Performance Analysis of Iterative Closest Point (ICP) Algorithm using Modified Hausdorff Distance

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Abstract - *Registration, tracking and reconstruction of 3-D models in real time has many applications in the fields of pose estimation, alignment, motion estimation, object recognition, handheld scanning and augmented reality. Many varied techniques and approaches have been used to tackle the problem of registration and reconstruction of 3-D images. Our work aims to design a method for accurate and computationally efficient registration of 3-D shapes, through comparison and application of various alignment techniques. The expected outcome is a system able to align and reconstruct geometrically precise complex 3-D models in a fast and efficient manner. We mainly focus on the Iterative Closest Point Algorithm, which is the most popular method used in this field through many different forms and variations. An effort has been made to analyze these various forms which are used for object registration through the use of Modified Hausdorff Distance (MHD). Usage of outlier removal methods such as Random Sampling Consensus (RANSAC) and accelerating techniques such as kd-Tree search are also studied. This system could act as a foundation for implementing the ICP algorithm along with depth images or Simultaneous Localization and Mapping (SLAM).*

Key Words: Iterative Closest Point(ICP), Object Modeling, Object Alignment, Pose Estimation, 3-D Registration, Modified Hausdorff Distance

1. INTRODUCTION

In recent years digitization of physical objects has become easy with the growing popularity of 3D scanners. Scanners use laser, light or x-rays to form a point cloud defining the shape of the object being scanned. Registration is useful in comparing two scans taken at different time/conditions and is also helpful in combining the information in two scans into a single one. This paper aims to design a method for accurate and computationally efficient registration of 3-D shapes; the expected outcome being a system able to align and reconstruct geometrically precise complex 3-D models as efficiently as possible. The Iterative Closest Point Algorithm of Besl and Mckay is one of the most popular method used for rigid transformation of roughly aligned data sets. It is widely used for the registration of free form surfaces where dense data is assumed, and a good initial estimate is available or can be easily estimated. The ICP algorithm has become the dominant method for aligning three dimensional models based purely on the geometry, and sometimes color, of the meshes. This system has been used for registering the outputs of 3D scanners, which typically only scan an object from one direction at a time. ICP starts with two meshes and an initial guess for their relative rigid-body transform, and iteratively refines the transform by repeatedly generating pairs of corresponding points on the meshes and minimizing an error metric. Many different variants of ICP have been utilized over the years for different purposes. We aim to analyze these variants and their performance and accuracy parameters in terms of Modified Hausdorff Distance which gives us a measure of the similarity between two-point sets.

The initial part of the work entails the alignment of 3D models using the various ICP algorithms and optimization. We utilize the basic ICP model for different point data sets and analyze the shortcomings and scope for improvement. Based on that, we optimize the algorithm for efficiency and performance.

1.1 Related Work

Most of the early work performed in the field of object tracking, object recognition and registration revolved around global shape matching or registration having very limited classes of shapes. O. D. Faugeras and M. Hebert were among the first to perform free-form shape matching using 3-D data using a Renault auto part in the early 1980s. [1] The works of P. J. Besl, Brian Curless and Marc Levoy laid down a mathematical background upon which further work in the field would be carried out.[2][3] The first proper method for registration of 3-D shapes was performed by P. J. Besl and Neil Mckay, when they introduced the Iterative Closest Point (ICP) algorithm in 1992, which is still used to this day in various optimized forms.[4][5] The ICP algorithm provides a solution to the free-from surface matching problem and provides a general unified solution to the point-set matching problem without correspondences and the free-form curve matching problem. Besl and Mckay's implementation required no extracted features, no surface or curve derivatives, and no pre-processing of data. The main application was to register digitized data from unfixtured rigid objects with an idealized geometric model prior to shape inspection. Many variants of the ICP algorithm have been used in diverse applications and with various parameters, some of which were thoroughly compared by Rusinkiewicz and Levoy in 2001. [6] Some of the most popular variants of ICP include Comprehensive ICP (CICP), Trimmed ICP (TICP), Picky ICP(PICP), Non-rigid ICP, Scaling

