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KINEMATIC ANALYSIS OF VARIOUS ROBOT CONFIGURATIONS

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* * *

Abstract - *Robots are very powerful elements of today's industry and they are capable of performing many different task and operations precisely and do not require common safety and comfort element that humans need, however it takes much efforts and many resource to make a robot function properly. Robotic arms are widely used in industrial manufacturing. There is no doubt that robots have increased in capability and performance through improved mechanisms, controller, software development, sensing, drive systems, and materials. The goal of this study is to analyze forward and inverse kinematics of robot manipulators. The study includes use of D-H parameters for studying of both DK and IK. The study models robot kinematics for 2R, 3R, 3R-1P, 5R, 6R using algebraic method along with RoboAnalyser and MATLAB. All results of these methods are compared and validated.*

Key Words: Forward and Inverse Kinematics, Robot Manipulator, D-H parameters, Arm Matrix, RoboAnalyser, MATLAB.

1. INTRODUCTION

Robot kinematics applies geometry to the study of the movement of multi-degree of freedom kinematic chains that form the structure of robotic systems. The emphasis on geometry means that the links of the robot are modelled as rigid bodies and its joints are assumed to provide pure rotation or translation.

Robot kinematics studies the relationship between the dimensions and connectivity of kinematic chains and the position, velocity and acceleration of each of the links in the robotic system, in order to plan and control movement and to compute actuator forces and torques. The relationship between mass and inertia properties, motion, and the associated forces and torques is studied as part of robot dynamics.

Forward kinematics uses the kinematic equations of a robot to compute the position of the end-effector from specified values for the joint parameters. The reverse process that computes the joint parameters that achieve a specified position of the end-effector is known as inverse kinematics. [5]

The controlling of robot manipulator has been challenging with higher DOF. Position and orientation analysis of robotic manipulator is an essential step to design and control. A robot manipulator consist a set of links connected together either in serial or parallel manner. The FK analysis is simple to do analysis of model and calculate the position using the joint angle. But the challenge is to analyze the IK solution using the position. So aim is to study complexity of the IK which increases with increase in the DOF. In this case we would be studying robot configurations i.e. 2R, 3R, 3R-1P, 5R, 6R where R and P stands for revolute and prismatic joints. The main motive of the study is to calculate the robot parameters i.e. study forward and inverse kinematics using algebraic method and then validate this calculations with the outputs from RoboAnalyser and MATLAB.

2. LITERATURE REVIEW

The study of forward kinematics is easy as its analysis is simple to do. The challenge is to do analysis of inverse kinematics. The study of inverse kinematics can be done by various means. These various means i.e. algebraic method [1], [3], [4], using software tools such as RoboAnalyser and MATLAB [2] are studied by various authors. The algebraic method is the traditional way to study the kinematics of robot manipulator whereas RoboAnalyser and MATLAB are used to validate these mathematical results. Here we would be using all three ways to compare their results and validate the results.

3. METHODOLOGY

The steps followed to do this study are named and explained in the next lines along with flowchart as in Fig. 1:

1. Study the robot kinematics both forward and inverse kinematics of robot manipulators.
2. Collect information regarding forward and inverse kinematics for various robot configuration under study i.e. 2R, 3R, 3R-1P, 5R and 6R.
3. Collect formulae for this configurations to calculate their parameters for direct and inverse kinematics by algebraic method.
4. Study the RoboAnalyser for various robot configuration and using the same calculate the arm matrix and the configurations of the robot for direct and inverse kinematics.
5. Just like step 4 study the MATLAB for various robot configuration and using the same calculate the arm matrix and the configurations of the robot for direct and inverse kinematics.

6. Next compare the results of algebraic method and software results for each of the robot configuration.
7. Finally validate the results for all three ways of studying DK and IK.

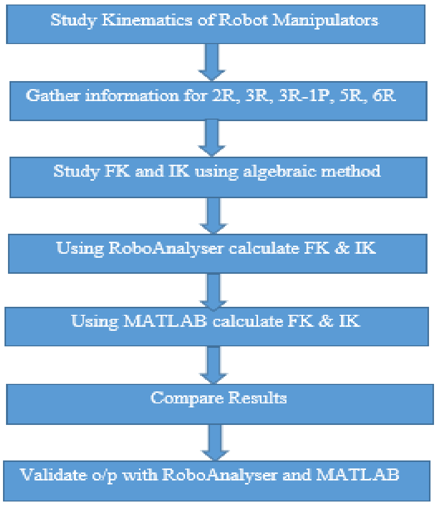


Fig-1: Work Methodology

4. KINEMATIC ANALYSIS OF VARIOUS ROBOT MANIPULATORS

4.1 2R Mechanism [Two Axis Planar Articulated Robot Arm] [5]

4.1.1 Algebraic Method

4.1.1.1 3D Model

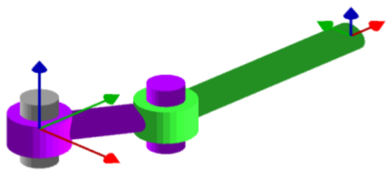


Fig-2: 3D Model of Two Axis Planar Articulated Robot Arm

4.1.1.2 Kinematic Parameters Table

Table-1: Kinematics Parameter Table

Joints	Rows of Table	Type of Joint	θ k	d k	a k	α k	SHP
1	$\theta_1, d_1, a_1, \alpha_1$	Base	θ_1	0	a_1	0	60^0
2	$\theta_2, d_2, a_2, \alpha_2$	Shoulder	θ_2	0	a_2	0	-60^0

4.1.1.3 Arm Matrix

$$T_0^2 = \begin{bmatrix} C_{12} & -S_{12} & 0 & a_1c_1 + a_2c_{12} \\ S_{12} & C_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.1.1.4 Calculations

Forward Kinematics

Joint	θ	d	a	α
Base	θ_1	0	a_1	0
Shoulder	θ_2	0	a_2	0

a_1=100 mm $\theta_1=60^0$

a_2=200 mm $\theta_2=30^0$

Solution

$P_x = a_1c_1+a_2c_{12}$

$P_y = a_1s_1+a_2s_{12}$

P_x = 50 mm P_y = 286.6025 mm

Inverse Kinematics

$$\begin{bmatrix} 0 & -1 & 0 & 50 \\ 1 & 0 & 0 & 286.6025 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution

$$\theta_2 = \pm cos\left[\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2 * a_1 * a_2}\right]$$

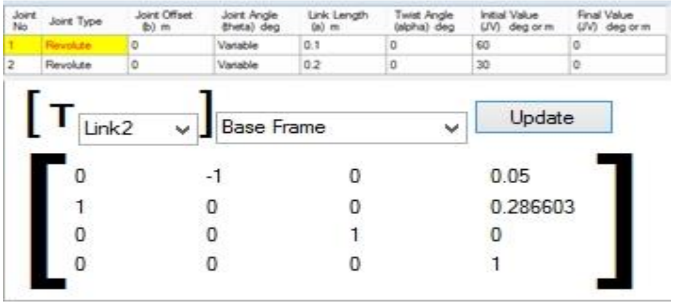
$$\theta_1 = \tan^{-1}\left[\frac{w_2(a_1 + a_2c_2) - w_1s_2a_2}{w_1(a_1 + a_2c_2) + w_2s_2a_2}\right]$$

