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# Cordial labelings in the context of triplication

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**Abstract -** *In this paper, we introduce the extended triplicate graph of a ladder and investigate the existence of cordial labeling, total cordial labeling, product cordial labeling, total product cordial labeling and prime cordial labeling for the extended triplicate graph of a ladder graph by presenting algorithms.*

***Key Words***: **Ladder graph, Triplicate graph, Graph labelings.**

## 1.INTRODUCTION

Graph theory has various applications in the field of computer programming and networking, marketing and communications, business administration and so on. Some major research topics in graph theory are Graph coloring, Spanning trees, Planar graphs, Networks and Graph labeling. Graph labeling has been observed and identified for its usage towards communication networks. That is, the concept of graph labeling can be applied to network security, network addressing, channel assignment process and social networks [3].

In 1967, Rosa introduced the concept of graph labeling [4]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of mapping is the set of vertices (edges) then the labeling is called a vertex(an edge) labeling.

In 1987, Cahit introduced the notion of cordial labeling [2]. A graph G is said to admit a cordial labeling if there exists a function $f : V \to \{0, 1\}$ such that the induced function $f^* : E \to \{0, 1\}$ defined as $f^*(v_iv_j) = | f(v_i) - (f(v_j) |$ or $(f(v_i) + f(v_j))$ (mod 2) satisfies the property that the number of vertices labeled '0' and the number of vertices labeled '1' differ by atmost one and the number of edges labeled '0' and the number of edges labeled '1' differ by atmost one. A graph G is said to admit a total cordial labeling if there exists a function $f : V \to \{0, 1\}$ such that the induced function $f^* : E \to \{0, 1\}$ defined as $f^*(v_iv_j) = | f(v_i) - (f(v_j) |$ or $(f(v_i) + f(v_j))$ (mod 2) satisfies the property that the number of vertices and edges labeled with '0' and the number of vertices and edges labeled with '1' differ by atmost one.

In 2004, Sundaram, Ponraj and Somasundaram have introduced the concept of product cordial labeling [ 5,6 ]. A graph G is said to admit product cordial labeling if there exists a function $f : V \to \{0, 1\}$ such that the induced function $f^* : E \to \{0, 1\}$ defined as $f^*(v_iv_j) = \{(f(v_i) \times f(v_j) \mid v_iv_j \in E\}$ satisfies the property that the number of vertices labeled '0' and the number of vertices labeled '1' differ by atmost 1 and number of edges labeled '0' and the number of edges labeled '1' differ by atmost 1. A graph that admits product cordial labeling is called product cordial graph.

A graph is called **total product cordial** graph if there exists a function $f : V \to \{0, 1\}$ such that the induced function $f^* : E \to \{0, 1\}$ defined as $f^*(v_iv_j) =\{(f(v_i) \times f(v_j) \mid v_iv_j \in E\}$ satisfies the property that the number of 0's on the vertices and edges taken together differ by atmost one with the number of 1's on the vertices and edges taken together.

In 2011, Bala and Thirusangu introduced the concept of the extended triplicate graph of a path $P_n$ (($ETG(P_n)$) and proved many results on this newly defined concept [1]. Let $V = \{ v_1, v_2,\ldots,v_{n+1}\}$ and $E = \{ e_1, e_2, \ldots, e_n\}$ be the vertex and Edge set of a path $P_n$. For every $v_i \in V$, construct an ordered triple $\{v_i , v_i' , v_i''\}$ where $1\le i \le n+1$ and for every edge $v_iv_j \in E$, construct four edges $v_iv_j'$, $v_j' v_i''$, $v_jv_i'$ and $v_i' v_j''$ where $j = i +1$, then the graph with this vertex set and edge set is called a Triplicate Graph of a path $P_n$. It is dentoted by $TG(P_n)$. Clearly the Triplicate graph $TG(P_n)$ is disconnected. Let $V_1 = \{v_1, v_2 \ldots,v_{3n+1}\}$ and $E_1 = \{ e_1, e_2,\ldots, e_{4n}\}$be the vertex and edge set of $TG(P_n)$. If n is odd, include a new edge $(v_{n+1} , v_1)$ and if n is even, include a new edge $(v_n ,v_1)$ in the edge set of $TG(P_n)$. This graph is called the Extended Triplicate of the path $P_n$ and it is denoted by $ETG(P_n)$.