ICP(SICP) and Generalized ICP (GICP), Of these, CICP, PICP and TICP vary from the original ICP algorithm (denoted as OICP for the rest of this paper) in the first step of the algorithm: choosing correspondence points in the two data sets. The OICP algorithm did not impose any restrictions on corresponding point search; it considered every point in the scene surface for closest point calculation. However, for speeding up the convergence there has been a lot of interest regarding selection of points used for estimating transformation vectors. Trimmed ICP or TICP [8] selects only a predefined number of estimated matched pairs based on some given criterion. Picky ICP or PICP [9] rejects all points previously estimated to correspond to one reference point except one with the smallest distance. Comprehensive ICP or CICP [7] uses an enhanced implementation of the ICP algorithm using a comprehensive look-up matrix to find the best correspondence pairs.

Other ICP variants have come about as a result of negating the various drawbacks of the original algorithm of Besl and Mckay. Non-Rigid ICP [10] extends the ICP framework to non-rigid restoration, using an adjustable stiffness parameter, while retaining the convergence properties of the original algorithm. Scaling ICP or SICP [11] integrates a scale matrix with boundaries into the original ICP algorithm for scaling registration. Generalized ICP or GICP [12] combines the ICP and point-to-plane ICP algorithms into a new single probabilistic framework. This allowed for far greater robustness than the standard ICP approach. The latest techniques include Learning Anisotropic ICP, Weighted Average ICP, and ICP using Bi-unique Correspondences. Learning Anisotropic ICP (LA-ICP) [13] presents an online learning approach to 3D object registration that vastly improves the performance of Iterative Closest Point (ICP) methods. Weighted Average ICP (WA-ICP) [14] uses a new weighting approach to establish correspondence. ICP using Bi-Unique Correspondences [15] guarantees the uniqueness of corresponding pairs by searching multiple closest points.

The latest trend in the field of 3D object registration and modeling is the use of RGB-D cameras which capture RGB images along with per-pixel depth information. This has been facilitated mainly by the increasing cheapness and availability of such cameras to the general public. RGB-D allows for greater resolution than Laser scanning but at the cost of reduced accuracy. An in-depth look into using depth cameras for 3D modelling in conjunction with using the ICP algorithm can be found in the work of Peter Henry et al in the field of RGB-D mapping [16]. It utilizes a novel joint optimization algorithm combining both matching using visual features and shape alignment. Sparse feature detection is carried out on the RGB images using a Scale Invariant Feature Transform (SIFT) feature detector and extractor. Random Sampling Consensus or RANSAC is used to find the best rigid transformation between the obtained feature sets. Then the ICP algorithm is carried out on the dense point clouds obtained from the depth data. The two results obtained are properly weighted and added to give us a finely refined 3D model of the indoor environment, which would not be possible using a single approach. Several others have tried to carry forward with the work done in this paper with some newer and interesting approaches [17] [18] [19] [20]. A team comprising of several researchers at Microsoft also have used the depth sensors present in the Xbox Kinect camera to create KinectFusion. [21], which enables a user holding and moving a standard Kinect camera to rapidly create detailed 3-D environments of an indoor scene. The depth data is used to track the 3D pose of the sensor and recreate the precise 3D models of the scene in real-time due to the use of a novel GPU based pipeline. The work done by the KinectFusion team does not require any explicit feature detection and in fact allows for user interaction with dynamically changing scenes. This allows for some exciting new usages in the field of low-cost handheld scanning, object segmentation through direct interaction and in the field of Geometry Aware Augmented Reality. Further progress in this particular project was made by the team of Qin Ye, where they further improved upon the KinectFusion algorithm by combining epipolar constraints with point-to-point constraints. [22]

Our work combines the work of [7] using the Comprehensive ICP (CICP) algorithm in conjunction with RANSAC which has not been done before. Also, we have identified Modified Hausdorff distance, described in detail in [23] as a parameter for analyzing our observations along with the usual parameters such as number of iterations and computation time. Thus, we have come to conclusions regarding our alignment performance through a novel approach that has not been implemented before.

