"A Simplified Description of FUZZY TOPSIS Method for Multi Criteria Decision Making"

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Abstract – *To achieve success in any project, a quality of work and various decision modes plays an important role. Usually, many factors affect the project work and with few similar options for work done creates complications in decision making. Hence, cause the rising of Optimization Techniques to have a full proof decisions and conclusions. Fuzzy TOPSIS (Technique for Order Preference by Similarities to Ideal Solution) is one of the best method to get ideal solution among the similar options. Also it can be used to automate the process and overcome ambiguity, uncertainty in selection process. So, we have described method of Fuzzy TOPSIS for a multi-criteria group decision making scenario.*

Key Words: Optimization Techniques, Similar Options, Fuzzy TOPSIS, multi-criteria decision making.

1. INTRODUCTION

In general, Multi Criteria Group Decision Making (MCGDM) problems are frequently evaluated. To solve problems related to decision making several optimization methods are used in practice. But, in case where decision activity is based on similar options it becomes critical to analyse various factors, alternatives with similar category.

One simple example, a group of three person (say A, B and C) intends to determine which mobile phone is to buy based on certain criteria. Let say they have various criteria like price, model quality, screen size, battery life and memory etc. But each person among A, B and C may give different importance to different criteria. So, now it becomes challenge to decision makers (i.e. A, B and C) to find which alternative best meets the group's criteria.

Fuzzy TOPSIS is a method that can help in objective and systematic evaluation of alternatives on multiple criteria. Here, we provide a simplified description of required Fuzzy theory details and an example scenario has been worked out to illustrate the Fuzzy TOPSIS steps.

2. FUZZY TOPSIS THEORY DETAILS

The **Technique for Order of Preference by Similarity to Ideal Solution** (**TOPSIS**) is a multi-criteria decision analysis method, which was originally developed by Hwang and Yoon in 1981 with further developments by Yoon in 1987 and Hwang, Lai and Liu in 1993. TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution (PIS) and the longest geometric distance from the negative ideal solution (NIS). It is a method of compensatory aggregation that compares a set of alternatives by identifying weights for each criterion.

As the parameters or criteria are often of incongruous dimensions in multi-criteria problems it may create problems in evaluation. So, to avoid this problem a need of Fuzzy system is necessary. Using Fuzzy numbers in TOPSIS for criteria analysis make it simple for evaluation. Hence, Fuzzy TOPSIS is simple, realistic form of modelling and Compensatory method which include or exclude alternative solutions based on hard cut-offs.

2.1 Definitions

The definitions of fuzzy TOPSIS have been adapted from sources. These definitions are presented as follows.

Definition 1: A fuzzy set $\bar{a}$ in a universe of discourse X is characterized by a membership function $\mu\bar{a}(x)$ that maps each element x in X to a real number in the interval [0, 1]. The function value $\mu\bar{a}(x)$ is termed the grade of membership of x in $\bar{a}$. The nearer the value of $\mu\bar{a}(x)$ to unity, the higher the grade of membership of x in $\bar{a}$.

Definition 2: A triangular fuzzy number is represented as a triplet $\bar{a} = (a1, a2, a3)$. The membership function $\mu\bar{a}(x)$ of triangular fuzzy number $\bar{a}$.

A fuzzy set $\bar{a}$, membership function $\mu\bar{a}$ that maps each element x in X to a real number in the interval [0, 1]. A triangular fuzzy number is represented $\bar{a} = (a1, a2, a3)$

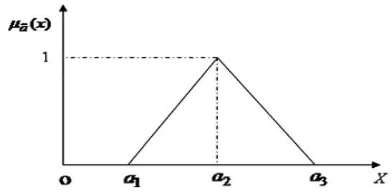

Fig -1: Triangular Fuzzy number system

$a2$gives the maximal grade of μ $a$that μ a=1
$a1$gives the minimal grade of μ $a$that μ a=0
$a1$and $a3$are the lower and upper bounds of the available area for the evaluation data.

$$\mu\bar{a}(x)=\begin{cases} x - a1 / a2 - a1 & if\ a1 \le x \le a2 \\ a3 - x / a3 - a2 & if\ a2 \le x \le a3 \\ 0 & Otherwise \end{cases}$$

$$\mu\bar{a}(x)=\begin{cases} 0, & x < a1 \\ x - a1 / a2 - a1 & if\ a2 \ge x \ge a1 \\ x - a2 / a3 - a2 & if\ a2 \ge x \ge a3 \\ 0 & x > a3 \end{cases} \quad .....(1)$$

