Effect of thermal-diffusion, dissipation and chemical reaction on unsteady flow of a viscous electrically conducting fluid in a vertical channel with heat sources with travelling thermal waves on the boundaries

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Introduction:

There are many transport processes in nature and in many industries where flows with free convection currents caused by the temperature differences are affected by the differences in concentration or material constitutions. In a number of engineering applications foreign gases are injected to attain more efficiency, the advantage being the reduction in wall shear stress, the mass transfer conductance or the rate of heat transfer. Gases such as H_2, H_2O, CO_2, etc., are usually used as foreign gases in air flowing past bodies. So the problems of heat and mass transfer past vertical bodies in boundary layer flows have been studied by many of whom the names of Somers[13],Gill et al[4], Adeams and Lowell[1] and Gebhart and Peera[3] are worth mentioning. The mass transfer phenomenon in unsteady free convective flow past infinite vertical porous plate was also studied by Soudalgekar and Wavre[15]and Hossain and Begum[6].

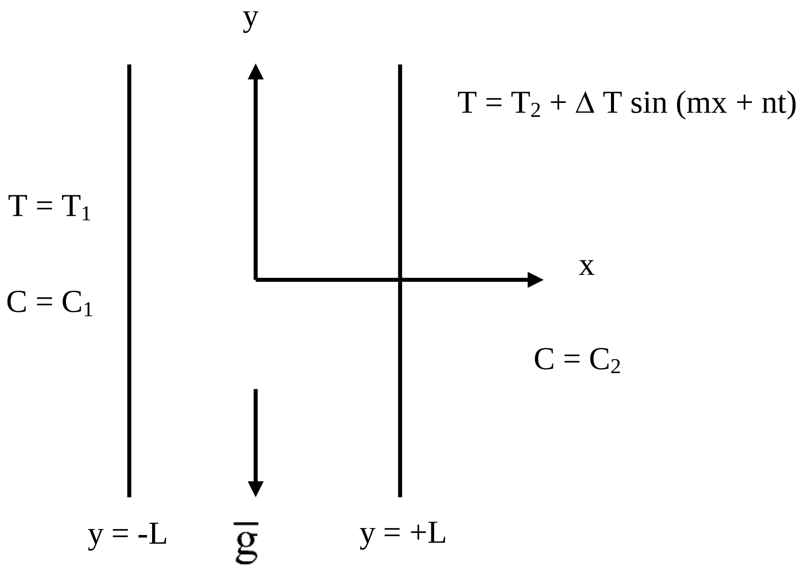
The combined effects of thermal and mass diffusion in channel flows have been studied in the recent times by a few authors notably. Nelson and wood [10, 11]. Lee et al[7], Miyatake and Fujii [8,9], Sparrow et al [16] and others [12,15,19],Nelson and Wood[11] have presented numerical analysis of developing laminar flow between vertical parallel plates for combined heat and mass transfer natural convection with uniform wall temperature and concentration boundary conditions. At intermediate Rayleigh numbers it is observed that the parallel plate heat and mass transfer is higher than that for a single plate. Yan and Lin [21] have examined the effects of the latent heat transfer associated with the liquid film vaporization on the heat transfer in the laminar forced convection channel flows. Results are presented for an air-water system under various conditions. The effects of system temperature on heat and mass transfer are investigated. Recently Atul Kumar Singh et al [2] investigated the heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Convection fluid flows generated by travelling thermal waves have also received attention due to applications in physical problems. The linearised analysis of these flows has shown that a traveling thermal wave can generate a mean shear flow within a layer of fluid, and the induced mean flow is proportional to the square of the amplitude of the wave. From a physical point of view, the motion induced by travelling thermal waves is quite interesting as a purely fluid-dynamical problem and can be used as a possible explanation for the observed four –day retrograde zonal motion of the upper atmosphere of Venus. Also, the heat transfer results will have a definite bearing on the design of oil or gas –fired boilers.

Vajravelu and Debnath [18] have made an interesting and a detailed study of non-linear convection heat transfer and fluid flows, induced by travelling thermal waves. The traveling thermal wave problem was investigated both analytically and experimentally by Whitehead [20] by postulating series expansion in the square of the aspect ratio (assumed small) for both the temperature and flow fields. Whitehead [20] obtained an analytical solution for the mean flow produced by a moving source. Theoretical predictions regarding the ratio of the mean flow velocity to the source speed were found to be good agreement with experimental observations in Mercury which therefore justified the validity of the asymptotic expansion a posteriori. Tanmay Basak et al [17] have analyzed the natural convection flows in a square cavity filled with a porous matrix for uniformly and non-uniformly heated bottom wall and adiabatic top wall maintaining crust temperature of cold vertical walls Darcy – Forchheimer model is used to simulate the momentum transfer in the porous medium. Guria and Jana [5] have discussed the two dimensional free and forced convection flow and heat transfer in a vertical wavy channel with traveling thermal waves embedded in a porous medium. The set of non-linear ordinary differential equations are solved analytically. The velocity and temperature fields have been obtained using perturbation technique.

In this chapter we consider the unsteady thermal convection due to the imposed traveling thermal wave boundary through a vertical channel bounded by flat walls. The effects of free convective heat and mass transfer flow have been discussed by solving the governing unsteady non-linear equations under perturbation scheme. The velocity, temperature and concentration have been analyzed for different variations of the governing parameters. The shear stress, the rate of heat and mass transfer have been evaluated and tabulated for these sets of parameters.

Formulation of the Problem:

We consider the motion of viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls. The thermal buoyancy in the flow field is created by a travelling thermal wave imposed on the boundary wall at y=+L while the boundary at y = -L is maintained at constant temperature T_1. The walls are maintained at constant concentrations. A uniform magnetic field of strength H_0 is applied transverse to the walls. Assuming the magnetic Reynolds number to be small, we neglect the induced magnetic field in comparison to the applied magnetic field. Assuming that the flow takes place at low concentration we neglect the Duffer effect .



Physical Model of the Problem

The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are taken into account to the transport of heat by conduction and convection in the energy equation. Also the kinematic viscosity ν, the thermal conductivity k are treated as constants. We choose a rectangular Cartesian system 0(x,y) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at y = ± L.

The equations governing the unsteady flow and heat transfer are

Equation of linear momentum

$$\rho_e\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\rho g-(\sigma\mu_e^2 H_{o)}^2 u \qquad (2.1)$$

$$\rho_e\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (2.2)$$

Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2.3)$$

Equation of energy

$$\rho_e C_p (\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = \lambda(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) + Q$$
$$+ \mu((\frac{\partial u}{\partial y})^2 + (\frac{\partial v}{\partial x})^2) + (\frac{\mu}{\lambda k} + \sigma\mu_e^2 H_o^2)(u^2 + v^2) - \frac{\partial(q_R)}{\partial y} \quad (2.4)$$

Equation of Diffusion

$$(\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y}) = D_1(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}) - k_1(C - C_e) + k_{11}(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) \quad (2.5)$$

Equation of state

$$\rho - \rho_e = -\beta\rho_e(T - T_e) - \beta^* \rho_e(C - C_e) \quad (2.6)$$

where ρ_e is the density of the fluid in the equilibrium state, T_e, C_e are the temperature and Concentration in the equilibrium state, (u,v) are the velocity components along O(x,y) directions, p is the pressure, T ,C are the temperature and Concentration in the flow region,ρis the density of the fluid,μ is the constant coefficient of viscosity, Cp is the specific heat at constant pressure, λ is the coefficient of thermal conductivity ,k is the permeability of the porous medium,D_1 is the molecular diffusivity, k_{11} is the cross diffusivity, β is the coefficient of thermal expansion, β^* is the volume expansion with mass fraction, k1 is the chemical reaction coefficient, Q is the strength of the constant internal heat source, σ is electrical conductivity of the medium, μe is magnetic permeability and qr is the radiative heat flux.
Invoking Rosseland approximation for radiative heat flux

$$q_r = -\frac{4\sigma^\bullet}{3\beta_R}\frac{\partial(T'^4)}{\partial y} \quad (2.6a)$$

Expanding T'^4 in Taylor's series about T_e neglecting higher order terms $T'^4 \cong 4T_e^3 T' - 3T_e^4$ (2.6b)

Where $\sigma^\bullet$ is the Stefan-Boltzmann constant, β_R is the Extinction coefficient.
In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial x} - \rho_e g \quad (2.7)$$

Where $p = p_e + p_D$, p_D being the hydrodynamic pressure.