2. THEORY

2.1 The ICP Algorithm

First let us take a look at the Iterative Closest Point (ICP) Algorithm. The ICP algorithm is a general-purpose, representation-independent method for the accurate and computationally efficient registration of 3-D shapes including free form curves and surfaces. Its main application is to register digitized data from unfixtured rigid objects with an idealized geometric model prior to shape detection. The basic tenets of the ICP algorithm are:

- Can be used for efficient registration of 3D shapes
- Allows for full six degrees of freedom (6DOF)
- Independent of shape representation.
- Given “model” shape; Sensed “data” shape
- Algorithm requires no extraction of features, no curve or surface derivatives, no pre-processing of 3D data (except for removal of statistical outliers)
- Always converges monotonically to the nearest local minimum of a mean square distance metric
- Works best when we already have an initial estimate of the relative pose.

Let us take a look at the principle and the steps involved in the Iterative Closest Point(ICP) algorithm.

Principle: A data shape P is moved through translation and rotation to be in best alignment with a model shape X.

STEPS OF THE ICP ALGORITHM:

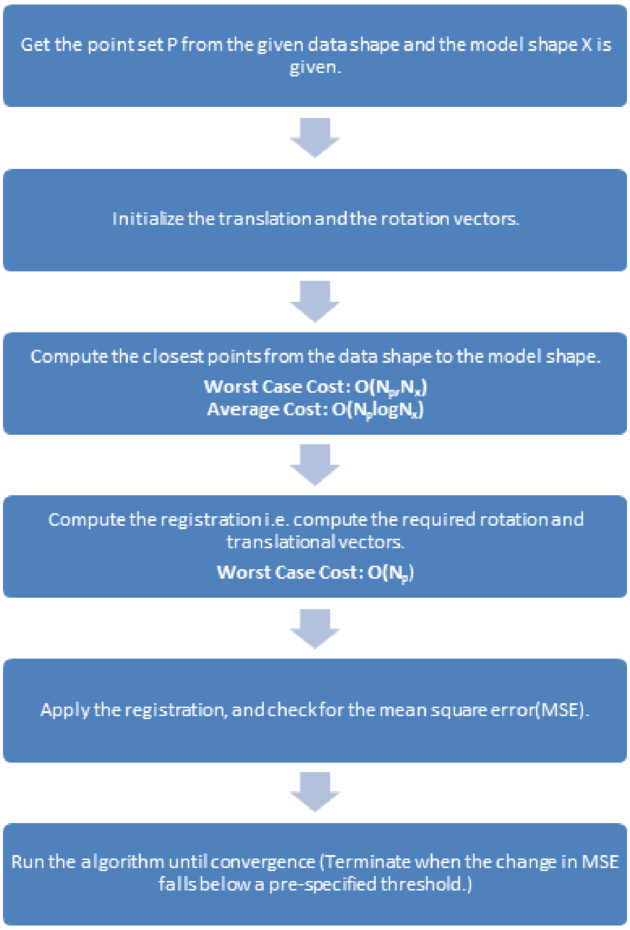

Chart -1: Flowchart describing the different steps of ICP algorithm

Several other variants of the original ICP (OICP) algorithm have also been used in recent times in for several different variations and for different applications. Some of the more popular ones are-

- **CICP**: Enhanced implementation of the ICP algorithm using a comprehensive look-up matrix to find the best correspondence pairs. It results in reducing the minimum MSE between the two data sets after registration.
- **TICP:** Selects only a predefined number of estimated matched pairs based on some given criterion.
- **PICP:** Rejects all points previously estimated to correspond to one reference point except one with the smallest distance.
- **SICP:** Integrates a scale matrix with boundaries into the original ICP algorithm for scaling registration. The scale matrix is introduced directly into the least squares problem with the constraint condition that the scale matrix is bounded.
- **Non-Rigid ICP:** Extending the ICP framework to non-rigid restoration, using an adjustable stiffness parameter, while retaining the convergence properties of the original algorithm. This is extremely useful for registration of human faces or animals.
- **GICP:** GICP developed at Stanford combines the ICP and point-to-plane ICP algorithms into a new single probabilistic framework. It utilises a new 'plane-to-plane' approach modelling the planar surface structure from both scans instead of just the model scans, which is the method followed in the typical 'point-to-plane' method. This allows for far greater robustness than the standard ICP approach.