In 2014 , Thirusangu et.al proved some results on Duplicate Graph of Ladder Graph [7].

A ladder graph $L_n$ is a planar undirected graph with 2n vertices and 3n− 2 edges. It is obtained as the cartesian product of two path graphs, one of which has only one edge: $L_{n,1} = P_n \times P_1$, where n is the number of rungs in the ladder.

Motivated by the study, the present work is aimed to provide label for the extended triplicate graph of a ladder graph and prove the existence of cordial labeling, total cordial labeling, product cordial labeling

and total product cordial labeling for the extended triplicate graph of a ladder graph.
Throughout this work, graph G = (V, E), we mean a simple, finite, connected and undirected graph with p vertices and q edges.

K.Thirusangu and E.Bala (2011) introduced the concept of triplicate graph and proved many results on this newly defined concepts.

Motivated by the study, the present work is aimed to provide label for the extended triplicate graph of a ladder graph and prove the existence of cordial labeling, total cordial labeling, product cordial labeling and total product cordial labeling for the extended triplicate graph of a ladder graph.

Throughout this work, graph G = (V, E), we mean a simple, finite, connected and undirected graph with p vertices and q edges.

## 2. STRUCTURE OF THE EXTENDED TRIPLICATE GRAPH OF LADDER

In this section we discuss about the structure of the extended triplicate graph of ladder by presenting algorithm.

***Algorithm 2.1:***

***Input*** *ladder graph* $L_n$
***procedure*** triplicate of graph $L_n$
for i = 1 to n **do**
$V \leftarrow \{v_i \cup v_i' \cup v_i'' \cup u_i \cup u_i' \cup u_i''\}$
end for
for i = 1 to n-1 **do**
$E_1 \leftarrow (v_i v_{i+1}') \cup (v_i' v_{i+1}'') \cup (u_i u_{i+1}') \cup (u_i' u_{i+1}'')$
end for
for i = 2 to n **do**
$E_2 \leftarrow (v_i v_{i-1}') \cup (u_i u_{i-1}') \cup (u_i' u_{i-1}'') \cup (v_i' v_{i-1}'')$
end for
for i=1 to n do
$E_3 \leftarrow (v_i u_i') \cup (u_i v_i') \cup (u_i' v_i'') \cup (v_i' u_i'')$
end for
$E \leftarrow E_1 \cup E_2 \cup E_3$
***end procedure***
**output :** Triplicate graph of ladder $L_n$

From the above algorithm 2.1, the triplicate graph of a ladder denoted by $TG(L_n)$ is a disconnected graph with 6n vertices and 12n - 8 edges. To make it as a connected graph, for convenience, we include an edge $v_n'' u_n''$ to the edge set E as defined in the above algorithm. Thus the graph so obtained is called an extended triplicate graph of ladder $L_n$ and is denoted by $ETG(L_n)$. By the construction, it is clear that, the graph $ETG(L_n)$ has 6n vertices and 12n - 7 edges.

***Illustration 2.1:***

The structure of extended triplicate graph of ladder $ETG(L_4)$ is given in figure 1.

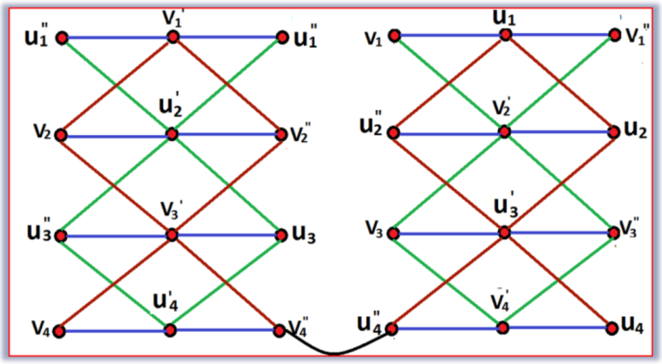

**Fig-1 :** $ETG(L_4)$

### *3 CORDIAL AND TOTAL CORDIAL LABELING*

In this section, we present an algorithm and prove the existence of cordial and total cordial labeling of the extended triplicate graph of ladder ($ETG(L_n)$).