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$q = \frac{1}{L}\int_{-L}^{L} u\, dy. \quad (2.8)$$

The boundary conditions for the velocity and temperature fields are
u = 0, v = 0, $T=T_1$, $C = C_1$ on y = -L
$u=0,\ v=0, T = T_2 + \Delta T_e\, Sin(mx+nt), C = C_2$ on y = L (2.9)

Where $\Delta T_e = T_2 - T_1$ and $Sin(mx+nt)$ is the imposed traveling thermal wave

In view of the continuity equation we define the stream function ψ as

u = $-\psi_y$, v = ψ_x (2.10)

Eliminating pressure p from equations (2.2) & (2.3) and using the equations governing the flow in terms of ψ are

$$[(\nabla^2\psi)_t + \psi_x(\nabla^2\psi)_y - \psi_y(\nabla^2\psi)_x] = \nu\nabla^4\psi - \beta g(T-T_0)_y - \beta^{\bullet} g(C-C_0)_y$$
$$-(\frac{\sigma\mu_e^2 H_o^2}{\rho_e})\frac{\partial^2\psi}{\partial y^2}$$

(2.11)

$$\rho_e C_p(\frac{\partial T}{\partial t} - \frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}) = \lambda\nabla^2 T + Q + \mu((\frac{\partial^2\psi}{\partial y^2})^2 + (\frac{\partial^2\psi}{\partial x^2})^2)$$
$$+(\frac{\mu}{k} + (\frac{\sigma\mu_e^2 H_o^2}{\rho_e}))((\frac{\partial\psi}{\partial x})^2 + (\frac{\partial\psi}{\partial y})^2)) + \frac{16\sigma^{\bullet} T_e^3}{\beta_R\lambda}\frac{\partial^2 T}{\partial y^2}$$

(2.12)

$$(\frac{\partial C}{\partial t} - \frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}) = D_1\nabla^2 C - k_1 C + K_{11}\nabla^2 T \quad (2.13)$$

Introducing the non-dimensional variables in (2.11) - (2.13) as

$$x' = mx,\ y' = y/L, t' = t\nu m^2, \Psi' = \Psi/\nu, \theta = \frac{T-T_e}{\Delta T_e}, C' = \frac{C-C_1}{C_2-C_1} \quad (2.14)$$

The governing equations in the non-dimensional form (after dropping the dashes) are

$$\delta R(\delta(\nabla_1^2\psi)_t + \frac{\partial(\psi,\nabla_1^2\psi)}{\partial(x,y)}) = \nabla_1^4\psi - (\frac{G}{R})(\theta_y + NC_y) - M^2\frac{\partial^2\psi}{\partial y^2} \quad (2.15)$$

The energy equation in the non-dimensional form is

$$\delta P(\delta\frac{\partial\psi}{\partial t} - \frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}) = \nabla_1^2\theta + \alpha + (\frac{PR^2E_c}{G})((\frac{\partial^2\psi}{\partial y^2})^2 + \delta^2(\frac{\partial^2\psi}{\partial x^2})^2)$$
$$+(D^{-1} + M^2)(\delta^2(\frac{\partial\psi}{\partial x})^2 + (\frac{\partial\psi}{\partial y})^2)) + \frac{4}{3N_1}\frac{\partial^2\theta}{\partial y^2}$$

(2.16)

The Diffusion equation is

$$\delta Sc(\delta\frac{\partial C}{\partial t} - \frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}) = \nabla_1^2 C - (kSc)C + \frac{ScSo}{N}\nabla_1^2\theta \quad (2.17)$$

where

$R = \frac{qL}{\nu}$ (Reynolds number), $G = \frac{\beta g\Delta T_e L^3}{\nu^2}$ (Grashof number), $P = \frac{\mu c_p}{k_1}$ (Prandtl number), $E_c = \frac{\beta g L^3}{C_p}$ (Eckert number), $\delta = mL$ (Aspect ratio), $\gamma = \frac{n}{\nu m^2}$ (Non-dimensional thermal wave velocity), $Sc = \frac{\nu}{D_1}$ (Schimdt Number), $N = \frac{\beta^*\Delta C}{\beta\Delta T}$ (Buoyancy ratio), $So = \frac{k_{11}\beta^{\bullet}}{\nu\beta}$ (Soret Parameter), $M^2 = \frac{\sigma\mu_e^2 H_o^2 L^2}{\nu^2}$ (Hartmann Number), $N_1 = \frac{\beta_R\lambda}{4\sigma_{\bullet} T_e^3}$ (Radiation parameter), $k = \frac{k_1 L^2}{D_1}$ (Chemical reaction parameter), $N_2 = \frac{3N_1}{3N_1+4}$ $P_1 = PN_2$ $\alpha_1 = \alpha N_2$

$$\nabla_1^2 = \delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The corresponding boundary conditions are

$$\psi(+1) - \psi(-1) = -1$$

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad at\ y = \pm 1 \tag{2.18}$$

$\theta(x, y) = 1$, $C(x, y) = 0$ on y = -1

$\theta(x, y) = Sin(x + \gamma t)$, C(x, y) = 1 on y = 1

$$\frac{\partial \theta}{\partial y} = 0, \frac{\partial C}{\partial y} = 0 \quad at \quad y = 0 \tag{2.19}$$

The value of ψ on the boundary assumes the constant volumetric flux is consistent with the hypothesis (2.8). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t.

Analysis of the Flow

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio δ to be small.

We adopt the perturbation scheme and write

$$\psi(x, y, t) = \psi_0(x, y, t) + \delta\psi_1(x, y, t) + \delta^2\psi_2(x, y, t) + \ldots\ldots\ldots\ldots$$
$$\theta(x, y, t) = \theta_0(x, y, t) + \delta\,\theta_1(x, y, t) + \delta^2\,\theta_2(x, y, t) + \ldots\ldots\ldots\ldots$$
$$C(x, y, t) = C_0(x, y, t) + \delta C_1(x, y, t) + \delta^2\,C_2(x, y, t) + \ldots\ldots\ldots\ldots \tag{3.1}$$

On substituting (3.1) in (2.16) - (2.17) and separating the like powers of δ the equations and respective conditions to the zeroeth order are

$$\psi_{0,\,yyyyy} - M_1^2\psi_{0,\,yy} = \frac{G}{R}(\theta_{0,y} + NC_{O,Y}) \tag{3.2}$$

$$\theta_{o,yy} + \alpha_1 + \frac{P_1E_cR^2}{G}(\psi_{o,yy})^2 + \frac{P_1E_cM_1^2}{G}(\psi_{o,y}^2) = 0 \tag{3.3}$$

$$C_{O,YY} - (kSc)C_0 = -\frac{ScSo}{N}\theta_{0,yy} \tag{3.4}$$

with $\psi_0(+1) - \Psi(-1) = -1$,

$\psi_{0,y} = 0$, $\psi_{0,x} = 0$ at $y = \pm 1$ (3.5)