2.2 Comprehensive ICP(CICP)

The CICP algorithm has been used extensively in our work hence a closer inspection of the algorithm is warranted. In the CICP algorithm, a novel effective evaluation matrix called a comprehensive look-up matrix was introduced for the purpose of correspondence search. This new method makes sure that each selected point on the scene surface has a definite and unique match in the reference surface.

In the original ICP algorithm, searching of correspondence of points is done based on a vector search approach within a P-M distance matrix as shown below in figure 2, where $d_{i,j}$ is the distance between p_i and m_j. CICP differs in that it sorts the $d_{i,j}$ distances in ascending order within the P-M distance matrix. Also, once either m_j or p_i has been assigned a correspondence, those points are no longer considered. This helps to ensure unique association between points in the scene surface and the reference surface. The algorithm can be summarized as follows, for model shape P and data shape X.

1. For each point p_i € P (i=1,…,N_p), compute Euclidian distance to each point m_j € X(j=1,…,N_m). Then for N_p times, loop:
    a. Look for location (i,j) corresponding to minimum distance in the look up matrix
    b. Assign p_i to m_j as correspondence pair
    c. Remove this correspondence pair from further consideration.
2. Compute the transformation parameters (R, T) and transform P accordingly.
3. Compute the MSE between reference and transformed data sets. If MSE is above a threshold and number of iterations k is less than maximum allowed number of iterations, new iteration starts, else the iterative procedure stops.

Although the CICP algorithm consumes more time to compute the transformation, it converges faster in terms of number of iterations and also gives more accurate results.

2.3 Modified Hausdorff Distance(MHD)

Hausdorff distance (Huttenlocheret al. 1993) is a measure of similarity between two point sets. Its objective is to find out the greatest mismatch between two point sets. The biggest advantage of using Hausdorff distance over other parameters as a similarity metric is that it cannot be zero until both the sets are exactly matched.

Given two point sets P = {p1, p2 ,……, pn} and Q={q1,q2,….,qm}, the Hausdorff distance is defined as

$$\boxed{\begin{array}{l}(P, Q) = \max(h(P, Q), h(Q, P)), \text{ where} \\ h(P, Q) = ||p-q|| \text{ for } \max(p \in P) \min(q \in Q)\end{array}}$$

Here h (P, Q) is the maximum value of the Forward

Hausdorff distances (FHD), and h (Q, P) is the maximum value of the Reverse or Backward Hausdorff Distances (BHD). FHD is the set of distances of the nearest points in P for every point in Q and BHD is the set of distances of the nearest points in Q for every point in P.

Hausdorff Distance however is not very robust to outliers in the point maps. Thus, Dubuisson and Jain in 1994 proposed the Modified Hausdorff Distance (MHD).

$$\boxed{MHD = \max(\text{mean}(h(P, Q)), \text{mean}(h(Q, P)))}$$

MHD is more robust to outliers and it increases monotonically as the similarity between the two data sets decreases. So, MHD has proven to be a better choice for a similarity metric.

2.4 Random Sampling Consensus (RANSAC):

Random sampling consensus or RANSAC is an iterative method to estimate parameters of a mathematical model from a set of observed data that contains outliers, when outliers not to be given any influence on the values of the estimates. Thus, it is basically an outlier detection method. RANSAC is a learning technique which estimates the parameters of a model through random sampling of observed data.

3. EXPERIMENTAL RESULTS

Using the MRPT library, we were able to generate objects, introduce an alignment error, and analyze the performance of ICP algorithm on these objects. Objects generated were a sphere and two concentric disks. Scans of the objects were performed by raytracing followed by conversion to point cloud format.