$\theta_1 = 59.99999863^0$

$\theta_2 = \pm 30.00006631^0$

4.1.2 Using RoboAnalyser

4.1.2.1 Forward Kinematics



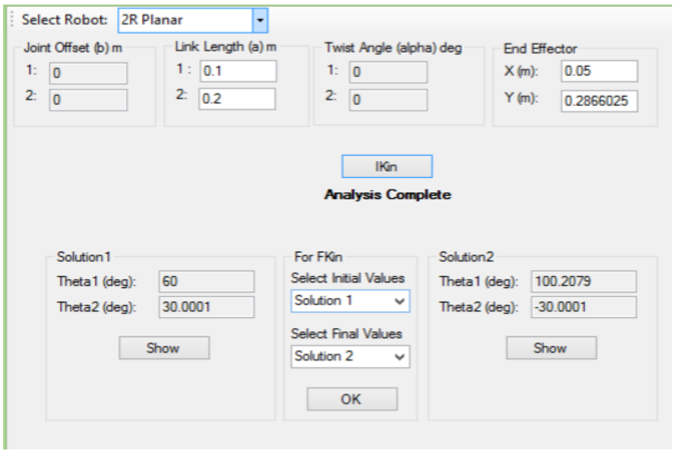
Joint No	Joint Type	Joint Offset (b) m	Joint Angle (theta) deg	Link Length (a) m	Twist Angle (alpha) deg	Initial Value (JV) deg or m	Final Value (JV) deg or m
1	Revolute	0	Variable	0.1	0	60	0
2	Revolute	0	Variable	0.2	0	30	0

[T Link2] Base Frame — Update

$$\begin{bmatrix} 0 & -1 & 0 & 0.05 \\ 1 & 0 & 0 & 0.286603 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig-3: Results of Two Axis Planar Articulated Robot Arm for Direct Kinematics

4.1.2.2 Inverse Kinematics



Select Robot: 2R Planar

Joint Offset (b) m
1: 0
2: 0

Link Length (a) m
1: 0.1
2: 0.2

Twist Angle (alpha) deg
1: 0
2: 0

End Effector
X (m): 0.05
Y (m): 0.2866025

Analysis Complete

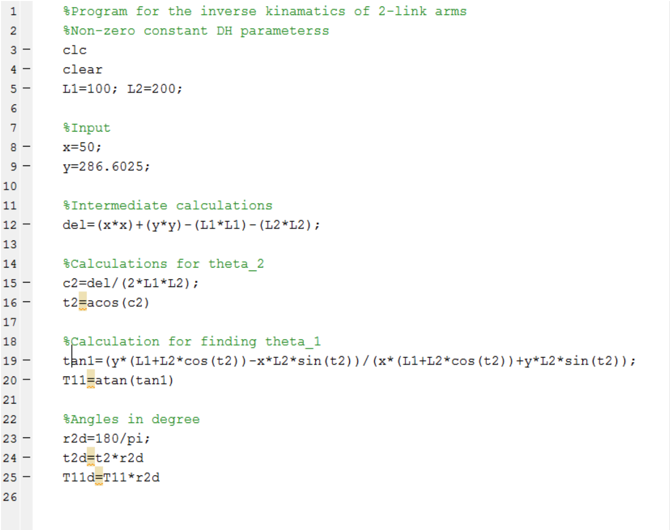
Solution1
Theta1 (deg): 60
Theta2 (deg): 30.0001

For FKin
Select Initial Values: Solution 1
Select Final Values: Solution 2

Solution2
Theta1 (deg): 100.2079
Theta2 (deg): -30.0001

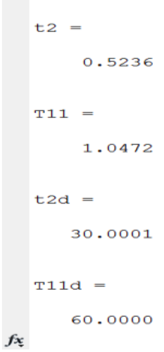
Fig-4: Results of Two Axis Planar Articulated Robot Arm for Inverse Kinematics

4.1.3 Using MATLAB



```
%Program for the inverse kinamatics of 2-link arms
%Non-zero constant DH parameterss
clc
clear
L1=100; L2=200;

%Input
x=50;
y=286.6025;

%Intermediate calculations
del=(x*x)+(y*y)-(L1*L1)-(L2*L2);

%Calculations for theta_2
c2=del/(2*L1*L2);
t2=acos(c2)

%Calculation for finding theta_1
tan1=(y*(L1+L2*cos(t2))-x*L2*sin(t2))/(x*(L1+L2*cos(t2))+y*L2*sin(t2));
T11=atan(tan1)

%Angles in degree
r2d=180/pi;
t2d=t2*r2d
T11d=T11*r2d

```

Fig-5: MATLAB Program for IK of Two Axis Planar Articulated Robot Arm



```
t2 =

    0.5236


T11 =

    1.0472


t2d =

   30.0001


T11d =

   60.0000

fx
```

Fig-6: MATLAB Results for IK of Two Axis Planar Articulated Robot Arm

4.2 3R Mechanism [3-Axis Planar Articulated Arm i.e. Mini Drafter] [5]

4.2.1 Algebraic Method

4.2.1.1 3D Model

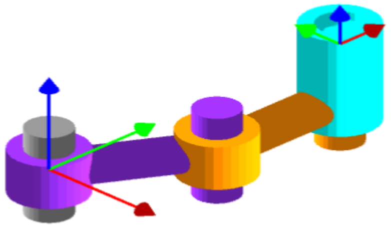


Fig-7: 3D Model of 3-Axis Planar Articulated Arm

4.2.1.2 Kinematics Parameter Table

Table-2: Kinematics Parameter Table

Joint	Rows of Table	Type	θ	d	a	α	SHP
1	θ1,d1,a1,α1	Base	θ1	0	a1	0	60^0
2	θ2,d2,a2,α2	Shoulder	θ2	0	a2	0	-60^0
3	θ3,d3,a3,α3	Roll	θ3	d3	0	0	0

4.2.1.3 Arm Matrix

$$T_0{}^3 = \begin{bmatrix} C_{123} & -S_{123} & 0 & a_1c_1 + a_2c_{12} \\ S_{123} & C_{123} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.2.1.4 Calculations:

Forward Kinematics

Joint	θ	d	a	α
Base	θ1	0	a1	0
Shoulder	θ2	0	a2	0
Roll	θ3	d3	0	0

$a_1=a_2=100$ mm
$d_3=50$ mm
$\theta_1=60^0$
$\theta_2=30^0$
$\theta_3=0^0$

Solution

$$T_0{}^3 = \begin{bmatrix} 0 & -1 & 0 & 50 \\ 1 & 0 & 0 & 186.6025 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P_x = 50$ mm

$P_y = 186.6025$ mm

Inverse Kinematics

$$\begin{bmatrix} 0 & -1 & 0 & 50 \\ 1 & 0 & 0 & 186.6025 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution

$$\theta_2 = \pm cos[\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2 * a_1 * a_2}]$$

$$\theta_1 = \pm \tan^{-1}[\frac{w_2(a_1 + a_2 c_2) + w_1 s_2 a_2}{w_1(a_1 + a_2 c_2) - w_2 s_2 a_2}]$$

$\theta_{123} = \theta_1 + \theta_2 + \theta_3$
$\theta_3 = \theta_{123} - \theta_1 + \theta_2$