$\theta_o = 1$, $C_0 = 0$ on $y = -1$

$\theta_o = Sin(x + \gamma t), C_0 = 1$ on $y = 1$ (3.6)

and to the first order are

$$\psi_{1,\,yyyyy} - M_1^2\psi_{1,\,yy} = \frac{G}{R}(\theta_{1,Yy} + NC_{1,Y}) + (\psi_{0,\,y}\,\psi_{0,\,xyy} - \psi_{0,\,x}\psi_{0,\,yyy}) \tag{3.7}$$

$$\theta_{1,yy} = (\psi_{0,x}\theta_{o,y} - \psi_{0,y}\theta_{ox}) + \frac{2P_1E_cR^2}{G}(\psi_{0,yy}.\psi_{1,yy}) + \frac{2P_1E_cM_1^2}{G}(\psi_{0,y}.\psi_{1,y}) \tag{3.8}$$

$$C_{1,yy} - (kSc)\,C_1 = (\psi_{0,x}C_{o,y} - \psi_{0,y}C_{ox}) - \frac{Sc\,So}{N}\theta_{1,yy} \tag{3.9}$$

with

$\psi_1(+1) - \psi_1(-1) = 0$

$\psi_{1,y} = 0$, $\psi_{1,x} = 0$ at $y = \pm 1$ (3.10)

$\theta_1(\pm 1) = 0$, $C_1(\pm 1) = 0$ at $y = \pm 1$ (3.11)

Assuming $Ec \ll 1$ to be small we take the asymptotic expansions as

$$\psi_0(x,y,t)=\psi_{00}(x,y,t)+Ec\psi_{01}(x,y,t)+.........$$
$$\psi_1(x,y,t)=\psi_{10}(x,y,t)+Ec\psi_{11}(x,y,t)+ \quad ....$$
$$\theta_0(x,y,t)=\theta_{00}(x,y,t)+\theta_{01}(x,y,t)+.............$$
$$\theta_1(x,y,t)=\theta_{10}(x,y,t)+\theta_{11}(x,y,t)+...........$$
$$C_0(x,y,t)=C_{00}(x,y,t)+C_{01}(x,y,t)+...........$$
$$C_1(x,y,t)=C_{10}(x,y,t)+C_{11}(x,y,t)+.......... \qquad (3.12)$$

Substituting the expansions (3.12) in equations (3.2)-(3.4) and separating the like powers-of Ec we get the following

$$\theta_{00,yy} = -\alpha_1\,,\ \theta_{00}(-1)=1,\theta_{00}(+1)=SinD_1 \qquad (3.13)$$

$$C_{00,yy} -(kSc)C_{00} = -\frac{Sc\,So}{N}\theta_{00,yy}\,,\ C_{00}(-1)=0,C_{00}(+1)=1 \qquad (3.14)$$

$$\psi_{00,yyyy}-M_1^2\psi_{00,yy}=\frac{G}{R}(\theta_{00,y}+NC_{00,y}),$$

(3.15)

$$\psi_{00}(+1)-\psi_{00}(-1)=1,\psi_{00,y}=0,\psi_{00,x}=0 \quad at \quad y=\pm 1$$

$$\theta_{01,yy}=-\frac{P_1R}{G}\psi^2{}_{00,yy}-\frac{P_1M_1^2}{G}\psi^2_{00,y} \quad , \quad \theta_{01}(\pm 1)=0 \qquad (3.16)$$

$$C_{01,yy} \quad -(kSc)C_{01}=-\frac{Sc\,So}{N}\theta_{01,yy} \quad , \quad C_{01}(-1)=0,C_{01}(+1)=0 \qquad (3.17)$$

$$\psi_{01,yyyy}-M_1^2\psi_{01,yy}=\frac{G}{R}(\theta_{01,y}+NC_{01,y})$$

(3.18)

$$\psi_{01}(+1)-\psi_{01}(-1)=0,\psi_{01,y}=0,\psi_{01,x}=0 \quad at \quad y=\pm 1$$

$$\theta_{10,yy}=GP_1(\psi_{00,y}\theta_{00,x}-\psi_{00,x}\theta_{00,y}) \qquad \theta_{10}(\pm 1)=0 \qquad (3.19)$$

$$C_{10,yy}-(kSc)C_{10}=Sc(\psi_{00,y}C_{00,x}-\psi_{00,x}C_{00,y})-\frac{Sc\,So}{N}\theta_{10,yy} \quad C_{10}(\pm 1)=0 \qquad (3.20)$$

$$\psi_{10,yyyy}-M_1^2\psi_{10,yy}=\frac{G}{R}(\theta_{10,y}+NC_{10,Y})++(\psi_{00,y}\psi_{00,xyy}-\psi_{00,x}\psi_{00,yyy})$$

(3.21)

$$\psi_{10}(+1)-\psi_{10}(-1)=0,\psi_{10,y}=0,\psi_{10,x}=0 \quad at \quad y=\pm 1$$

$$\theta_{11,yy}=P_1(\psi_{00,y}\theta_{01,x}-\psi_{01,x}\theta_{00,y}+\theta_{00,x}\psi_{01,y}$$

(3.22)

$$-\theta_{01,y}\psi_{0,x})-\frac{2P_1R^2}{G}\psi_{00,yy}\psi_{10,yy}-\frac{2P_1M_1^2}{G}\psi_{00,y}\psi_{10,y}\ ,\ \theta_1(\pm 1)=0$$

$$C_{11,yy}-(kSc)C_{11}=Sc(\psi_{00,y}C_{01,x}-\psi_{01,x}C_{00,y}+C_{00,x}\psi_{01,y}-C_{01,y}\psi_{0,x})-\frac{Sc\,So}{N}\theta_{11,yy} \qquad (3.23)$$

$$\psi_{11,yyyy} - M_1^2 \psi_{1,yy} = \frac{G}{R}(\theta_{11,y} + NC_{11,Y}) + (\psi_{00,y}\psi_{11,xyy}$$

$$- \psi_{00,x}\psi_{01,yyy} + \psi_{01,y}\psi_{00,xyy} - \psi_{01.x}\psi_{00,yyy}) \qquad (3.24)$$

$$\psi_{11}(+1) - \psi_{11}(-1) = 0, \psi_{11,y} = 0, \psi_{11,x} = 0 \quad at \quad y = \pm 1$$

Solution of the Problem:

Solving the equations (3.13) - (3.24) subject to the relevant boundary conditions we obtain

$$\theta_{oo}(y,t) = (\frac{\alpha_1}{2})(1-y^2) + (\frac{Sin(D_1)-1}{2})y + (\frac{Sin(D_1)+1}{2})$$

$$C_{00} = 0.5(\frac{Sinh(\beta_1 y)}{Sinh(\beta_1)} - \frac{Cosh(\beta_1 y)}{Cosh(\beta_1)}) + a_3(1 - \frac{Cosh(\beta_1 y)}{Cosh(\beta_1)})$$

$$\psi_{oo}(y,t) = a_{11}Cosh(M_1 y) + a_{12}Sinh(M_1 y) + a_{13}y + a_{14} +$$

$$+ a_6 y + a_7 y^2 + a_8 y^3 - a_9 Ah(\beta_1 y) - a_{10}Ch(\beta_1 y)$$

$$\theta_{01}(y,t) = 0.5a_{19}(y^2 - 1) + a_{18}(ySh(M_1 y) - Sh(M_1)) + (\frac{a_{20}}{4M_1^2})(Ch(2M_1 y)$$

$$- Ch(2M_1)) + (\frac{a_{21}}{4\beta_1^2})(Ch(2\beta_1 y) - Ch(2\beta_1)) + (\frac{a_{22}}{M_1^2}$$

$$- (\frac{2a_{24}}{M_1^3}))(Ch(M_1 y) - Ch(M_1)) + (\frac{a_{23}}{\beta_1^2})(Ch(\beta_1 y) - Ch(\beta_1))$$

$$+ (\frac{a_{24}}{M_1^2})(ySh(M_1 y) - Sh(M_1))$$

$$C_{01}(y,t) = a_{31}(1 - \frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + a_{32}(y - \frac{Sh(\beta_1 y)}{Sh(\beta_1)}) + a_{33}(y^2 - \frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + a_{34}(y^4 - \frac{Ch(\beta_1 y)}{Ch(\beta_1)})$$

$$+ a_{35}(y^6 - \frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + a_{36}(Ch(2M_1 y) - Ch(2M_1)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + a_{37}(Ch(2\beta_1 y)$$

$$- Ch(2\beta_1)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + a_{38}(Ch(M_1 y) - Ch(M_1)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + a_{39}(ySh(\beta_1 y)$$

$$- Sh(2\beta_1)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + a_{40}(y^2 Ch(M_1 y) - Ch(M_1)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + a_{41}(Ch(\beta_2 y)$$

$$- Ch(\beta_2)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + a_{42}(Ch(\beta_3 y) - Ch(\beta_3)\frac{Ch(\beta_1 y)}{Ch(\beta_1)})$$

$$\psi_{01}(y,t) = a_{58} + a_{57}y + a_{55}Ch(M_1 y) + a_{56}Sh(M_1 y) + \phi(y)$$

$$\phi(y) = a_{43} + a_{44}y + a_{45}y^3 + a_{46}y^5 + a_{47}Sh(2M_1 yt) + a_{48}Sh(2\beta_1 y) + a_{49}Sh(M_1 y)$$
$$+ a_{50}yCh(M_1 y) + a_{51}yCh(\beta_1 y) + a_{52}y^2 Sh(M_1 y) + a_{53}Sh(\beta_2 y) + a_{54}Sh(\beta_3 y)$$

$$\theta_{10}(y,t) = a_{77}(y^2-1) + a_{78}(y^3-y) + a_{79}(y^4-1) + a_{80}(y^5-y) + (a_{81}+ya_{87})(Ch(\beta_1 y) - Ch(\beta_1)$$
$$+(a_{82}+ya_{88})(Sh(\beta_1 y) - Sh(\beta_1)) + (a_{83}+ya_{85})(Ch(M_1 y) - Ch(M_1))$$
$$+(a_{84}+ya_{86})(Sh(M_1 y) - ySh(M_1))$$

$$C_{10}(y,t) = b_2(Ch(2\beta_1 y) - Ch(2\beta_1)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + b_3(Sh(2\beta_1 y) - Sh(2\beta_1)\frac{Sh(\beta_1 y)}{Sh(\beta_1)})$$
$$+b_4(yCh(\beta_1 y) - Ch(\beta_1)\frac{Sh(\beta_1 y)}{Sh(\beta_1)}) + b_5(ySh(\beta_1 y) - Sh(\beta_1)\frac{Ch(\beta_1 y)}{Ch(\beta_1)})$$
$$+b_6(y^2-1)Ch(\beta_1 y) + b_7(y^2-1)Sh(\beta_1 y) + b_8 y(y^2 Ch(\beta_1 y) - Ch(\beta_1)\frac{Ch(\beta_1 y)}{Ch(\beta_1)})$$
$$+b_9 y(y^2 Sh(\beta_1 y) - Sh(\beta_1)\frac{Ch(\beta_1 y)}{Ch(\beta_1)}) + b_{10}(Sh(\beta_2 y) - Sh(\beta_2)\frac{Sh(\beta_1 y)}{Sh(\beta_1)})$$
$$+b_{11}(Sh(\beta_3 y) - Sh(\beta_3)\frac{Sh(\beta_1 y)}{Sh(\beta_1)}) + b_{12}(Ch(\beta_2 y) - Ch(\beta_2)\frac{Ch(\beta_1 y)}{Ch(\beta_1)})$$
$$+b_{13}(Ch(\beta_3 y) - Ch(\beta_3)\frac{Ch(\beta_1 y)}{Ch(\beta_1)})$$

$$\psi_{10} = b_{80}Ch(M_1 y) + b_{81}Sh(M_1 y) + b_{82}y + b_{83} + \phi_1(y)$$
$$\phi_1(y) = b_{53}y^2 + b_{54}y^3 + b_{55}y^4 + b_{56}y^5 + b_{57}y^6 + (b_{58} + b_{64}y + b_{66}y^2 + b_{71}y^3 + b_{73}y^4)Sh(\beta_1 y)$$
$$+(b_{59} + b_{65}y + b_{67}y^2 + b_{70}y^3 + b_{72}y^4)Ch(\beta_1 y) + (b_{60}y + b_{63}y^2)Ch(M_1 y)$$
$$+(b_{61}y + b_{62}y^2)Sh(M_1 y) + b_{74}ySh(2M_1 y) + b_{75}yCh(2M_1 y) + b_{76}Sh(\beta_2 y)$$
$$+b_{77}Sh(\beta_3 y) + b_{78}Ch(\beta_2 y) + b_{79}Ch(\beta_3 y)$$

where

$a_1, a_2, ... a_{105}, b_1, b_2, ... b_{79}$, are constants.

On substituting the zeroeth order and first order solutions of velocity, temperature and concentration expressions in (3.1) equations we get the solution for velocity, temperature and concentration.

SHEAR STRESS, NUSSELT NUMBER and SHERWOOD NUMBE

The shear stress on the channel walls is given by

$$\tau = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=\pm L}$$

which in the non- dimensional form reduces to

$$\tau = \left(\frac{\frac{\tau}{\mu U}}{a}\right) = (\psi_{yy} - \delta^2 \psi_{xx})$$
$$= [\psi_{00,yy} + Ec\psi_{01,yy} + \delta(\psi_{10,yy} + Ec\psi_{11,yy} + O(\delta^2)]_{y=\pm 1}$$

and the corresponding expressions are

$$(\tau)_{y=+1} = b_{90} + \delta b_{91} + O(\delta^2)$$
$$(\tau)_{y=-1} = b_{92} + \delta b_{93} + O(\delta^2)$$

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w}\left(\frac{\partial \theta}{\partial y}\right)_{y=\pm 1}$$

and the corresponding expressions are

$$(\mathrm{Nu})_{y=+1} = \frac{(\mathrm{d}_{65} + \mathrm{Ecd}_{67} + \delta \quad \mathrm{d}_{69})}{(\theta_m - \mathrm{Sin}(\mathrm{D}_1)}$$

$$(Nu)_{y=-1} = \frac{(d_{66} + Ecd_{68} + \delta d_{70})}{(\theta_m - 1)}$$

$$\theta_m = d_{71} + Ecd_{72} + \delta d_{73}$$

The local rate of mass transfer coefficient (Sherwood number) (Sh) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m - C_w}\left(\frac{\partial C}{\partial y}\right)_{y=\pm 1}$$

and the corresponding expressions are

$$(Sh)_{y=+1} = \frac{(d_{74} + \delta d_{76})}{(C_m - 1)}$$

$$(Sh)_{y=-1} = \frac{(d_{75} + \delta d_{77})}{(C_m)}$$

$$C_m = d_{78} + \delta d_{79}$$

where d_1,……….d_{77} constants given in the appendix.