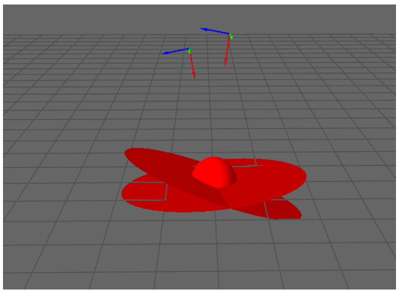

Fig -1: Generated Objects

Firstly, let us take a look at the result of ICP for a given alignment error (x displacement, y displacement, z displacement, yaw, pitch, roll) and the parameters used:

SCENE 1: (0.15,-0.07, 0.10, -0.03, 0.1, 0.1)

- Maximum Iterations = 500
- Minimum Absolute Step(translation) = 0.000001
- Minimum Absolute Step (Rotation) = 0.000001
- Threshold Distance = 0.400000
- Threshold Angle = 0.000000 deg
- Smallest Threshold Distance = 0.100000
- Pitch and Yaw set for scanning and raycasting: 125

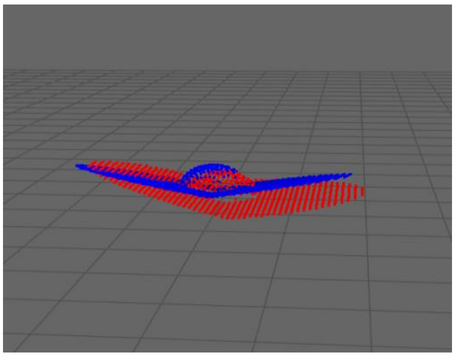

Fig -2: Unaligned Scans(Scene-1)

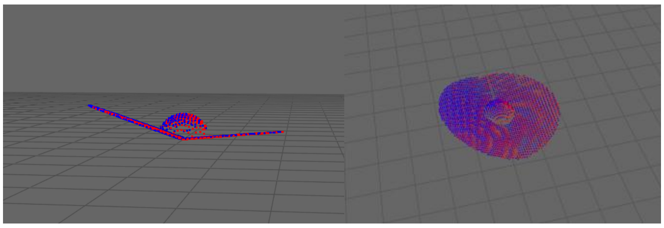

Fig -3: Scans aligned using CICP algorithm (Scene 1)

Results obtained from the above:

- Size of Scan 1: 1847
- Size of Scan 2: 2390
- ICP run took 0.017835 secs.
- Goodness: 99.621% (Goodness is a [0, 1] range indicator of percentage of correspondences. It is an inbuilt measure in the MRPT library.)

- No. of iterations= 73
- **MHD is: 0.0338736**
-

Keeping the parameters constant including the size of the point sets M1 and M2, the experiment is repeated with two different and increasingly greater alignment errors and the results obtained are as follows:

SCENE 2: (0.65, 0.15, 0.22, -0.65, -0.05, 0.15)
SCENE 3: (1.5, 0.85, 1.50, 0.35, -1.5, -2.15)

Table-1: Analyzing performance of ICP for different alignments

Parameter	Computation Time	No. of Iterations	MHD
Scene 1	0.017835	73	0.0338736
Scene 2	0.06654	182	0.0367668
Scene 3	0.064447	244	0.0341597

Next, we considered the precision of the scans and the effect it has on the computation time, iterations and MHD performance. While performing the scans of the 3D objects, two parameters YAW and PITCH are initialized and this indicates the precision of scanning. These parameters determine how accurately the scans of the 3-D objects are taken and this has a definite impact on the outcome of the algorithm.