***Algorithm 3.1***

***procedure*** (cordial labeling for $ETG(L_n)$)
for i = 1 to n **do**
$V \leftarrow \{v_i \cup v_i' \cup v_i'' \cup u_i \cup u_i' \cup u_i''\}$
end for
for i = 1 to n **do**
$u_i'' \leftarrow v_i \leftarrow 0$
$v_i' \leftarrow \begin{cases} 1, & i \equiv 1 (mod\ 2) \\ 0, & otherwise \end{cases}$
$v_i'' \leftarrow u_i \leftarrow 1$
$u_i' \leftarrow \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$
end for
***end procedure***
***output*** labeled vertices of $ETG(L_n)$

***Theorem 3.1***

The extended triplicate graph of a ladder graph admits cordial labeling.

**Proof:**

We know that, the extended triplicate graph of a ladder has 6n vertices and 12n – 7 edges. Consider the

arbitrary vertex $v_i \in V$. To label the vertices, using algorithm 3.1, define a map f: $V \rightarrow \{0,1\}$ . Clearly the number of vertices labeled with '0' is 3n and '1' is 3n.
Thus the number of vertices labeled with '0' and the number of vertices labeled with '1' differ by atmost one.
In order to get the labels for the edges, define the induced map $f^* : E \rightarrow \{0,1\}$such that for any $v_iv_j \in E$, $f^*(v_iv_j) = (f(v_i) + f(v_j)) \pmod 2$. Thus,

(i) For $1 \le i \le n - 1$, the edges receives the following labels:

$$v_i v'_{i+1} = v'_i v''_{i+1} = u_i u'_{i+1} = u'_i u''_{i+1} = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$$

(ii) For $2 \le i \le n$ , the edges receives the labels as follows:

$$v'_i v''_{i-1} = v_i v'_{i-1} = u_i u'_{i-1} = u'_i u''_{i-1} = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$$

(iii) For $1 \le i \le n$ , the edges receives the labels as follows:

$$v_i u'_i = u_i v'_i = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$$

$$u'_i v''_i = v'_i u''_i = \begin{cases} 1, & i \equiv 1 (mod\ 2) \\ 0, & otherwise \end{cases}$$

(iv) $v''_n u''_n = 1$

Clearly the number of edges labeled with '0' is 6n – 4 and '1' is 6n – 3.
Thus, the number of edges labeled with '0' and '1' differ by atmost one.
Hence ETG($L_n$) admits cordial labeling.

***Theorem 3.2***

Extended triplicate graph of ladder admits total cordial labeling.

**Proof:**

By theorem 3.1 , using the map f on V and there by the induced map f* on E, the total number of vertices and edges labeled together with '0' and '1' is 9n-4 and 9n-3 respectively.

Thus for all *n*, the number of zeroes on the vertices and edges taken together differ by atmost 1 with the number of one's on vertices and edges taken together.

Hence the extended triplicate graph of ladder admits total cordial labeling.

***Illustration 3.1***

ETG($L_4$) with its cordial labeling is given below in figure 2.

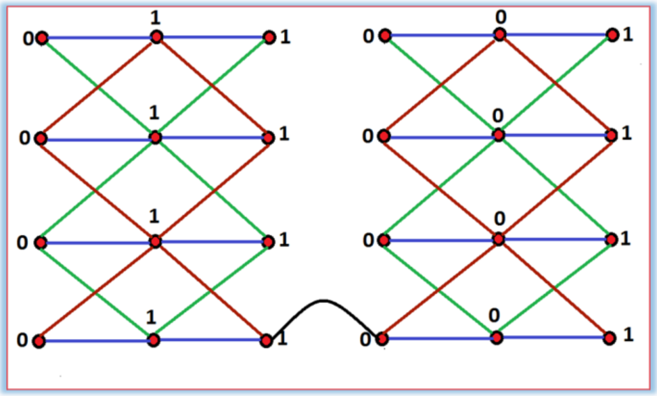

Fig-2: ETG($L_4$) and its cordial labeling

## *4 PRODUCT CORDIAL AND TOTAL PRODUCT CORDIAL LABELING*

In this section we present an algorithm and prove the existence of product cordial and total product cordial labelings for the extended triplicate graph of ladder (*ETG($L_n$)*) .