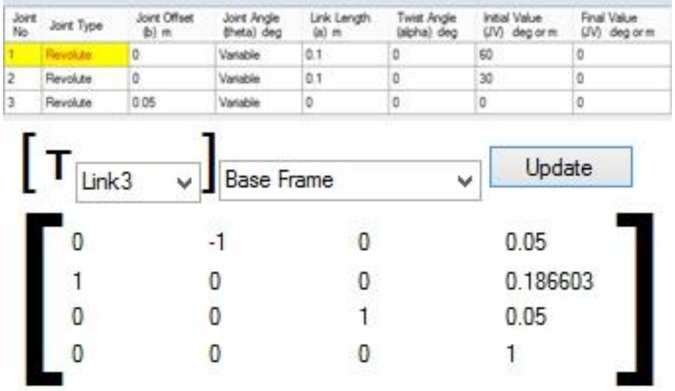
$\theta_1 = 59.9999938^0$

$\theta_2 = \pm 30.00008634^0$

$\theta_3 = 0^0$

4.2.2 Using RoboAnalyser

4.2.2.1 Forward Kinematics



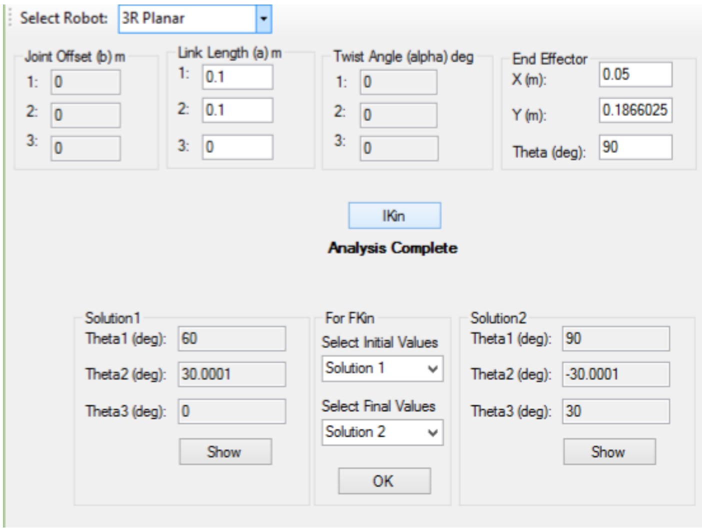
Joint No	Joint Type	Joint Offset (b) m	Joint Angle (theta) deg	Link Length (a) m	Twist Angle (alpha) deg	Initial Value (JV) deg or m	Final Value (JV) deg or m
1	Revolute	0	Variable	0.1	0	60	0
2	Revolute	0	Variable	0.1	0	30	0
3	Revolute	0.05	Variable	0	0	0	0

[T Link3] Base Frame

$$\begin{bmatrix} 0 & -1 & 0 & 0.05 \\ 1 & 0 & 0 & 0.186603 \\ 0 & 0 & 1 & 0.05 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig-8: Results of 3-Axis Planar Articulated Arm for Forward Kinematics

4.2.2.2 Inverse Kinematics



Select Robot: 3R Planar

Joint Offset (b) m
1: 0
2: 0
3: 0

Link Length (a) m
1: 0.1
2: 0.1
3: 0

Twist Angle (alpha) deg
1: 0
2: 0
3: 0

End Effector
X (m): 0.05
Y (m): 0.1866025
Theta (deg): 90

Analysis Complete

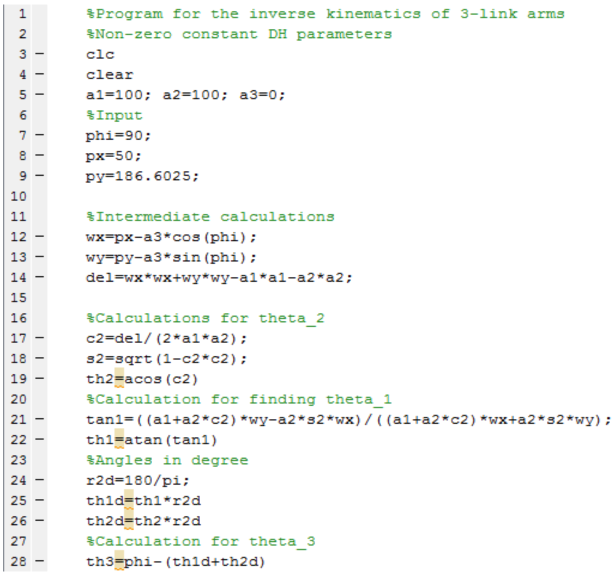
Solution1
Theta1 (deg): 60
Theta2 (deg): 30.0001
Theta3 (deg): 0

For FKin
Select Initial Values: Solution 1
Select Final Values: Solution 2

Solution2
Theta1 (deg): 90
Theta2 (deg): -30.0001
Theta3 (deg): 30

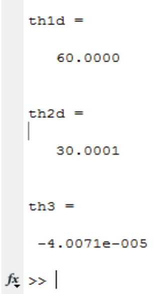
Fig-9: Results of 3-Axis Planar Articulated Arm for Inverse Kinematics

4.2.3 Using MATLAB



```
%Program for the inverse kinematics of 3-link arms
%Non-zero constant DH parameters
clc
clear
a1=100; a2=100; a3=0;
%Input
phi=90;
px=50;
py=186.6025;

%Intermediate calculations
wx=px-a3*cos(phi);
wy=py-a3*sin(phi);
del=wx*wx+wy*wy-a1*a1-a2*a2;

%Calculations for theta_2
c2=del/(2*a1*a2);
s2=sqrt(1-c2*c2);
th2=acos(c2)
%Calculation for finding theta_1
tan1=((a1+a2*c2)*wy-a2*s2*wx)/((a1+a2*c2)*wx+a2*s2*wy);
th1=atan(tan1)
%Angles in degree
r2d=180/pi;
th1d=th1*r2d
th2d=th2*r2d
%Calculation for theta_3
th3=phi-(th1d+th2d)
```

Fig-10: MATLAB program for IK of 3-Axis Planar Articulated Arm



```
th1d =

   60.0000


th2d =

   30.0001


th3 =

  -4.0071e-005

>> 
```

Fig-11: MATLAB Results for IK of 3-Axis Planar Articulated Arm

4.3 3R-1P Mechanism [4-Axis Adept-1 SCARA Robot] [5]

4.3.1 Algebraic Method

4.3.1.1 3D Model

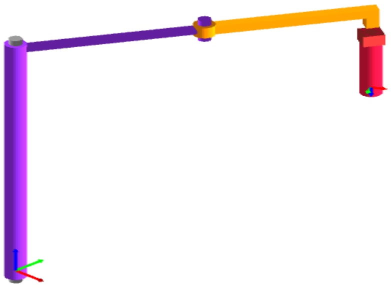


Fig-12: 3D Model of 4-Axis Adept-1 SCARA Robot

4.3.1.2 Kinematic Parameters Table

Table-3: Kinematics Parameter Table

Joints	Rows of KP Table	Types of Joint	θk	dk	ak	αk
1	θ1,d1,a1,α1	Base	θ1	d1	a1	±π
2	θ2,d2,a2,α2	Elbow	θ2	0	a2	0
3	θ3,d3,a3,α3	Vertical Extension	$\theta3=0^0$	d3	0	0
4	θ4,d4,a4,α4	Roll	θ4	d4	0	0

4.3.1.3 Arm Matrix

$$T_0^4 = \begin{bmatrix} C_{124} & S_{124} & 0 & a_1c_1 + a_2c_{12} \\ S_{124} & -C_{124} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & d_1 - q_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.3.1.4 Calculations:

Forward Kinematics

Joint	θ	d	a	α
Base	θ1	877	425	±π
Shoulder	θ2	0	375	0
VE	0	d3	0	0
Roll	θ4	200	0	0

$\theta_1=60^0$, $\theta_2=0^0$
$\theta_4=30^0$
$d_3=100$ mm

Solution

$$T_0^4 = \begin{bmatrix} 0.866 & 0.5 & 0 & 400 \\ 0.5 & -0.866 & 0 & 692.82 \\ 0 & 0 & -1 & 577 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P_x = 400$ mm
$P_y = 692.82$ mm
$P_z = 577$ mm