2.2 The distance between fuzzy triangular numbers

Let $\bar{a} = (a1, a\bar{2}, a3)$ and $b = (b1, b2, b3)$ be two triangular Fuzzy numbers. The distance between them is given using the vertex method by:

$$d(\bar{a}, \bar{b}) = \sqrt{1/3\,[(a1 - b1)^2 + (a2 - b2)^2 + (a3 - b3)^2]} \quad .....(2)$$

2.3 Fuzzy Set Theory

Conversion scales are applied to transform the linguistic terms into fuzzy numbers. Usually apply a scale of 1 to 9 for rating the criteria and the alternatives. The intervals are chosen so as to have a uniform representation from 1 to 9 for the fuzzy triangular numbers used for the five linguistic ratings.

Table -1
FUZZY RATINGS FOR LINGUISTIC VARIABLES

Fuzzy number	Alternative Assessment	QA Weights
(1,1,3)	Very Poor (VP)	Very Low (VL)
(1,3,5)	Poor (P)	Low (L)
(3,5,7)	Fair (F)	Medium (M)
(5,7,9)	Good (G)	High (H)
(7,9,9)	Very Good (VG)	Very High (VH)

3. FUZZY TOPSIS METHOD

The technique called fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Situation) can be used to evaluate multiple alternatives against the selected criteria. In the TOPSIS approach an alternative that is nearest to the Fuzzy Positive Ideal Solution (FPIS) and farthest from the Fuzzy Negative Ideal Solution (FNIS) is chosen as optimal. An FPIS is composed of the best performance values for each alternative whereas the FNIS consists of the worst performance values. Here, we have presented relevant steps of fuzzy TOPSIS as below.

3.1 Steps to illustrate Fuzzy TOPSIS method using an example scenario

Let's say, the decision group has K members and the i^{th} alternative on j^{th} criterion. The fuzzy rating and importance weight of the k^{th} decision maker, about the i^{th} alternative on j^{th} criterion.

Step 1: ALTERNATIVES RATINGS B Y DECISION MAKERS

Here, we have two alternatives such as A1 and A2 for comparison with four criteria such as C1, C2, C3 and C4 also we have three decision makers namely DM1 and DM2. Now, decision makers rate the alternatives as shown in table-2.

Table -2
ALTERNATIVE RATING

Criteria	A1		A2	
	DM1	DM2	DM1	DM2
C1	F	F	G	G
C2	VG	VG	G	VG
C3	P	F	P	P
C4	F	F	P	P

Step 2: CRITERIA WEIGHTAGE BY DECISION MAKERS

Table -3
CRITERIA WEIGHTAGE

Criteria	DM1	DM2
C1	H	M
C2	VH	H
C3	VH	H
C4	M	L

Step 3: APPLY FUZZY NUMBERS (REFER TABLE NO. 1)

Table -4
FUZZY NUMBERS FOR ALTERNATIVE RATING

Criteria	A1		A2	
	DM1	DM2	DM1	DM2
C1	F(3,5,7)	F(3,5,7)	G(5,7,9)	G(5,7,9)
C2	VG(7,9,9)	VG(7,9,9)	G(5,7,9)	VG(7,9,9)
C3	P(1,3,5)	F(3,5,7)	P(1,3,5)	P(1,3,5)
C4	F(3,5,7)	F(3,5,7)	P(1,3,5)	P(1,3,5)

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Table -5
FUZZY NUMBERS FOR CRITERIA WEIGHTAGE

Criteria	DM1	DM2
C1	H(5,7,9)	M(3,5,7)
C2	VH(7,9,9)	H(5,7,9)
C3	VH(7,9,9)	H(5,7,9)
C4	M(3,5,7)	L(1,3,5)

Step 4: AGGREGATED ALTERNATIVE AND CRITERIA WEIGHTAGE FUZZY DECISION MATIX

$\bar{x}^k_{ij} = (a^k_{ij}, b^k_{ij}, c^k_{ij})$
$w^k_j = (w^k_{j1}, w^k_{j2}, w^k_{j3})$

$$a_{ij} = \min_k\{a^k_{ij}\},\ b_{ij} = 1/K \sum_{k=1}^{K} b^k_{ij},\ c_{ij} = \max_k\{c^k_{ij}\} \quad .....(3)$$

$$w_{j1} = \min_k\{w_j k1\},\ w_{j2} = 1/K \sum_{k=1}^{K} w_j k2,\ w_{j3} = \max_k\{w_j k3\} \quad .....(4)$$