***Algorithm 4.1***
***procedure*** (product cordial labeling for ETG($L_n$))
for i = 1 to n **do**
  $V \leftarrow \{v_i \cup v'_i \cup v''_i \cup u_i \cup u'_i \cup u''_i\}$
end for
for i = 1 to n **do**

$$u''_i \leftarrow u_i \leftarrow v'_i \leftarrow \begin{cases} 1, & i \equiv 1 (mod\ 2) \\ 0, & otherwise \end{cases}$$

$$v''_i \leftarrow v_i \leftarrow u'_i \leftarrow \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$$

end for
***end procedure***
***output*** labeled vertices of ETG($L_n$)

***Theorem 4.1***

The extended triplicate graph of a ladder admits product cordial labeling.

**Proof:**

The extended triplicate graph of twig has 6n vertices and 12n - 7 edges. Using algorithm 4.1, define the function f: $V \rightarrow \{0,1\}$ to label the vertices. Thus the number of vertices labeled with '0' is 3n and '1' is 3n.

To obtain the edge labels, define the induced function $f^* : E \rightarrow \{0,1\}$ such that for any $v_iv_j \in E$, $f^*(v_iv_j) = (f(v_i) \times f(v_j)) \pmod 2$.

(i) For $1 \le i \le n - 1$, the edges receives the following labels:

$$v_i v'_{i+1} = u'_i u''_{i+1} = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$$

$$u_i u'_{i+1} = v'_i v''_{i+1} = \begin{cases} 1, & i \equiv 1 (mod\ 2) \\ 0, & otherwise \end{cases}$$

(ii) For $2 \le i \le n$, the edges receives the labels as follows:

$$v_i v'_{i-1} = u'_i u''_{i-1} = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$$

$$v'_i v''_{i-1} = u_i u'_{i-1} = \begin{cases} 1, & i \equiv 1 (mod\ 2) \\ 0, & otherwise \end{cases}$$

(iii) For $1 \le i \le n$, the edges receives the labels as follows:

$$v_i u'_i = u'_i v''_i = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & otherwise \end{cases}$$

$$v'_i u''_i = u_i v'_i = \begin{cases} 1, & i \equiv 1 (mod\ 2) \\ 0, & otherwise \end{cases}$$

(iv) $v''_n u''_n = 0$

Clearly the number of edges labeled with '0' is 6n – 3 and '1' is 6n – 4
Thus, for all $n$, the number of edges labeled with '0' and '1' differ by atmost one.
Hence ETG($L_n$) admits product cordial labeling.

***Illustration 4.1:***

ETG($L_4$) with its product cordial labeling is given below in figure 3 .

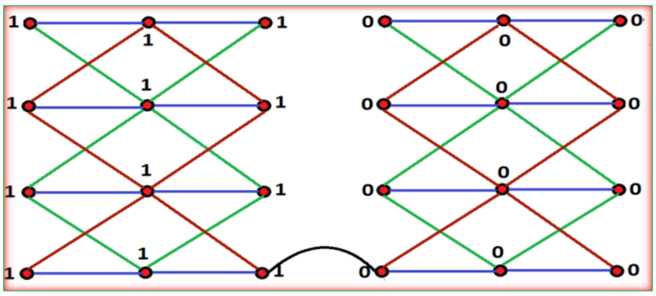

Fig-3: ETG($L_4$) and its product cordial labeling

***Theorem 4.2***

The extended triplicate graph of ladder admits total product cordial labeling.

**Proof**:

By theorem 4.1, using the map f on V and there by the induced map f* on E, we have the total number of vertices and edges labeled together with '0' and '1' is 9n-3 and 9n-4 respectively.

Thus for all $n$, the number of zeroes on the vertices and edges taken together differ by atmost 1 with the number of one's on vertices and edges taken together.

Hence the extended triplicate graph of ladder admits total product cordial labeling.

### *5 CONCLUSION*

In this paper, we have introduced and proved the existence of cordial labeling, total cordial labeling, product cordial labeling and total product cordial labeling for the extended triplicate graph of ladder by presenting algorithms.

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