Inverse Kinematics

$$\begin{bmatrix} 0.866 & 0.5 & 0 & 400 \\ 0.5 & -0.866 & 0 & 692.82 \\ 0 & 0 & -1 & 577 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution

$$\theta_2 = \pm cos[\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2 * a_1 * a_2}]$$

$$\theta_4 = \theta_{124} - \theta_1 - \theta_2$$

$$\theta_{124} = tan^{-1}\left[\frac{R_{21}}{R_{11}}\right]$$

$$\theta_4 = \theta_{124} - \theta_1 - \theta_2$$

p_3 or $w_3=d_1-q_3-d_4$

q_3 i.e $d_3 = d_1-d_4-w_3$

$$\theta_1 = \pm 60.00002314°$$

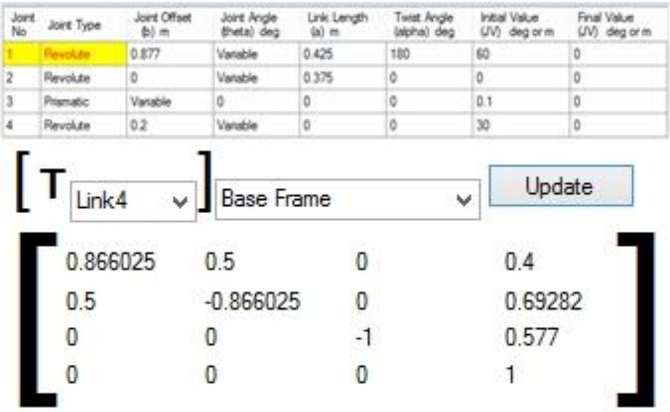
$$\theta_2 = \pm 0.0025°$$

$$\theta_4 = 29.99747686°$$

d_3=100 mm

4.3.2 Using RoboAnalyser

4.3.2.1 Forward Kinematics



Joint No	Joint Type	Joint Offset (b) m	Joint Angle (theta) deg	Link Length (a) m	Twist Angle (alpha) deg	Initial Value (JV) deg or m	Final Value (JV) deg or m
1	Revolute	0.877	Variable	0.425	180	60	0
2	Revolute	0	Variable	0.375	0	0	0
3	Prismatic	Variable	0	0	0	0.1	0
4	Revolute	0.2	Variable	0	0	30	0

$$\begin{bmatrix} 0.866025 & 0.5 & 0 & 0.4 \\ 0.5 & -0.866025 & 0 & 0.69282 \\ 0 & 0 & -1 & 0.577 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig-13: Results of 4-Axis Adept-1 SCARA Robot for Forward Kinematics

4.3.2.2 Inverse Kinematics

Due to certain limitations inverse kinematics for this configuration couldn't be completed by RoboAnalyser. So using Arm Matrix from DK we have calculated IK values using algebraic method. Similarly for 5R and 6R is done.

Results

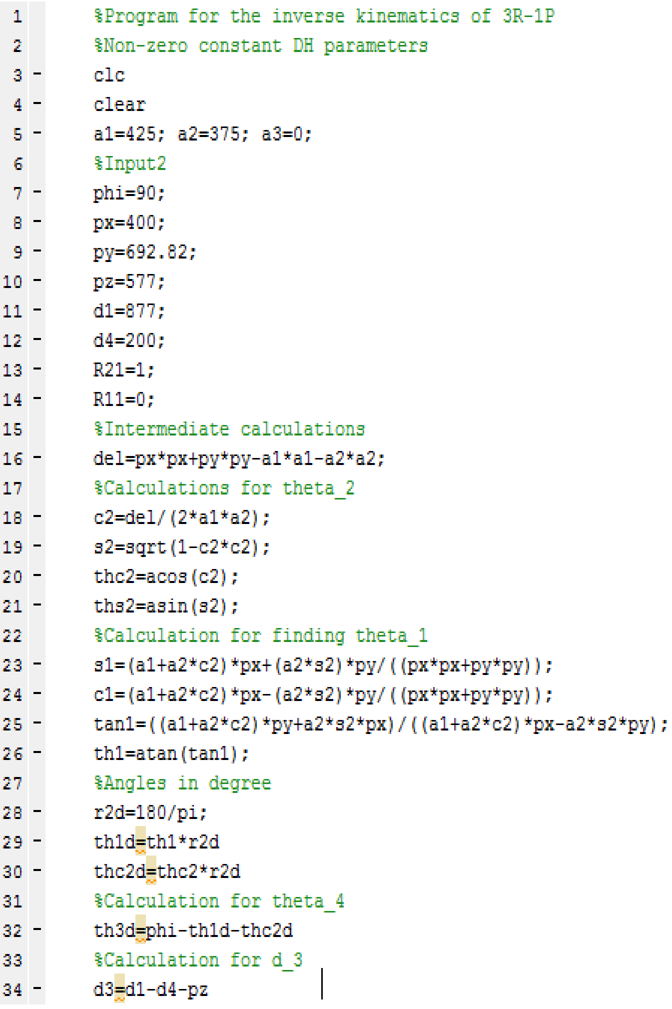
$\theta_1 = 60^0$

$\theta_2 = 0^0$

$\theta_4 = 30.0001^0$

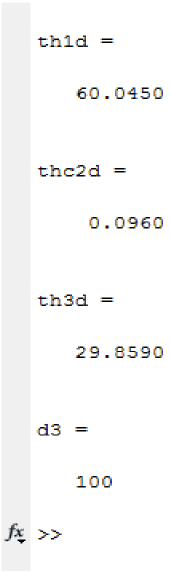
d_3=100 mm

4.6.3.3 Using MATLAB



```
%Program for the inverse kinematics of 3R-1P
%Non-zero constant DH parameters
clc
clear
a1=425; a2=375; a3=0;
%Input2
phi=90;
px=400;
py=692.82;
pz=577;
d1=877;
d4=200;
R21=1;
R11=0;
%Intermediate calculations
del=px*px+py*py-a1*a1-a2*a2;
%Calculations for theta_2
c2=del/(2*a1*a2);
s2=sqrt(1-c2*c2);
thc2=acos(c2);
ths2=asin(s2);
%Calculation for finding theta_1
s1=(a1+a2*c2)*px+(a2*s2)*py/((px*px+py*py));
c1=(a1+a2*c2)*px-(a2*s2)*py/((px*px+py*py));
tan1=((a1+a2*c2)*py+a2*s2*px)/((a1+a2*c2)*px-a2*s2*py);
th1=atan(tan1);
%Angles in degree
r2d=180/pi;
th1d=th1*r2d
thc2d=thc2*r2d
%Calculation for theta_4
th3d=phi-th1d-thc2d
%Calculation for d_3
d3=d1-d4-pz
```

Fig-14: MATLAB program for IK of 4-Axis Adept-1 SCARA Robot



```
th1d =

   60.0450


thc2d =

    0.0960


th3d =

   29.8590


d3 =

   100

fx >>
```

Fig-15: MATLAB Results for IK of 4-Axis Adept-1 SCARA Robot

4.4 5R Mechanism [Five Axis Articulated Microbot α-II Robot Arm] [5]