Table -6
AGGREGATED FUZZY DECISION MATRIX FOR ALTERNATIVE

Criteria	A1	A2
C1	(3.000,5.000,7.000)	(5.000,7.000,9.000)
C2	(7.000,9.000,9.000)	(5.000,8.000,9.000)
C3	(1.000,4.000,7.000)	(1.000,3.000,5.000)
C4	(3.000,5.000,7.000)	(1.000,3.000,5.000)

Example – C1 A1 – from Table -4 (use Eq.3)

1. $a_{ij} = \min_k\{a^k_{ij}\}$ = 3.000 [i.e. minimum value of first place (**3**,5,7 & **3**,5,7)]
2. $b_{ij} = 1/K \sum_{k=1}^{K} b^k_{ij}$ = 5.000 [i.e. average of values at middle place (3,**5**,7 & 3,**5**,7)]
3. $c_{ij} = \max_k\{c^k_{ij}\}$ = 7.000 [i.e. maximum value of last place (3,5,**7** & 3,5,**7**)]

Table -7
AGGREGATED FUZZY DECISION MATRIX FOR CRITERIA WEIGHTAGE

Criteria	Agg. Weightage
C1	(3.000,6.000,9.000)
C2	(5.000,8.000,9.000)
C3	(5.000,8.000,9.000)
C4	(1.000,4.000,7.000)

[Same procedure of work as did for table -6 (use Eq.4)]

Step 5: FUZZY MULTI CRITERIA GROUP DECISION MAKING (GDM) AND PROCESS OF NORMALIZING

As we are working on various criteria for decision making, some might be benefit criteria and some might be cost criteria. Aim is to maximize benefit and minimize the cost. A fuzzy multi criteria Group Decision Making (GDM) problem which can be concisely expressed in matrix format as:

$$\bar{D} = \begin{matrix} & C_1 & C_2 & & C_n \\ A_1 & \bar{x}_{11} & \bar{x}_{12} & \dots & \bar{x}_{1n} \\ A_2 & \bar{x}_{21} & \bar{x}_{22} & \dots & \bar{x}_{2n} \\ A_m & \bar{x}_{m1} & \bar{x}_{m2} & \dots & \bar{x}_{mn} \end{matrix} \quad .....(5)$$

$$\bar{W} = (\bar{w}_1, \bar{w}_2, \dots, \bar{w}_n) \quad .....(6)$$

NORMALIZING

$$\bar{R} = [\bar{r}_{ij}]_{m\times n},\ i = 1, 2, \dots, m;\ j = 1, 2, \dots, n \quad .....(7)$$

$$\left.\begin{array}{l}\bar{r}_{ij} = (a_{ij} / c^*_j,\ b_{ij} / c^*_j,\ c_{ij} / c^*_j) \text{ and} \\ c^*_j = \max_i c_{ij} \text{ (benefit criteria)}\end{array}\right\} \quad .....(8)$$

$$\left.\begin{array}{l}\bar{r}_{ij} = (\bar{a}_j / c_{ij},\ \bar{a}_j / b_{ij},\ \bar{a}_j / a_{ij}) \text{ and} \\ \bar{a}_j = \min_i a_{ij} \text{ (cost criteria)}\end{array}\right\} \quad .....(9)$$

Table -8
NORMALIZED AGGREGATED FUZZY DECISION MATRIX FOR ALTERNATIVE

Criteria	A1	A2
C1	(0.429,0.600,1.000)	(0.333,0.429,0.600)
C2	(0.778,1.000,1.000)	(0.556,0.889,1.000)
C3	(0.143,0.571,1.000)	(0.143,0.429,0.714)
C4	(0.429,0.714,1.000)	(0.143,0.429,0.714)

[The ranges of normalized triangular fuzzy numbers belong to (0, 1)]

Weighted Normalized Fuzzy Decision Matrix,

$$\bar{P} = [\bar{p}_{ij}] \text{ where } \bar{p}_{ij} = \bar{r}_{ij} \times \bar{w}_j \quad .....(10)$$

Table -9
WEIGHTED NORMALIZED FUZZY DECISION MATRIX

Criteria	A1	A2
C1	(1.287,3.600,9.000)	(0.999,2.574,5.400)
C2	(3.890,8.000,9.000)	(2.780,7.112,9.000)
C3	(0.715,4.568,9.000)	(0.715,3.432,6.426)
C4	(0.429,2.856,7.000)	(0.143,1.716,4.998)

[Refer Table- 7, Table- 8 and Eq.10]

Step 6: FPIS AND FNIS

$A^+ = (p^+_1, p^+_2, \dots, p^+_n)$ where
$$p^+_j = \max_i \{p_{ij3}\},\ i = 1, 2 \dots, m;\ j = 1, 2, \dots, n \quad .....(11)$$
$A^- = (p^-_1, p^-_2, \dots, p^-_n)$ where
$$p^-_j = \min_i \{p_{ij}1\},\ i = 1, 2 \dots, m;\ j = 1, 2, \dots, n \quad .....(12)$$

Select the maximum value from each row as p^+ and select the minimum value from each row as p^-.

