Progressive improvements in basic Intensity-Duration-Frequency curves deriving approaches: A review

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Abstract - *Intensity–duration–frequency (IDF) curves are amongst the most useful tool in designing water resource structures and provide helpful probable information about rainfall storm occurrence. Distinct improvements are made in the basic idea to generate IDF curves in the past. This paper reviews methods which suggest the black box theory and also the methods suggesting single variable explaining theory and double variable explaining i.e. Copulas method. These improvements incorporated in the IDF curves propose better ideas to generate IDF curves and explain the physical phenomena of rainfall storm more comprehensively.*

Key Words: IDF curves, Copula method, Extreme events, Recurrence interval, Gumbel distribution.

1. INTRODUCTION

Rainfall intensity–duration–frequency curves are graphical depictions of the measure of statistical characteristic of rainfall storms that entail the design compatible rainfall intensity for various relevant long storms for a particular watershed area. To design a hydraulic structure, first it is to know that the amount of flood discharge it is able to accommodate. It is called design peak flood. To decide design peak flood, Rational method is used where peak flood is calculate as

$$Q_p = \frac{1}{3.6} C(i_{tc,p})A ----(1)$$

Where,

Q_p = Peak discharge (m³/s)

C = Coefficient of runoff

A =Drainage area (Km²)

$i_{tc,p}$ = mean intensity of rainfall (mm/h) for duration equal to concentration time and an expedience probability

$$P = \frac{1}{T}$$

Here, T is recurrence interval.

Rainfall intensity in the Rational formula is decided by IDF curve of required recurrence interval. To design a hydraulic structure, it is very important to know its criticality in terms of life period and factors affecting its stability. Some structures are very important like Dams and Spillways which are made for long life period and affected by many critical forces like hydrostatic force, uplift force, silt force etc. and some unpredictable affects like behaviour of climate in the particular watershed etc. Failure in such conditions may cause a catastrophe. While designing, structures are strengthened to counteract those forces. To combat the unpredictability, historical study of climatic factors is done. In case of IDF curves, these factors are intensity of rainfall and duration of rainfall. On the basis of past, these factors are used to generate IDF curves so that they incorporate all the possible condition for the upcoming life period of structures. These past data are analysed to develop IDF curves empirically first. With the time, more relevant analysis turned out. One of them explained the IDF curves using single variable rainfall intensity as a pronounced variable of rainfall phenomenon then other method illustrated both intensity and duration of rainfall of equal importance. Both variables are used simultaneously with the help of Copula method of statics. With these improvements, very relevant IDF curves are generated which not only significantly incorporate the past possibility but also enable to explain physical things up to a great extant.

2. LITERATURE REVIEW

2.1Black box Theory Phase

Very initially, Sherman and Bernard tried to established IDF curve. They used various parameters with the variables which accommodate the relationship in different special conditions without acknowledging physical understanding. To study rainfall intensity and duration relation, an empirical approach is applied. At very first, **Sherman (1931); [11],** derived empirical relation between these two variables and it is expressed as

$$R = \frac{a}{(D+b)^c} ----(2)$$

Where,

R represents average intensity of rainfall in inches per hour, duration of rainfall is governed by D in minutes for a particular return period. Here, three parameters a , b and c are used. These parameters are used to incorporate the effect of geographical location and recurrence interval.

Particularly, a considers the effect of geographical location and recurrence interval and b , c acknowledges different geographical locations.

After a year, **Bernard (1932); [3],** proposed a similar type of equation with little modification. He introduced the return period in the empirical formula as

$$R_D^T = \frac{a_0 T^{a_1}}{D^{a_2}} ----(3)$$

Where,
R_D^T is used for rainfall intensity achieved for D duration of rainfall storm and recurrence interval T and a_0, a_1, a_2 are constants.
With the time, this relation is made specific and constants are defined for some ranges. **Gert et al. (1987); [5]** and **Hargreaves (1988); [6],** fixed the range of a_1 between 0.18 and 0.26 and for a_2 between 0.7 and 0.85 for the rainfall events of duration less than 24 hours. **Kothoyari and Garde (1992); [7],** practiced the same study for the Indian region. They used rainfall data of 78 gauged stations and determine the values of these constants as a_0 =40.10, a_1 =0.20, a_2 =0.70 and above equation is generalized as