4.4.1 Algebraic Method

4.4.1.1 3D Model

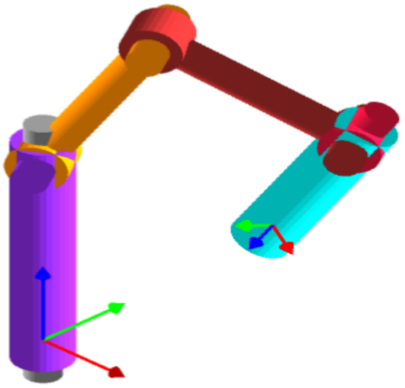


Fig-16: 3D Model of Five Axis Articulated Microbot α-II Robot Arm

4.4.1.2 Kinematics Parameter Table

Table-4: Kinematics Parameter Table

Joints	Rows of KP Table	Types of Joint	θk	dk	ak	αk
1	θ1,d1,a1,α1	Base	θ1	d1	0	-90^0
2	θ2,d2,a2,α2	Shoulder	θ2	0	a2	0
3	θ3,d3,a3,α3	Elbow	θ3	0	a3	0
4	θ4,d4,a4,α4	Tool Pitch	θ4	0	a4	-90^0
5	θ5,d5,a5,α5	Tool Roll	Θ5	d5	0	0

4.4.1.3 Arm Matrix

$T_0^5=$

$$\begin{bmatrix} C_1C_{234}C_5+S_1S_5 & -C_1S_5C_{234}+S_1C_5 & -C_1S_{234} & C_1(a_2c_2+a_3c_{23}-d_5S_{234}) \\ S_1C_{234}C_5-C_1S_5 & -S_1S_5C_{234}-C_1C_5 & -S_1S_{234} & S_1(a_2c_2+a_3c_{23}-d_5S_{234}) \\ -C_5S_{234} & S_{234}S_5 & -C_{234} & d_1-a_2s_2-a_3s_{23}-d_5C_{234} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.4.1.4 Calculations

Forward Kinematics

Type of Joint	θ	d	a	α
Base	θ1	215	0	-90^0
Shoulder	θ2	0	177.8	0
Elbow	θ3	0	177.8	0
Tool Pitch	θ4	0	a_4	-90^0
Tool Roll	Θ5	129.5	0	0

$\theta_1=30^0$
$\theta_2=-60^0$
$\theta_3=90^0$
$\theta_4=0^0$
$\theta_5=45^0$
$a_4=0$

Solution

$$T_0^5=\begin{bmatrix} 0.8839 & -0.1768 & -0.4330 & 154.265 \\ -0.3062 & -0.9186 & -0.25 & 89.065 \\ -0.3536 & 0.3536 & -0.8660 & 167.929 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P_x = 154.265$ mm
$P_y = 89.065$ mm
$P_z = 167.929$ mm

Inverse Kinematics

$$\begin{bmatrix} 0.8839 & -0.1768 & -0.4330 & 154.265 \\ -0.3062 & -0.9186 & -0.25 & 89.065 \\ -0.3536 & 0.3536 & -0.8660 & 167.929 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution

$$\theta_1 = tan^{-1}(\frac{w_2}{w_1})$$

$$\theta_{234} = tan^{-1}[\frac{-(C_1 w_4 + S_1 w_5)}{-w_6}]$$

$$-C_{234} = -0.8660$$

$$b_2 = d_1 - d_5 C_{234} - w_3$$

$$b_1 = C_1 w_1 + S_1 w_2 + d_5 s_{234}$$

$$\theta_3 = \pm cos^{-1}[\frac{b_1^2 + b_2^2 - a_2^2 - a_3^3}{2a_2 a_3}]$$

$$\theta_2 = \pm cos^{-1}[\frac{(a_2 + a_3 C_3)b_1 + (a_3 S_3)b_2}{||b||^2}]$$

$$||b||^2 = a_2^2 + a_3^2 + 2 * a_2 * a_3 * C_3$$

$$\theta_{234} = \theta_2 + \theta_3 + \theta_4$$

$$\theta_4 = \theta_{234} - \theta_2 - \theta_3$$

$$\theta_5 = tan^{-1}[\frac{R_{32}}{-R_{31}}]$$

$$\theta_1 = 30.00001692$$

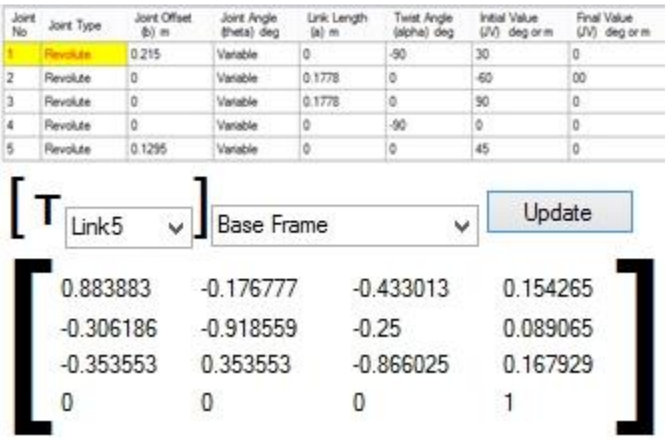
$$\theta_3 = 89.99974525°$$

$$\theta_2 = -59.99988837°$$

$$\theta_4 = 0°$$

4.4.2 Using RoboAnalyser

4.4.2.1 Forward Kinematics



Joint No	Joint Type	Joint Offset (b) m	Joint Angle (theta) deg	Link Length (a) m	Twist Angle (alpha) deg	Initial Value (JV) deg or m	Final Value (JV) deg or m
1	Revolute	0.215	Variable	0	-90	30	0
2	Revolute	0	Variable	0.1778	0	-60	00
3	Revolute	0	Variable	0.1778	0	90	0
4	Revolute	0	Variable	0	-90	0	0
5	Revolute	0.1295	Variable	0	0	45	0

$$\begin{bmatrix} 0.883883 & -0.176777 & -0.433013 & 0.154265 \\ -0.306186 & -0.918559 & -0.25 & 0.089065 \\ -0.353553 & 0.353553 & -0.866025 & 0.167929 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

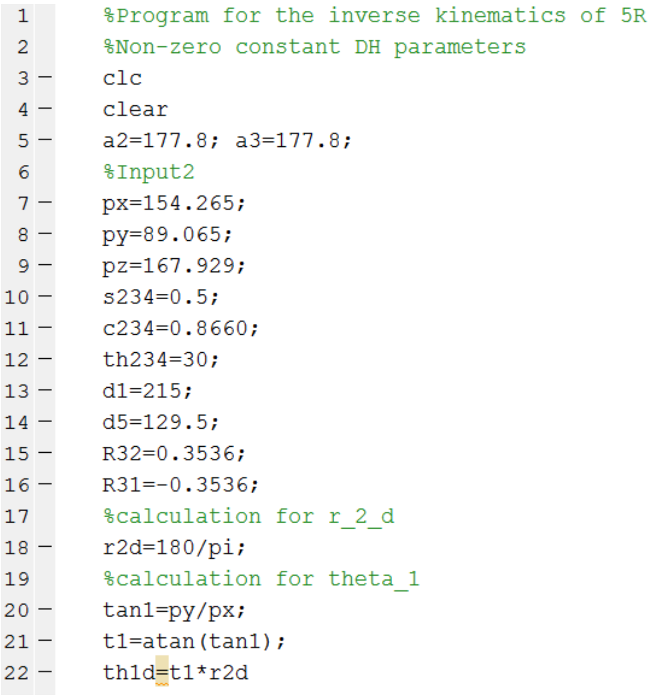
Fig-17: Results of Five Axis Articulated Microbot α-II Robot Arm for Forward Kinematics