$A^+ = [p^+_1(9,9,9,), \; p^+_2(9,9,9), p^+_3(9,9,9), p^+_4(7,7,7)]$ .....*(13)*

$A^- = [p^-_1(0.999,0.999,0.999), p^-_2(2.780,2.780,2.780), p^-_3(0.715,0.715,0.715), p^-_4(0.143,0.143,0.143)]$ .....*(14)*

Step 7: FPIS AND FNIS for each criteria

FPIS (A1) = d (pij, p1+) and
FNIS (A1) = d (pij, p1-)
Here, we have to use equation no. (2) To find distance of each criteria from FPIS AND FNIS for both alternatives.
So, using equation 2, 13, 14 and table no. 9 we get,

Table -10
DISTANCE OF CRITERIA OF EACH ALTERNATIVE FROM FPIS AND FNIS

Criteria	FPIS(A1)	FPIS(A2)	FNIS(A1)	FNIS(A2)
C1	5.436	6.279	4.860	2.698
C2	3.006	3.753	4.732	4.376
C3	5.425	5.952	5.275	3.651
C4	4.485	5.129	4.261	2.946

Example - from table 9

Criteria	A1
C1	(0.999,2.574,5.400)

From eq. 13, $p^+_1(9.000,9.000,9.000)$
From eq. 2
$d(\tilde{a}, \tilde{b}) = \sqrt{1/3\,[(a1 - b1)^2 + (a2 - b2)^2 + (a3 - b3)^2]}$

Now, for (C1-A1+)

$d = \sqrt{1/3\,[(0.999-9.000)^2 + (2.574-9.000)^2 + (5.400-9.000)^2]}$

$= \sqrt{1/3\,[(64.02) + (41.29) + (12.96)]}$

$= \sqrt{[39.423]}$

= 6.279 (likewise use same method to find remaining values of table-10)

Step 8: THE DISTANCE OF EACH WEIGHTED ALTERNATIVE

$d_i^+ = \sum_{j=1}^{n} d(\bar{p}_{ij}, p_j^+)$ .....*(15)*

$d_i^- = \sum_{j=1}^{n} d(\bar{p}_{ij}, p_j^-)$ .....*(16)*

$d_1^+ = 18.352$ $d_2^+ = 21.113$

$d_1^- = 19.128$ $d_2^- = 13.671$

Step 9: CLOSENESS COEFFICIENT OF EACH ALTERNATIVE

$CC_i = d^-_i / (d^-_i + d^+_i)$, i = 1, 2 . . . ,m .....*(17)*

$CC_1 = 19.128 / (18.352+19.128) = 0.510$

$CC_2 = 13.671 / (13.671+ 21.113) = 0.393$

Step 10: RANKING OF EACH ALTERNATIVE

Hence, the ranking order of A1 > A2,
A1 is the Best Choice considering the given criteria.

3.2 Summary

Above we describe the application of FUZZY TOPSIS for a scenario where there are 2 decision makers, 4 evaluation criteria C1 – C4, and rating scale is as shown in Table 1. Key input from decision makers is typically alternative rating and to identify the proper weightage to various criteria. As result of above Fuzzy TOPSIS steps, Closeness coefficients CC_i, of the two A1 and A2 come out to be 0.510 and 0.393 respectively. Hence the ranking order for the alternatives is A1 > A2, that is, A1 is the best choice considering the given criteria. The closeness coefficient scores for alternatives are numeric values and can be further utilized to indicate the degree of inferiority or superiority of the alternatives w.r.t each other.

4. CONCLUSION

As a result, use of optimization techniques in work helps in reducing human efforts. Fuzzy TOPSIS is a method which uses data in any form like numerical and linguistic etc. After data collection and converting it into Fuzzy system mode it reduces chances of errors caused by units of parameters which also creates problem for mathematical calculations. To overcome these challenges Fuzzy TOPSIS is an ideal method. Also using software like MATLAB and its Fuzzy tools we can create a model for Fuzzy TOPSIS. So, Fuzzy TOPSIS is a simple, superior and full proof solution to multi criteria decision making.

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[4] FUZZY TOPSIS by Mohammad Khalily Dermany

BIOGRAPHIES

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