (2015). Indium-zinc in situ composites were fabricated and their viscoelastic properties studied over 8.5 decades of frequency. Material with 5% indium by weight was found to have a stiffness damping product (the figure of merit for damping layers) of 1.9 GPa at 10 Hz; 3 times better than the peak of polymer damping layers and over a wider frequency range. Material with 15% indium had a stiffness damping product of 1.8 GPa. The indium segregated in a platelet morphology, particularly favorable for attaining high damping from a small concentration, as predicted by viscoelastic composite theory. The indium-zinc exhibited high damping and a high product of stiffness and damping in the linear range of behavior, at small oscillatory strain. This is in contrast to most high damping metals reported heretofore; these require substantial amplitude of vibration to achieve damping.

Ellie H. Fini (2016) The use of waste biomass has received a lot of attention. Accordingly, this paper studies low-temperature rheological characteristics of asphalt binders modified by biobinders made from four different biomass types (swine manure, corn stover, wood pellet, and miscanthus pellet) using numerical analysis and experimental tests. A three-point bending test was performed on each modified specimen using BBR equipment at temperatures ranging from -12 to -24°C.

The fractional viscoelastic model was used to calculate damping ratio and dissipated energy ratio, and based on these results, the specimen modified with CS was found to have the highest dissipated energy ratio, followed by BB, WP, and MP. The fractional viscoelastic model with two parameters is a simple and practical model in comparison with other viscoelastic models, such as the generalized Maxwell and Burgers model. In addition, crossover temperature was calculated for each specimen, showing that CS had the lowest crossover temperature followed by MP, BB, and WP. The main purpose of this research paper was to conduct a comparative study for evaluating the low-

temperature performance of four biomodified asphalt binders.

Fan Bai and Xinhua Yang (2017) The mechanical properties of asphalt mixture always exhibit large degrees of variability due to asphalt's random internal structure. A stochastic constitutive model is proposed to describe the random mechanical response of an uniaxially compressed asphalt mixture. In this model, the simplified Schapery's nonlinear viscoelastic model and the modified Swchartz's viscoplastic model are combined to estimate the material average deformation, and a random parameter with lognormal probability distribution is introduced to reflect the specimen dispersion. Based on statistical methods, the model is calibrated by the multiple-stress repeated creep-recovery test and the 1-h creep test, and then validated by the irregular loading test and the uniform repeated creep-recovery test. It is shown that the model can predict not only the average mechanical behavior but also the scattering range of the material responses at a certain probability. Following conclusions can be made: 1. The idealized asphalt mixture exhibits nonlinear creep and recovery behavior under cyclic loading; its viscoelastic and viscoplastic deformation trends can be well predicted with the simplified Schapery's nonlinear viscoelastic model and the modified Schwartz's viscoplastic model, respectively; 2. When the randomness of the viscoelastic strain is negligible, the time-independent random parameter with respect to the viscoplastic strain can approximately describe the scatter of the material time-dependent responses in replicated experiments; 3. The samples of the random parameter fall roughly on a straight line in the lognormal probability plot, which indicates that the random parameter can be approximately described by the lognormal probability distribution; and 4. The proposed stochastic viscoelastic-viscoplastic model can predict not only the average mechanical behavior but also the scattering range of the material responses at a certain probability.

3. CRITICAL APPRAISAL OF LITERATURE REVIEW

Serious efforts have been taken to develop the availability of viscoelastic dampers in civil engineering. The dynamic characteristics of viscoelastic damper has significantly improved the damping ratio of RCC as well as steel structure. Viscoelastic material is highly reflective to temperature, hence selecting the right viscoelastic material is the key. Creep and shrinkage failures are major concerns for the viscoelastic solid slabs used in the viscoelastic dampers. Material thickness and environment has a major role in repeatability of this viscoelastic damper. Majorly asphalt slabs has been considered because we can satisfactorily predict the constitutive behavior of asphalt all the way up to failure under various loading conditions including different stress-strain amplitudes, monotonic versus cyclic loadings, and different modes of loading.

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