Results and Discussion

The aim of this analysis is to discuss the effect of chemical reaction and dissipation on unsteady convective heat & mass transfer flow in a vertical channel on whose walls a travelling thermal wave is imposed. The equations governing the flow of heat & mass transfer are solved by using a regular perturbation technique with the aspect ratio δ as a perturbation parameter. The analysis has been carried out with prandtl number p = 0.71.

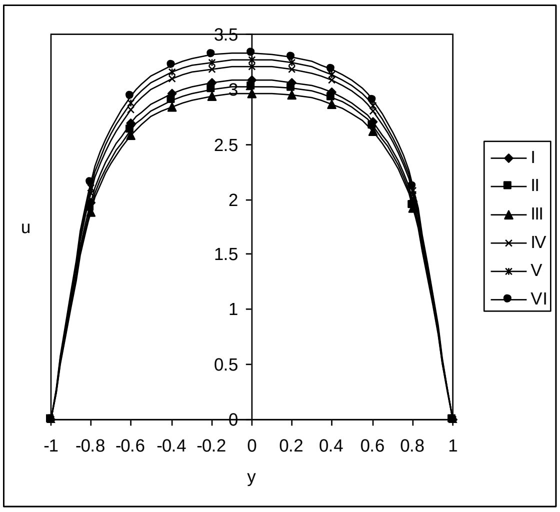


Fig. 1 : Variation of u with G & R

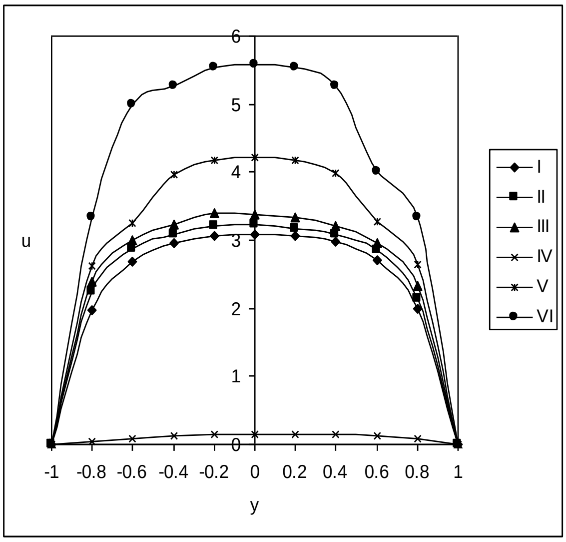


Fig. 2 : Variation of u with R & M

The axial velocity u is shown in figs(1 – 6) for different variations of G, R, M, S_o, N, N_0, γ, Ec and x + γt. Fig (1) represents u with Grashof number G. The actual axial flow is in the vertically upward direction and hence u < 0 represents a reversal flow. It is found that the axial velocity depreciates with increase in G > 0 and enhances with G < 0 with maximum attained at y = 0. An increase in the Reynolds number R or Hartmann number M leads to an enhancement in u thus higher the Lorentz force larger |u| in the flow region (fig 2).

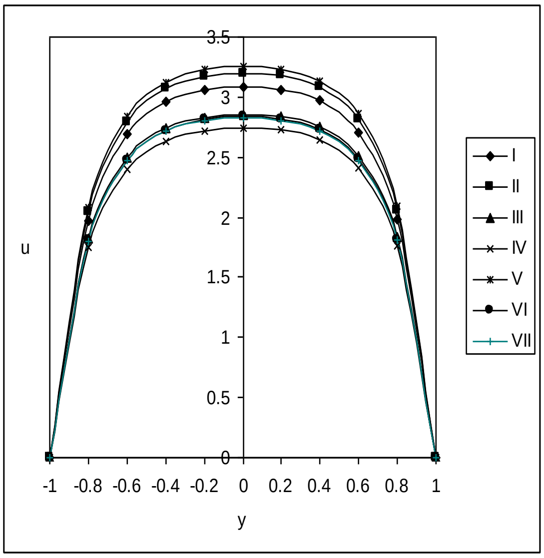


Fig. 3 : Variation of u with S_0 & N

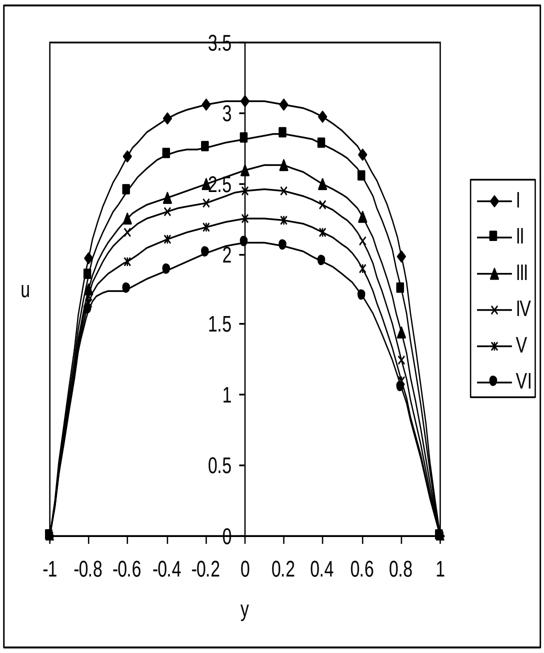


Fig. 4 : Variation of u with γ & Q_1

The effect of thermo diffusion on u is shown in fig (3). It is found from the analysis of the graph that the velocity enhances with increase in $S_o > 0$ and depreciates with $S_o < 0$. The variation of u with buoyancy ratio N shows that when the molecular buoyancy force dominates over the thermal buoyancy force the velocity enhances when the buoyancy forces act in the same direction and for the forces acting in the opposite direction it depreciates in the flow region (fig 3). From fig (4) it is observed that higher the radiative heat flux lesser the velocity in the entire flow region. Fig (4) represents u with chemical reaction parameter γ. An increase in γ results in a depreciation of the velocity u .

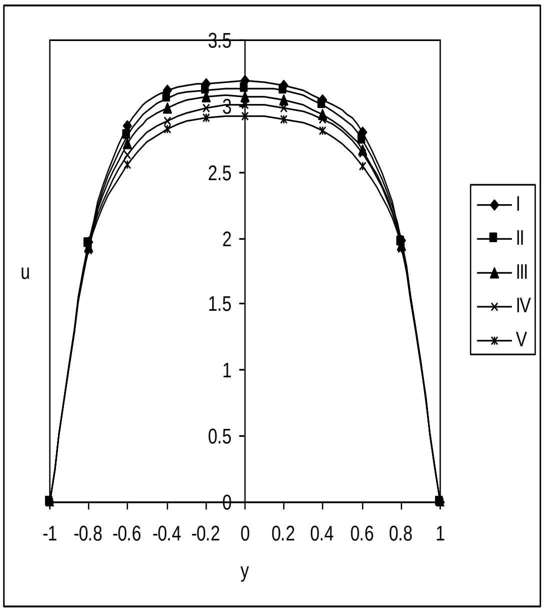


Fig. 5 : Variation of u with Ec

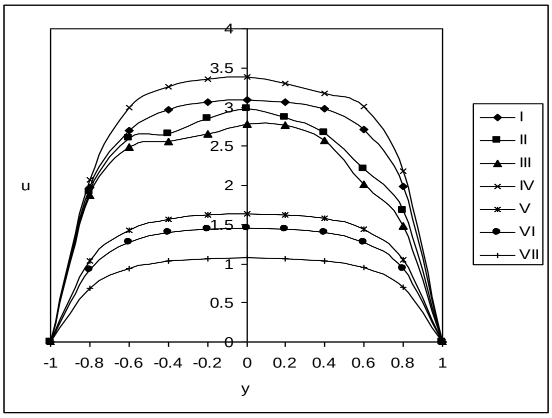


Fig. 6 : Variation of u with x+γt & k

With respect to Eckert number Ec we find that the axial velocity experiences depreciation with increase in Ec. Thus higher the dissipative energy lesser the actual velocity in the flow region (fig5). Fig (6) represents u with phase $x + \gamma t$ of the boundary temperature. An increase in $x + \gamma t \leq \pi$ results in a depreciation in u and for higher $x + \gamma t \geq 2\pi$ we notice an enhancement in u in the entire flow region.