3.4.2.2 Inverse Kinematics

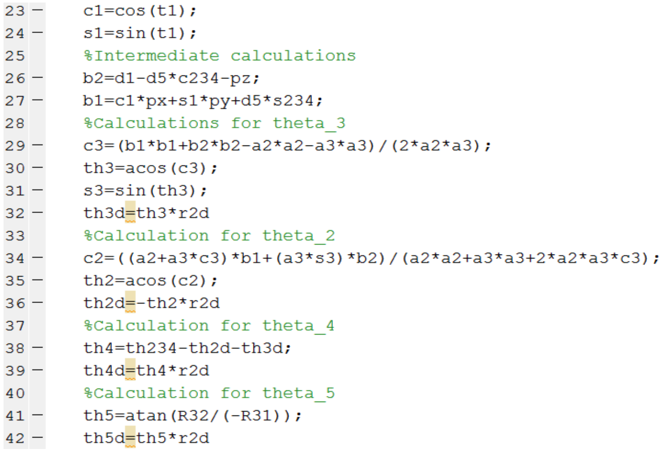
Results

$\theta_1 = 30.0001^0$
$\theta_2 = -59.999^0$
$\theta_3 = 90^0$
$\theta_4 = 0^0$
$\theta_5 = 45^0$

4.4.3 Using MATLAB

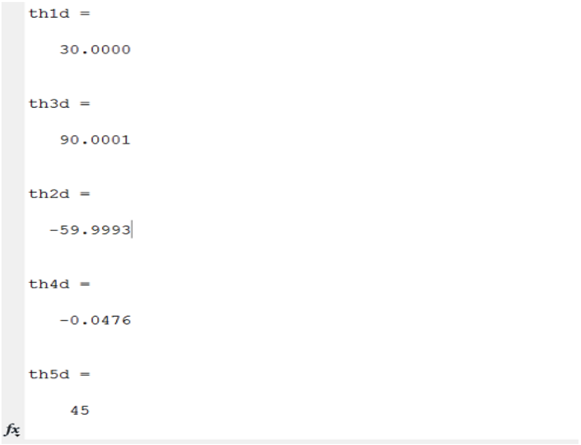


```
%Program for the inverse kinematics of 5R
%Non-zero constant DH parameters
clc
clear
a2=177.8; a3=177.8;
%Input2
px=154.265;
py=89.065;
pz=167.929;
s234=0.5;
c234=0.8660;
th234=30;
d1=215;
d5=129.5;
R32=0.3536;
R31=-0.3536;
%calculation for r_2_d
r2d=180/pi;
%calculation for theta_1
tan1=py/px;
t1=atan(tan1);
th1d=t1*r2d
```



```
    c1=cos(t1);
    s1=sin(t1);
    %Intermediate calculations
    b2=d1-d5*c234-pz;
    b1=c1*px+s1*py+d5*s234;
    %Calculations for theta_3
    c3=(b1*b1+b2*b2-a2*a2-a3*a3)/(2*a2*a3);
    th3=acos(c3);
    s3=sin(th3);
    th3d=th3*r2d
    %Calculation for theta_2
    c2=((a2+a3*c3)*b1+(a3*s3)*b2)/(a2*a2+a3*a3+2*a2*a3*c3);
    th2=acos(c2);
    th2d=-th2*r2d
    %Calculation for theta_4
    th4=th234-th2d-th3d;
    th4d=th4*r2d
    %Calculation for theta_5
    th5=atan(R32/(-R31));
    th5d=th5*r2d
```

Fig-18: MATLAB program for IK of Five Axis Articulated Microbot α-II Robot Arm



```
th1d =

   30.0000


th3d =

   90.0001


th2d =

 -59.9993


th4d =

   -0.0476


th5d =

    45
```

Fig-19: MATLAB Results for IK of Five Axis Articulated Microbot α-II Robot Arm

4.5 6R Mechanism [Puma 560 6R Robot] [2],[3]

4.5.1 Algebraic Method

4.5.1.1 3D Model

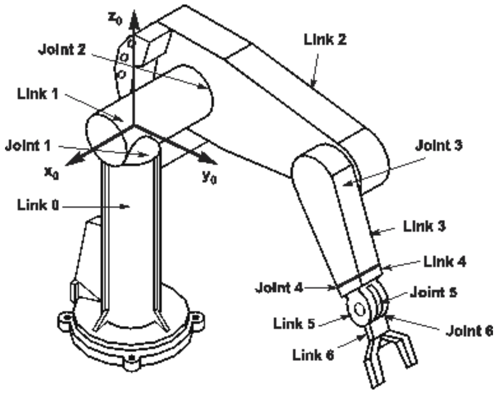


Fig-20: 3D Model of Puma 560 6R Robot

4.5.1.2 Kinematic Parameters Table

Table-5: Kinematics Parameter Table

Joints	Rows of KP Table	Types of Joint	θk	dk	ak	αk
1	θ1,d1,a1,α1	Base	θ1	0	0	-90^0
2	θ2,d2,a2,α2	Shoulder	θ2	d2	a2	0
3	θ3,d3,a3,α3	Elbow	θ3	d3	a3	90^0
4	θ4,d4,a4,α4	Tool Pitch	θ4	d4	0	-90^0
5	θ5,d5,a5,α5	Tool Yaw	θ5	0	0	90^0
6	θ6,d6,a6,α6	Tool Roll	θ6	d6	0	0

4.5.1.3 Arm Matrix

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & \vdots & p_x \\ r_{21} & r_{22} & r_{23} & \vdots & p_y \\ r_{31} & r_{32} & r_{33} & \vdots & p_z \\ \cdots & \cdots & \cdots & \vdots & \cdots \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix}$$

$$r_{11} = C_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] + S_1[S_4C_5C_6 + C_4S_6]$$
$$r_{21} = S_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] - C_1[S_4C_5C_6 + C_4S_6]$$
$$r_{31} = -S_{23}(C_4C_5C_6 - S_4S_6) - C_{23}S_5C_6$$
$$r_{12} = C_1[C_{23}(-C_4C_5S_6 - S_4C_6) + S_{23}S_5S_6] + S_1[-S_4C_5S_6 + C_4C_6]$$
$$r_{22} = S_1[C_{23}(-C_4C_5S_6 - S_4C_6) + S_{23}S_5S_6] - C_1[-S_4C_5S_6 + C_4C_6]$$
$$r_{32} = -S_{23}(C_4C_5S_6 - S_4C_6) + C_{23}S_5S_6$$
$$r_{13} = -C_1(C_{23}C_4S_5 + S_{23}C_5) - S_1S_4S_5$$
$$r_{23} = -S_1(C_{23}C_4S_5 + S_{23}C_5) + C_1S_4S_5$$

$$r_{33} = S_{23}C_4S_5 - C_{23}C_5$$

$$P_x = C_1(a_2C_2 + a_3C_{23} - S_{23}d_4) - d_3S_1$$
$$P_y = S_1(a_2C_2 + a_3C_{23} - S_{23}d_4) + d_3C_1$$
$$P_x = -a_2S_2 - a_3S_{23} - C_{23}d_4$$

For last 3 angles i.e. θ_4, θ_5, θ_6

$${}^{3}_{6}[R] = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & -C_4C_5 \\ S_5C_6 & -S_5S_6 & C_5 \\ -S_4C_5C_6 - C_4S_6 & S_4C_5S_6 - C_4C_6 & S_4S_5 \end{bmatrix}$$