Table -2: Analyzing different performance parameters with increase in precision of scans

PITCH and YAW value	Set Size for data set and model set	Computation Time(secs)	No. of Iterations	MHD
50	M1: 291 M2: 371	0.000549	18	0.0833314
75	M1: 656 M2: 849	0.008814	46	0.0561693
100	M1: 1176 M2: 1526	0.012861	70	0.0417946
125	M1: 1847 M2: 2390	0.017835	73	0.0338736
150	M1: 2672 M2: 3458	0.039534	99	0.0287003
175	M1: 3637 M2: 4713	0.075907	123	0.0237184
200	M1: 4762 M2: 6154	0.108322	164	0.0210493
225	M1: 6035 M2: 7803	0.126281	164	0.0185829
250	M1: 7461 M2: 9637	0.160194	197	0.0165332
275	M1: 9036 M2:11676	0.198515	214	0.0150802

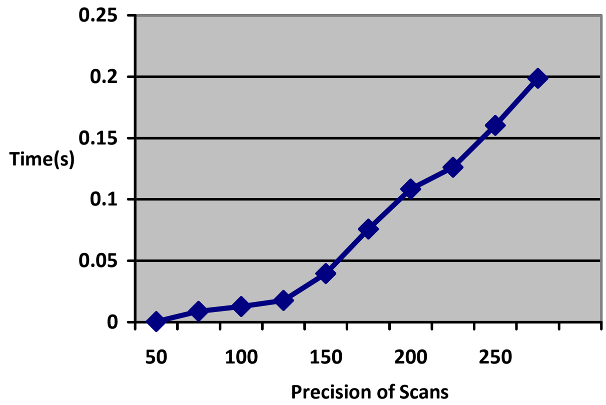

Fig -4: Graph showing computation time variance with increase in precision of scans (generated objects)

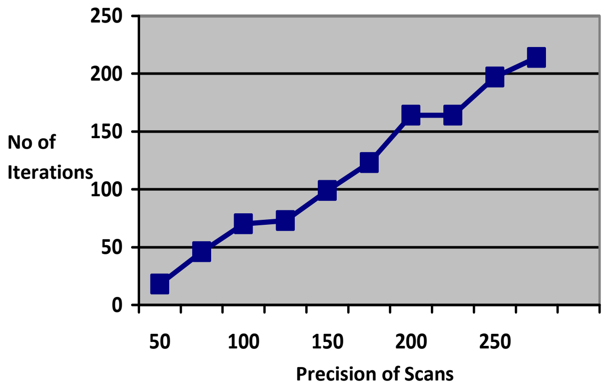

Fig -5: Graph showing variance of no of iterations with increase in precision of scans (generated objects)

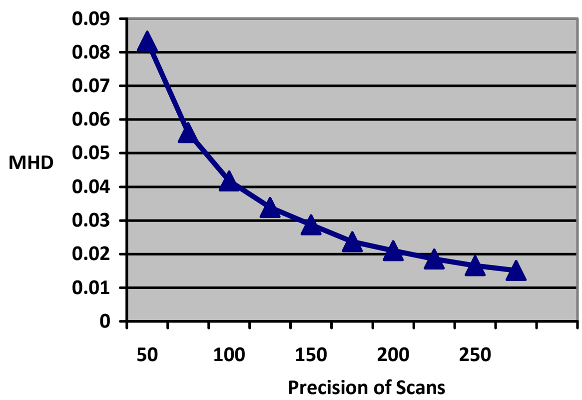

Fig -6: Graph showing variance of MHD with increase in precision of scans (generated objects)

Three crucial inferences can be made from these observations:

- With increase in the precision of scans, the computation time for the algorithm increases, all other parameters remaining constant. This is to be expected as the amount of data increases exponentially.

- With increase in the precision of scans, the number of iterations taken to reach convergence increases, all other parameters remaining constant. This is due to the increase in the depth of data available to the program.
- With increase in the precision of scans, MHD which is a measure of similarity between point sets decreases (Ideally MHD will be zero if the two point sets are identical). This is to be expected as more precision means more information about the two point sets are available. More data means a better approximation of rotation and translational vectors can be found, which translates to better alignment between the two point clouds.