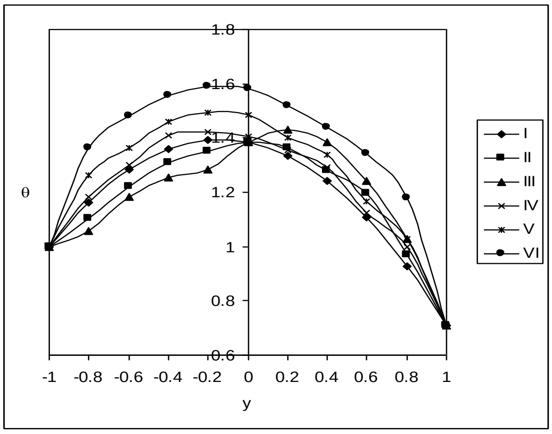


Fig. 7 : Variation of θ with G & R

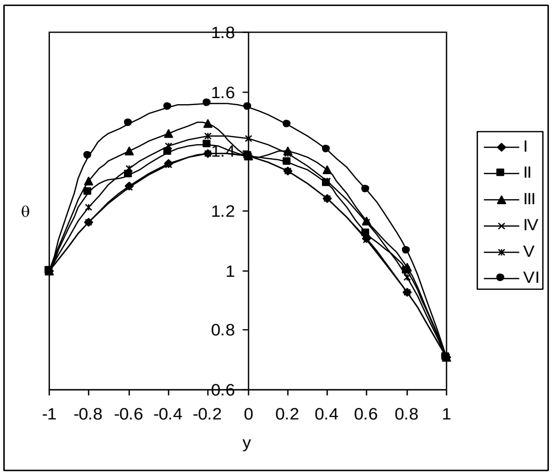


Fig. 8 : Variation of θ with R & M

The non-dimensional temperature distribution (θ) is shown in figs (7- 14) for different parametric values. We follow the convention that the non-dimensional temperature is positive or negative according as the actual temperature is greater / lesser than T_2. We notice that the non-dimensional temperature is positive for all parametric variations. Fig (7) represents θ with G it is found that the actual temperature depreciates in the left half and enhances in the right half with increase in G > 0 and for G < 0 it enhances in the left half and reduces in the right half. From fig (8) we find that the actual temperature enhances with increase in R or M. Thus the presence of the magnetic field enhances the actual temperature in the entire flow region. From fig (9) we find that the actual temperature enhances with increase in the strength of the heat sources and reduces with that of heat sink. An increase in Sc results in depreciation in θ thus lesser the molecular diffusivity smaller the actual temperature in the flow region (fig 10).

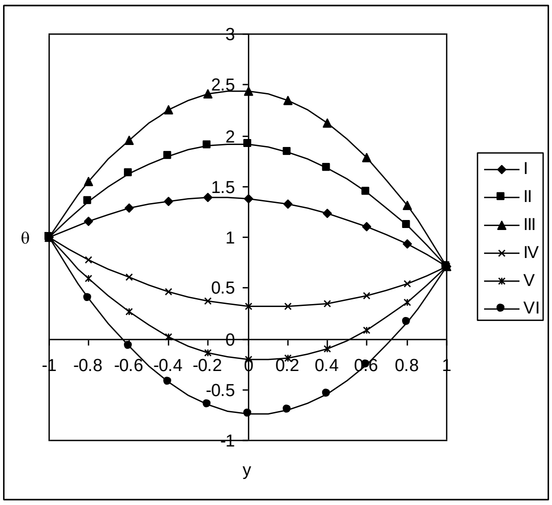


Fig. 9 : Variation of θ with α

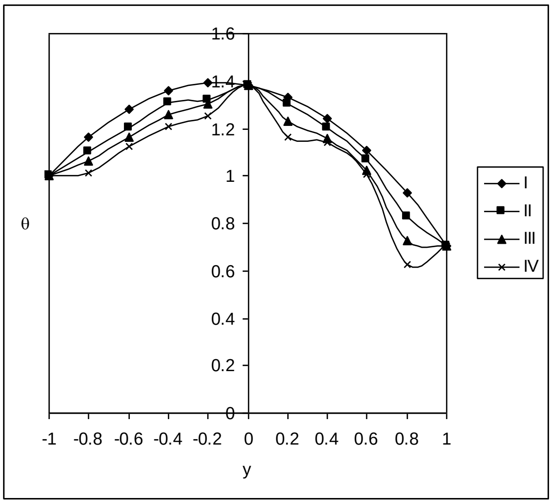


Fig. 10 : Variation of θ with Sc

With respect to S_0 we find that the actual temperature enhances with increase in $S_0 > 0$ and depreciates with $|S_0|$ (fig 11). The variation of θ with buoyancy ration N shows that the actual temperature enhances with increase in N > 0 and depreciates with N < 0 (fig 11). With respect to thermal radiation parameter N_1 we notice an enhancement in the actual temperature with increase in N_1 (fig 12). From fig (12) we find that an increase in the chemical reaction parameter γ leads to a depreciation in the actual temperature in the entire flow region. With respect to Ec we observe that higher the dissipative heat larger the actual temperature (fig 13). The variation of θ with phase x + γt of the boundary temperature exhibits that the actual temperature enhances with lower and higher values of x + γt and depreciates with intermediate value x + γt = π (fig 14).