4.5.1.4 Calculations

Forward Kinematics

Types of Joint	θ	d	a	α
Base	θ1	0	0	-90^0
Shoulder	θ2	500	700	0
Elbow	θ3	94.8	948	90^0
Tool Pitch	θ4	680	0	-90^0
Tool Yaw	Θ5	0	0	90^0
Tool Roll	Θ6	853	0	0

$\theta_1=60^0$
$\theta_2=30^0$
$\theta_3=45^0$
$\theta_4=0^0$
$\theta_5=90^0$
$\theta_6=0^0$

Solution

$$T_0^6 = \begin{bmatrix} -0.4829 & 0.8660 & -0.1294 & 15.27 \\ -0.8365 & -0.5 & -0.2241 & 216 \\ -0.2588 & 0 & 0.9659 & -1441.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3_6[R] = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$P_x = x = 15.27$ mm
$P_y = y = 216$ mm
$P_z = z = -1441.6$ mm

Inverse Kinematics

$$\theta_1 = 2\tan^{-1}[\frac{-x \pm \sqrt{x^2+y^2-{d_3}^2}}{y+d_3}]$$

$$\theta_3 = 2\tan^{-1}[\frac{-d_4 \pm \sqrt{{d_4}^2+{a_3}^2-K^2}}{K+a_3}]$$

Where, $K = (\frac{1}{2a_2})(x^2+y^2+z^2-{d_3}^2-{a_2}^2-{a_3}^2-{d_4}^2)$

$$\theta_2 = 2\tan^{-1}[\frac{-a_2-a_3c_3+d_4s_3 \pm \sqrt{{a_2}^2+{a_3}^2+{d_4}^2+2a_2(a_3c_3-d_4s_3)-z^2}}{z-(a_3s_3+d_4c_3)}]$$

$${}^3_6[R] = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\theta_5 = \cos^{-1}(r_{23})$$

$$\theta_6 = \tan^{-1}[\frac{-r22}{r21}]$$

$$\theta_4 = \sin^{-1}[\frac{r33}{S_5}]$$

$\theta_1 = 59.99287582^0$

$\theta_3 = 45.00048521^0$

$\theta_2 = 29.99863678^0$

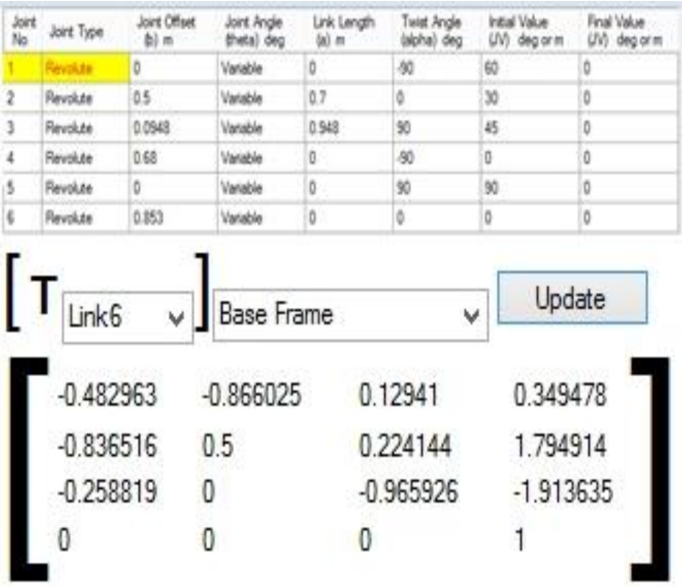
$\theta_4 = 0^0$

$\theta_5 = 90^0$

$\theta_6 = 0^0$

4.5.2 Using RoboAnalyser

4.5.2.1 Forward Kinematics



Joint No	Joint Type	Joint Offset (b) m	Joint Angle (theta) deg	Link Length (a) m	Twist Angle (alpha) deg	Initial Value (JV) deg or m	Final Value (JV) deg or m
1	Revolute	0	Variable	0	-90	60	0
2	Revolute	0.5	Variable	0.7	0	30	0
3	Revolute	0.0948	Variable	0.948	90	45	0
4	Revolute	0.68	Variable	0	-90	0	0
5	Revolute	0	Variable	0	90	90	0
6	Revolute	0.853	Variable	0	0	0	0

$$\begin{bmatrix} -0.482963 & -0.866025 & 0.12941 & 0.349478 \\ -0.836516 & 0.5 & 0.224144 & 1.794914 \\ -0.258819 & 0 & -0.965926 & -1.913635 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

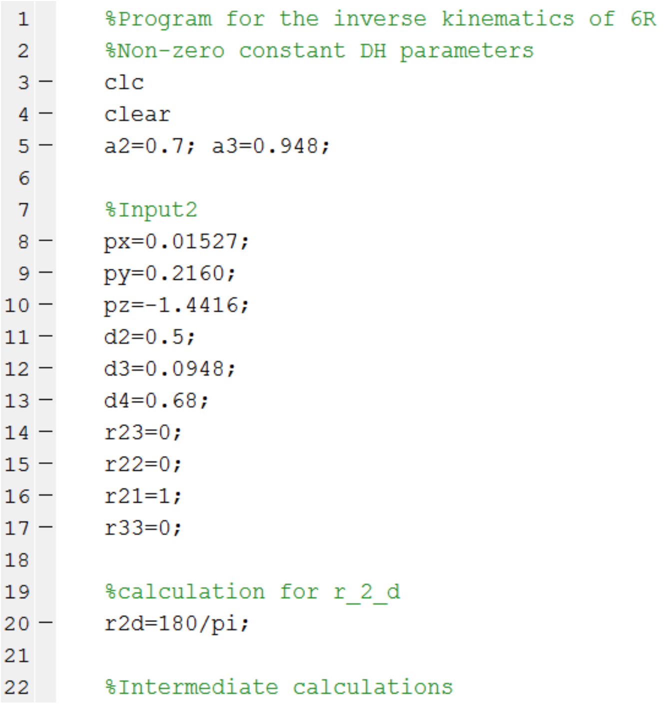
Fig-21: Results of Puma 560 6R Robot for Forward Kinematics

4.5.2.2 Inverse Kinematics

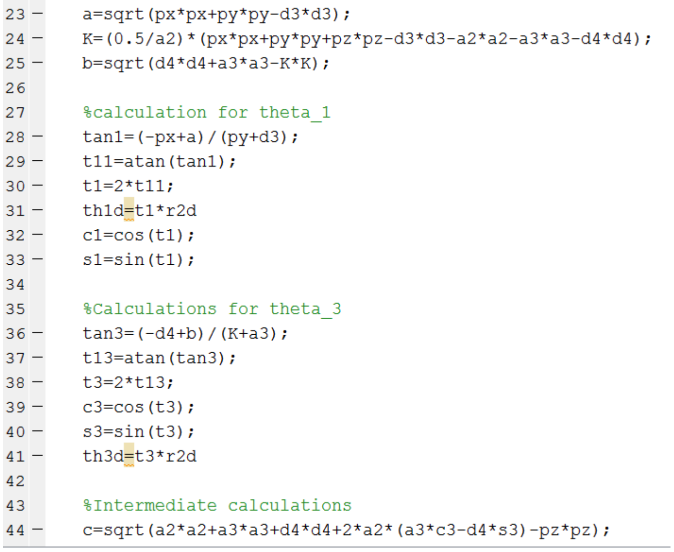
Results

$\theta_1=75.11463857^0$
$\theta 2= 46.3399-62.41i^0$
$\theta 3=35.65+103.96i^0$
$\theta_4=0^0$
$\theta_5=90^0$
$\theta_6=0^0$