$$R_D^T = a_0 \frac{T^{0.2}}{D^{0.7}} (V_{24}^2)^{0.33} ----(4)$$

Where,
V_{24}^2 shows the rainfall depth for 2 years and 24 hours and a_0 vary from 7.1 to 9.1 all over India.
With the time, these above relations are customized and tried to make them more generalized. **Bell (1969);[2],** suggested a generalized formula for IDF curve taking one hour duration and 10 years return period rainfall intensity P_1^{10} as index. **Cheng-lung Chen (1983); [4],** further proposed a generalized formula to derive IDF curve taking three base rainfall depths for any region of USA. For one hour 10 year return period P_1^{10}, 24 hour 10 year return period P_{24}^{10} and 24 hour rainfall duration 100 year return period P_1^{100} are taken as base index. **Bell (1969); [2] ,** used general formula of the type:

$$\frac{P_d^T}{P_d^{10}} = A \ln T + B ----(5)$$

Basically, equations suggested by **Bell (1969); [2]** and **Chen (1983); [4],** may be considered of type:

$$\frac{I_d^T}{I_{d'}^{T'}} = f_1(T) f_2(d) ----(6)$$

Where, T stands for the recurrence interval in years, d for the duration of rainfall; T' and d' stand for base constant recurrence interval in year and base duration of rainfall. I_d^T is the requested rainfall intensity for T years return period and d minutes duration of rainfall and $I_{d'}^{T'}$ is base rainfall intensity for base T' years return period and base d' minutes duration of rainfall. Function $f_1(T)$ is the only function of return period T and $f_2(d)$ is also the only function of rainfall duration d .

Bell (1969); [2], Chen (1983); [4] and **Koutsoyiannis et al.(1998); [8],** suggested the function $f_1(T)$ as the ratio of I_d^T to $I_d^{T'}$ as:

$$f_1(T) = \frac{I_d^T}{I_d^{T'}} = \frac{I_{d'}^T}{I_{d'}^{T'}} = \alpha + \beta \ln T ----(7)$$

And function $f_2(d)$ is suggested as the ratio of I_d^T to $I_{d'}^T$ as:

$$f_2(d) = \frac{I_d^T}{I_{d'}^T} = \frac{I_d^{T'}}{I_{d'}^{T'}} = \frac{a}{(d+b)^e} ----(8)$$

After merging equations (6), (7) and (5), generalized rainfall intensity formula for required duration of rainfall is generated considering $I_{d'}^{T'}$ as base rainfall intensity.

$$I_d^T = I_{d'}^{T'} (\alpha + \beta \ln T) \frac{a}{(d+b)^e} ----(9)$$

2.2 IDF curves using Uni-variate distribution function

With the time, studies arrived which used different probability distributions coupled with empirical equations to generalized IDF curves. To derive IDF curve, extreme rainfall event are analyzed so to fit extreme event rainfall intensity mostly Extreme value type-I i. e. Gumbel distribution is used. Various probability distributions are used to fulfill the purpose but Gumbel distribution was adopted mainly. **Baghirathan and Shaw (1978); [1],** utilized the Gumbel distribution to generate IDF relations for Sri Lanka. **Oyebande (1982); [9],** deduced IDF curves employing the Gumbel distribution for Nigeria. **Vieira and de Souza (1985); [13],** also used the Gumbel distribution to find out IDF curves for Ribierao Preto in Brazil. **Sreedharan et al. (1990); [12],** used the same distribution for Kerala region in India. Method adopted to derive IDF curve using this approach is as follow:

1) From the rainfall data of years of period, maximum rainfall intensity from each year is selected.

2) These values are arranged in decreasing order and highest value ranked first.
3) The return period is calculated using plotting position formula like Weibull's formula.

$$T = \frac{n+1}{m} ----(10)$$

Where, T stands for return period in year, n stands for the highest rank and m stands for rank value for observed rainfall intensity and the probability is obtained as:

$$P = \frac{1}{T}$$

4) The rainfall intensity is regressed with duration of rainfall.
5) After fitting regression, rainfall intensities series for different durations are calculated. Thus, means and standard deviations are calculated for different duration series.
6) Frequency factor K_T for required return period is calculated by applying Gumbel distribution as:

$$K_T = -\frac{\sqrt{6}}{\Pi}\left\{0.5772 + \ln\left[\ln\left[\frac{T}{T-1}\right]\right]\right\} --(11)$$

Where, T is return period.
7) Rainfall intensities are calculated for required duration series for corresponding return period using formula.

$$X_T = X_m + K_T s ----(12)$$

Where,
X_T is the intensity for required return period.
X_m is the mean intensity of rainfall
s is standard deviation.
K_T is frequency factor for given return period.

IDF curves made using Uni-variate approach are analyzed utilizing rainfall series predefined duration which is not a practical situation. With the time, more relevant approach is used. This approach incorporates not only event rainfall duration instead of fixed predefined duration but also enable to facilitate to show joint effect of both variables of rainfall event.