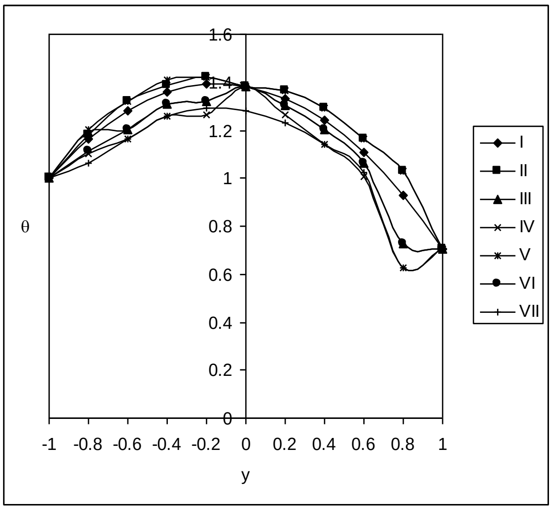


Fig. 11 : Variation of θ with S_0 & N

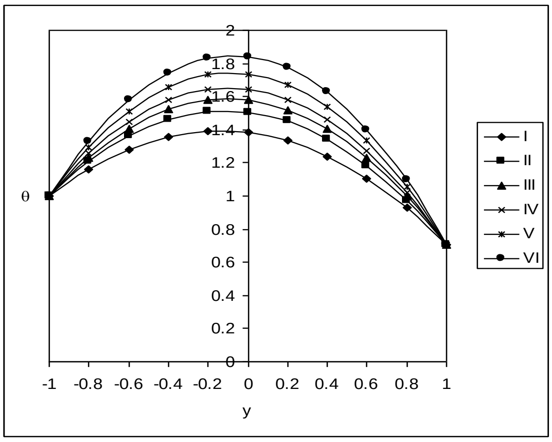


Fig. 12 : Variation of θ with γ & Q_1

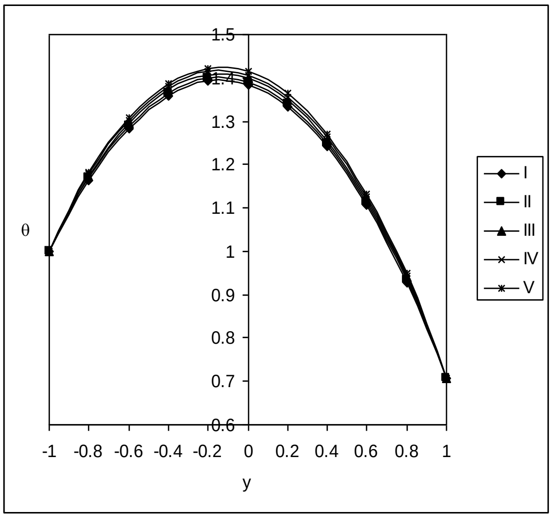


Fig. 13 : Variation of θ with Ec

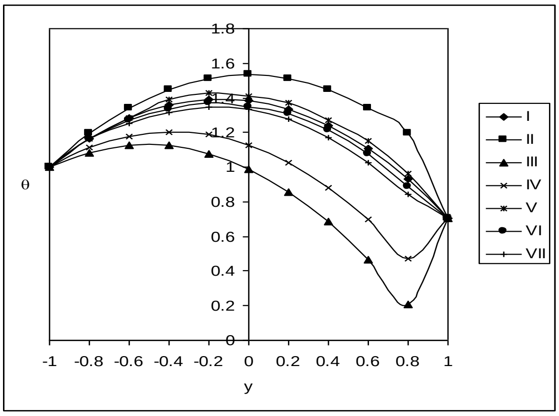


Fig. 14 : Variation of θ with x+γt & k

The concentration distribution (c) is shown in figs (15 - 20) for different parametric values. We follow the convention that the non-dimensional concentration is positive or negative according as the actual concentration is greater / lesser than C_2. Fig (15) represents the concentration c with G. It is found that the actual concentration depreciates with increase in |G| in the entire flow region. An increase in Schmidt number Sc results an enhancement in the actual concentration thus lesser the molecular diffusivity larger the actual concentration in the entire flow field (fig 17).

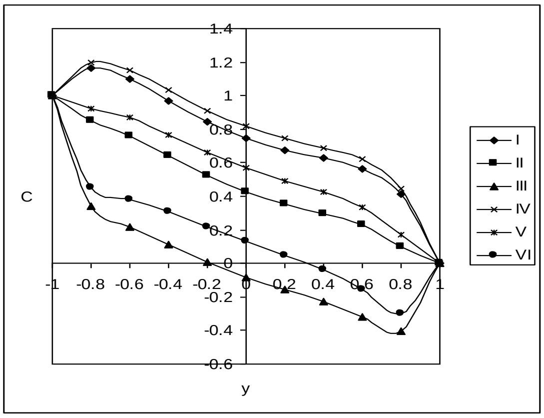


Fig. 15 : Variation of C with G & R

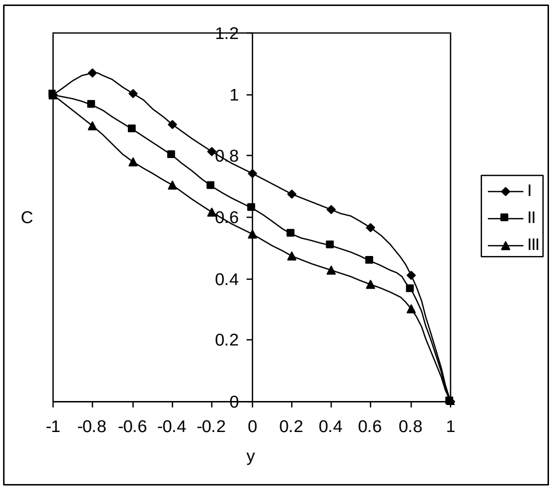


Fig. 16 : Variation of C with R

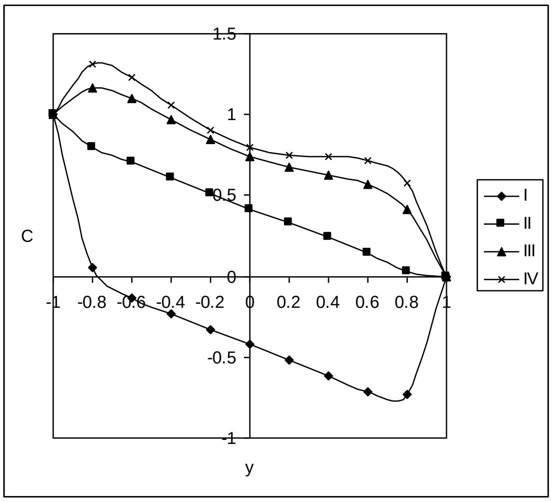


Fig. 17 : Variation of C with Sc

From fig (18) we find that the actual concentration enhances with $S_0 > 0$ and reduces with $|S_0|$. The variation of c with buoyancy ratio N shows that when the molecular buoyancy forces dominates over the thermal buoyancy forces the actual temperature depreciates irrespective of the directions of the buoyancy forces (fig 19). The variation of c with N_1 shows that an increase in $N_1 \leq 5$ leads to an enhancement in the actual concentration while for higher $N_1 \geq 10$ we notice a depreciation in the actual concentration (fig 20).

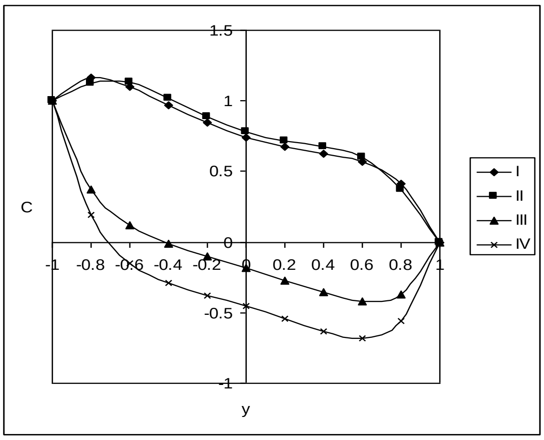


Fig. 18 : Variation of C with S_0

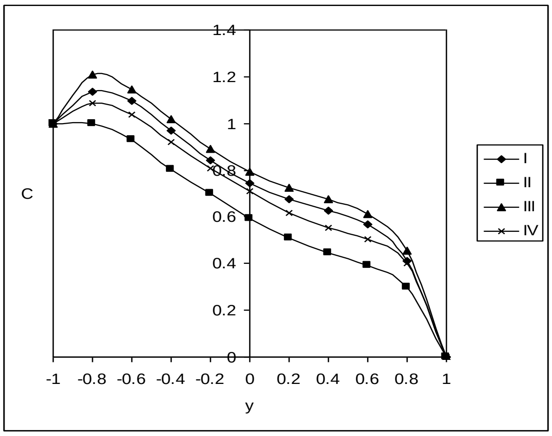


Fig. 19 : Variation of C with N

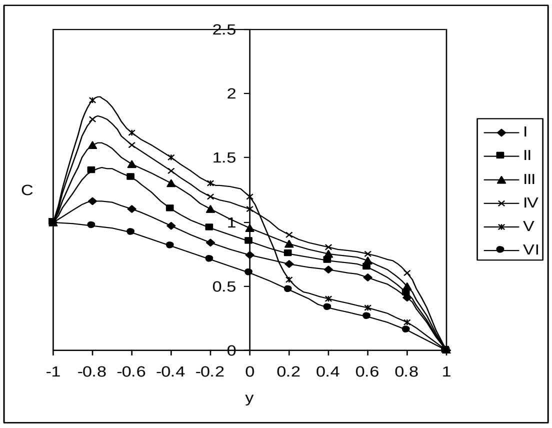


Fig. 20 : Variation of C with N_1

Nusselt Number (Nu) at y = +1 (Table 1)