Effect of Ransac:

The effectiveness of RANSAC can be highlighted by implementing our ICP algorithm with and without RANSAC enable and noting the changes. In our case we have also enabled "Only Unique Correspondences" which imply all correspondences between the data set and the model set will be unique. Let us consider the scans obtained at 0 degrees and 45 degrees of the Stanford Bunny taken from the Stanford 3-D Scanning Repository.

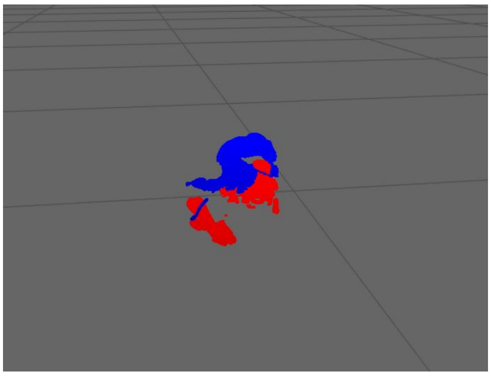

Fig -7: Scanned Models at 45 and 90 degrees of alignment

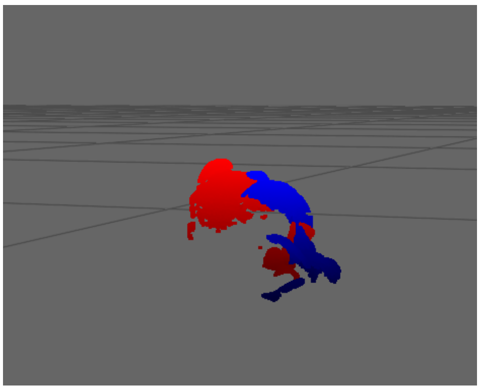

Fig -8: Scans aligned without using RANSAC

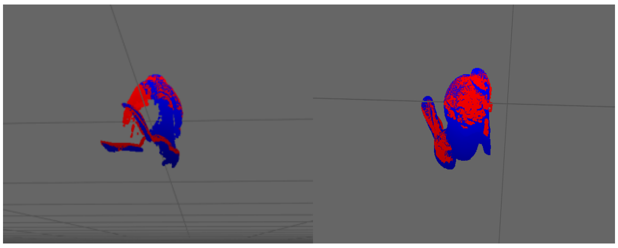

Fig -9: Scans aligned using RANSAC

Table -3: Analyzing the effect of RANSAC on performance

	Computation Time(s)	No of iterations	MHD
Without RANSAC	1.56418	132	0.0265503
With RANSAC	3.16925	423	0.00471862

From the observations, we can easily infer that

- RANSAC helps in fine-tuning the alignment
- Although RANSAC takes considerably more computation time and higher number of iterations, the performance is likewise much better.
- This better performance is highlighted by the difference in MHD, where RANSAC enabled ICP shows a much lower MHD than ICP without RANSAC.

4. CONCLUSION

This paper provides a general-purpose method for the accurate and efficient registration and alignment of 3-D objects and environments without the use of feature extraction, through the use of the Iterative Closest Point algorithm. We have introduced an efficient variant of the algorithm based on Comprehensive ICP, which uses a complete look up distance matrix to ensure unique correspondences between the two point sets. RANSAC or Random Sampling Consensus has also been used in conjunction to refine and fine tune our alignments, and the effect of using ICP with and without RANSAC has clearly been elaborated. Since we used an initial zero estimate of the translation and rotation vectors, the effect of alignment errors on the results of our algorithm has been shown as well. As is expected, with increase in alignment error above around 60 degrees, the performance drops significantly in the absence of a good initial guess. As a parameter for analyzing the performance of our algorithm for different scenarios, we used the Modified Hausdorff Distance as a performance metric. MHD gives us a measure of similarity between two point sets and we have used it to show how the performance of matching decreases as the alignment error increases, and precision of scans decreases. Finally, we also tested our algorithm in a special case where the data set is a subset of our model data, and found the algorithm to be working perfectly in such a situation as well. In future, we hope to build on the work done by incorporating our algorithm with RGB-D data to reconstruct environments and incorporate SLAM into our system.

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