2.3 IDF curves using Copulas Method

Copula method is actually a modern statistics method. It facilitates to analyze many random variables at a time and also shows the joint effect of these variables in terms of joint distribution simultaneously. Every random variable that explains the physical phenomena is fitted into different probability distributions and the most suitable distribution is selected as marginal distribution. . It is a mapping which assigns joint cumulative distribution $H(x, y)$ from marginal distributions. Where, the marginal cumulative distributions $G(y)$ and $F(x)$ can be defined as a subset of collection of random variable y and random variable x respectively. If x & y are two random variables with $F(x) = P(X \le x)$ and $G(y) = P(Y \le y)$, then there exists a copula 'C' as

$$H(x, y) = C(F(x), G(y)) ----(13)$$

To find the Copula C, it is inverted as,

$$C(x, y) = H(F_X^{-1}(x), F_Y^{-1}(y)) ----(14)$$

There are various Copula families which are defined over limited values of correlation coefficient and this correlation coefficient has fixed relationship with Copula's parameter. Kendall's tau τ is one such correlation coefficient and it is defined for N observations as

$$\tau_N = \binom{N}{2}^{-1} \sum_{i<j} sign[(x_i - x_j)(y_i - y_j)] ----(15)$$

Where, N =number of observations;
Sign =1 if $x_i < x_j$ and $y_i < y_j$;
Sign=0 if $(x_i - x_j)(y_i - y_j) = 0$; otherwise
Sign=-1 and i, j =1,2,3……N.

Singh et al. (2007);[10], first suggested the idea to generate IDF curves with the use of Copulas Statistics. Supporting the fact that correlation between rainfall intensity and rainfall duration is negative so the Frank Archimedean Copula is selected. It is also found mathematically simple. It is related with Copula's parameter θ as:

$$\tau = 1 - \frac{4}{\theta}(D_1(-\theta) - 1) ----(16)$$

Where, D_1 is first order Debye function. Kth order Debye function of positive agreement is defined as:

$$D_k(\theta) = \frac{k}{\theta^k}\int_0^\theta \frac{t^k}{\exp(t) - 1} dt ----(17)$$

This is applicable for positive θ. For negative agreement, following equation is used.

$$D_k(-\theta) = \frac{k}{\theta^k}\int_0^\theta \frac{t^k}{\exp(t) - 1} dt + \frac{k\theta}{k+1} ----(18)$$

Singh et al. (2007); [10], suggested the generation of IDF curves using Frank Archimedean Copula is as follows:

1) From the available data, annul maximum events are found out and their corresponding rainfall intensity and duration are note down.
2) Rainfall intensity and rainfall duration are fitted into suitable probability distribution and their cumulative distribution is found out.
3) Correlation coefficient Kendall's tau τ is calculated for rainfall intensity and duration. Hence, Frank Archimedean Copula's parameter θ is calculated.
4) Using cumulative probability distribution of rainfall intensity and rainfall duration and Frank Archimedean Copula's parameter θ, joint distribution function is generated as:

$$C_{U,V}(u,v) = -\frac{1}{\theta}\ln\frac{h(\theta)}{h(\theta)-h(\theta u)h(\theta v)} ----(19)$$

Here, $h(x)$ is generating function as:

$$h(x) = 1-\exp(-x)$$

5) To derive IDF curve, rainfall intensities are calculated over particular rainfall duration using conditional distribution

$$C_{R|D=d} = \frac{[\exp(-\theta r)-1]\exp(-\theta d)}{[\exp(-\theta d)-1][\exp(-\theta r)-1]+[\exp(-\theta)-1]} --(20)$$

6) Conditional distributions for different durations are calculated and conditional distribution is related with return period as

$$T(R \mid D=d) = \frac{1}{1-C_{R|D=d}} ----(21)$$

3. CONCLUSIONS

Empirical method is tried on various regions of world and every time new equation comes into consideration which depends on the importance given to different variables in the empirical equations. Many trails are made to generalize the equation but equation is generalized up to some extent with little constraint of rainfall duration and return period of past statistical data. It is also observed that very less physical interpretation of storm phenomena is explained. Uni-variate method is found out to be more appropriate as it is found out to be generalized method and statistically analyses intensity of rainfall and more logically relate with return period variable but little unrealistic as rainfall intensity time series of fixed duration is considered as it doesn't simulate the real phenomena. Copula method is found out to be the most appropriate method out of three. It significantly explains phenomena statistically and physically as it analyses both the variables rainfall intensity and rainfall duration simultaneously. It analyses real rainfall duration which solve the unrealistic method of Uni-variate method. It also shows correlation between both variables which shows hydraulic sensitivity of region.

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