G	I	II	III	IV	V	VI	VII
5	-3.4316	-3.4419	-3.4113	-3.4013	-3.4483	-3.4067	-3.4018
10	-3.4315	-3.4525	-3.3908	-3.3714	-3.4653	-3.3811	-3.3711
-5	-3.4336	-3.4238	-3.4536	-3.4637	-3.4177	-3.4577	-3.4626
-10	-3.4355	-3.4164	-3.4750	-3.4952	-3.4046	-3.4829	-3.4924
S_0	0.5	1	-0.5	-1	0.5	0.5	0.5
N	1	1	1	1	2	-0.5	-0.8

Nusselt Number (Nu) at y = +1 (Table 2)

G	I	II	III	IV	V	VI	VII	VIII
5	-3.4316	-3.3508	-3.3068	-3.2850	-3.4201	-3.4177	-3.4168	-3.4162
10	-3.4315	-3.3534	-3.3109	-3.2900	-3.4187	-3.4162	-3.4151	-3.4145
-5	-3.4336	-3.3468	-3.2997	-3.2763	-3.4244	-3.4226	-3.4218	-3.4213
-10	-3.4355	-3.3453	-3.2967	-3.2728	-3.4274	-3.4258	-3.4251	-3.4247
γ	0.5	1.5	2.5	3.5	0.5	0.5	0.5	0.5
Ec	0.01	0.01	0.01	0.01	0.03	0.05	0.07	0.09

Nusselt Number (Nu) at y = -1 (Table 3)

G	I	II	III	IV	V
5	0.01642	0.01662	0.01686	5.692	0.0002059
10	0.01675	0.01689	0.01706	4.105	0.0002074
-5	0.01579	0.01609	0.01622	28.64	0.0002030
-10	0.01549	0.01566	0.01579	26.36	0.0002016
R	35	70	140	35	35
M	5	5	5	2	10

Nusselt Number (Nu) at y = -1 (Table 4)

G	I	II	III	IV	V	VI
5	0.01642	0.03432	0.05320	-0.01673	-0.03208	-0.04669
10	0.01675	0.03635	0.05878	-0.01573	-0.02928	-0.04136
-5	0.01579	0.03085	0.04495	-0.01906	-0.03935	-0.06196
-10	0.01549	0.02935	0.04121	-0.02043	-0.04416	1.025
α	2	4	6	-2	-4	-6

Nusselt Number (Nu) at y = -1 (Table 5)

G	I	II	III	IV	V	VI	VII
5	0.01642	0.01583	0.01774	0.01849	0.01562	0.01779	0.01810
10	0.01675	0.01557	0.01975	0.02170	0.01518	0.01990	0.02068
-5	0.01579	0.01638	0.01473	0.01425	0.01661	0.01471	0.01451
-10	0.01549	0.01667	0.01357	0.01279	0.01717	0.01354	0.01322
S_0	0.5	1	-0.5	-1	0.5	0.5	0.5
N	1	1	1	1	2	-0.5	-0.8

Nusselt Number (Nu) at y = -1 (Table 6)

G	I	II	III	IV	V	VI	VII	VIII
5	0.01642	0.02429	0.03465	0.04721	0.006503	0.004526	0.003680	0.003209
10	0.01675	0.02442	0.03448	0.04659	0.006646	0.004632	0.003770	0.003291
-5	0.01579	0.02404	0.03498	0.04850	0.006233	0.004325	0.003508	0.003055
-10	0.01549	0.02391	0.03515	0.04917	0.006104	0.004230	0.003427	0.002981
γ	0.5	1.5	2.5	3.5	0.5	0.5	0.5	0.5
Ec	0.01	0.01	0.01	0.01	0.03	0.05	0.07	0.09

Sherwood Number (Sh) at y = +1 (Table 7)

G	I	II	III	IV	V	VI	VII
5	0.2373	0.9772	2.7537	6.5879	2.7630	2.7730	2.7037
10	0.1186	0.8293	2.5206	6.2952	2.5406	2.5606	2.5006
-5	0.5099	1.3308	3.3734	7.5662	3.3934	3.4034	3.3034
-10	0.6677	1.5447	3.7910	8.3463	3.8010	3.8910	3.7210
Sc	0.24	0.6	1.3	2.01	1.3	1.3	1.3
X + γt	π/4	π/4	π/4	π/4	π/2	π	2π

Sherwood Number (Sh) at y = +1 (Table 8)

G	I	II	III	IV	V	VI	VII
5	2.7537	11.6979	-1.6445	-2.4195	2.8774	2.5639	2.5254
10	2.5206	9.5680	-1.5415	-2.2041	2.7508	2.1595	2.0852
-5	3.3734	22.4202	-1.8354	-2.7914	3.2269	3.5884	3.6306
-10	3.7910	43.3627	-1.9227	-2.9487	3.4673	4.2557	4.3453
S_0	0.5	1	-0.5	-1	0.5	0.5	0.5
N	1	1	1	1	2	-0.5	-0.8

Sherwood Number (Sh) at y = -1 (Table 9)

G	I	II	III	IV	V	VI	VII
5	-3.4068	-6.1587	-38.0109	21.5489	-39.0102	-36.0109	-40.0106
10	-2.7225	-5.1629	-26.1105	23.7607	-24.0106	-24.1105	-29.1102
-5	-5.4914	-9.3837	-693.4395	17.4442	-69.9390	-69.0395	-70.4390
-10	-7.1963	-12.2512	88.6413	15.6600	88.9413	88.0413	90.6416
Sc	0.24	0.6	1.3	2.01	1.3	1.3	1.3
X + γt	π/4	π/4	π/4	π/4	π/2	π	2π

Sherwood Number (Sh) at y = -1 (Table 10)

G	I	II	III	IV	V	VI	VII
5	-38.0109	25.2294	2.9137	4.3411	-38.1488	-37.5806	-37.4610
10	-26.1105	36.7319	2.7838	4.1211	-26.4646	-25.1051	-24.8328
-5	-693.4395	15.1708	3.1254	4.6622	-85.3574	-51.2132	-48.7740

-10	88.6413	12.5544	3.2089	4.7719	91.8665	64.2855	58.5327
S_0	0.5	1	-0.5	-1	0.5	0.5	0.5
N	1	1	1	1	2	-0.5	-0.8

Sherwood Number (Sh) at y = -1 (Table 11)

G	I	II	III	IV	V	VI	VII	VIII
5	-38.0109	119.9237	183.2364	-208.4978	8.4288	6.3849	5.7430	5.4289
10	-26.1105	144.1174	245.5538	-339.4614	9.3180	6.8622	6.1176	5.7577
-5	-693.4395	73.1573	198.0641	-217.7907	7.0603	5.5986	5.1127	4.8700
-10	88.6413	56.9593	164.8282	-208.6530	6.5290	5.2748	4.8482	4.6333
γ	0.5	1.5	2.5	3.5	0.5	0.5	0.5	0.5
Ec	0.01	0.01	0.01	0.01	0.03	0.05	0.07	0.09

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