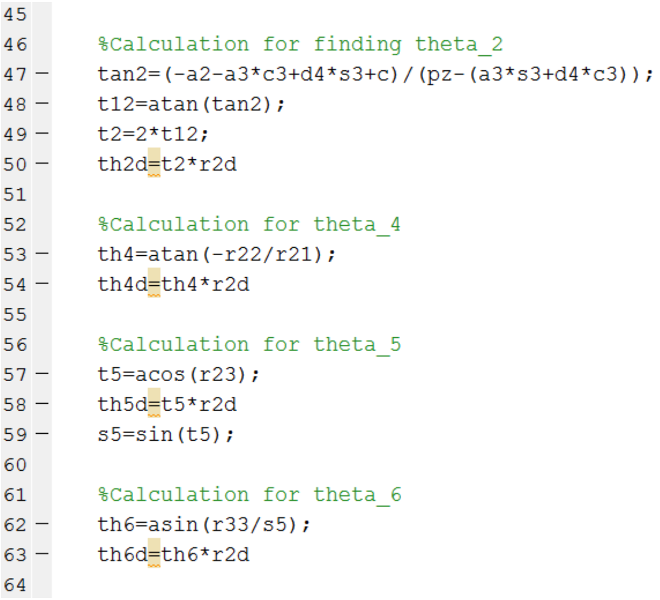
4.5.3 Using MATLAB



```
%Program for the inverse kinematics of 6R
%Non-zero constant DH parameters
clc
clear
a2=0.7; a3=0.948;

%Input2
px=0.01527;
py=0.2160;
pz=-1.4416;
d2=0.5;
d3=0.0948;
d4=0.68;
r23=0;
r22=0;
r21=1;
r33=0;

%calculation for r_2_d
r2d=180/pi;

%Intermediate calculations
```

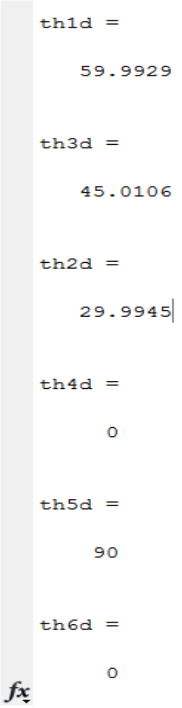


```
a=sqrt(px*px+py*py-d3*d3);
K=(0.5/a2)*(px*px+py*py+pz*pz-d3*d3-a2*a2-a3*a3-d4*d4);
b=sqrt(d4*d4+a3*a3-K*K);

%calculation for theta_1
tan1=(-px+a)/(py+d3);
t11=atan(tan1);
t1=2*t11;
th1d=t1*r2d
c1=cos(t1);
s1=sin(t1);

%Calculations for theta_3
tan3=(-d4+b)/(K+a3);
t13=atan(tan3);
t3=2*t13;
c3=cos(t3);
s3=sin(t3);
th3d=t3*r2d

%Intermediate calculations
c=sqrt(a2*a2+a3*a3+d4*d4+2*a2*(a3*c3-d4*s3)-pz*pz);
```



```

%Calculation for finding theta_2
tan2=(-a2-a3*c3+d4*s3+c)/(pz-(a3*s3+d4*c3));
t12=atan(tan2);
t2=2*t12;
th2d=t2*r2d

%Calculation for theta_4
th4=atan(-r22/r21);
th4d=th4*r2d

%Calculation for theta_5
t5=acos(r23);
th5d=t5*r2d
s5=sin(t5);

%Calculation for theta_6
th6=asin(r33/s5);
th6d=th6*r2d

```

Fig-22: MATLAB program for IK of Puma 560 6R Robot



```
th1d =

   59.9929


th3d =

   45.0106


th2d =

   29.9945


th4d =

     0


th5d =

    90


th6d =

     0
```

Fig-23: MATLAB Results for IK of Puma 560 6R Robot

5. RESULTS AND DISCUSSION

Table-6: Comparison of Results

Robot Configuration	Angles (input in degrees)	Method for Study of IK (output in degrees ^{0})		
		Algebraic	Robo Analyser	MATLAB
2R Configuration	θ1=60	59.99999863	60	60
	θ2=30	30.00006631	30.0001	30.0001
3R Configuration	θ1=60	59.9999938	60	60
	θ2=30	30.00008634	30.0001	30.0001
	θ3=0	0	0	-0.00004
3R-1P Configuration	θ1=60	60.00002314	60	60.0450
	θ2=0	0.0025	0	0.0960
	d3=100	100	100	100
	θ4=30	29.99747686	30.0001	29.8590
5R Configuration	θ1=30	30.000001692	30.0001	30.000
	θ2=-60	-59.99988837	-59.999	-59.9993
	θ3=90	89.99974525	90	90.0001
	θ4=0	0	0	-0.0476
	θ5=45	45	45	45
6R Configuration	θ1=60	59.99287582	75.11463857	59.9929
	θ2=30	29.99863678	46.3399-62.41i	29.9945
	θ3=45	45.00048521	35.65+103.96i	45.0106
	θ4=0	0	0	0
	θ5=90	90	90	90
	θ6=0	0	0	0

Table-7: Percentage Error in Calculation

Robot Configuration	Angles (input in degrees)	Percentage Error (in %)		
		Algebraic	Robo Analyser	MATLAB
2R Configuration	θ1=60	2.28*10^{-6}	0	0
	θ2=30	2.21*10^{-4}	3.33*10^{-4}	3.33*10^{-4}
3R Configuration	θ1=60	1.03*10^{-5}	0	0
	θ2=30	2.878*10^{-4}	3.33*10^{-4}	3.33*10^{-4}
	θ3=0	0	0	-
3R-1P Configuration	θ1=60	3.85*10^{-5}	0	0.075
	θ2=0	-	0	-
	d3=100	0	0	0
	θ4=30	8.41*10^{-3}	3.33*10^{-4}	0.47
5R Configuration	θ1=30	5.64*10^{-6}	3.33*10^{-4}	0
	θ2=-60	1.86*10^{-3}	1.67*10^{-3}	1.167*10^{-3}
	θ3=90	2.83*10^{-4}	0	1.11*10^{-4}
	θ4=0	0	0	-
	θ5=45	0	0	0
6R Configuration	θ1=60	1.87*10^{-2}	25.19	0.01183
	θ2=30	4.54*10^{-3}	54.47-207.13i	0.01833
	θ3=45	1.07*10^{-3}	20.78+231.02i	1.078*10^{-3}
	θ4=0	0	0	0
	θ5=90	0	0	0
	θ6=0	0	0	0

So from above Table it is imminent that there's a minute percentage error in calculations by all the three ways of studying inverse kinematics i.e. by algebraic method, using RoboAnalyser and using MATLAB. In most of the cases the results of RoboAnalyser and MATLAB are same as compared to algebraic method. The difference in algebraic method is mostly due to the fact that during calculations most of the values were approximated.

Also from above ways of studying IK and its output, it is clear that the simplicity level of studying inverse kinematics goes on increasing with increasing robot configuration.

6. CONCLUSION

Since the forward kinematic analysis of any robot configuration is simple to do analysis of model and calculate the position using the joint angle, its study is not of much bother to us. However the greater challenge is to analyze the inverse kinematics solution of the robot configuration using the final position the robot. Thus the aim was to study complexity of the IK with increasing degrees of freedom.

So in the study this aim have been materialized by means of three ways for analyzing the inverse kinematics solution using algebraic method, using RoboAnalyser and using MATLAB. So the study of the complexity of various robot configurations with increasing degrees of freedom is done for robot configurations i.e. 2R, 3R, 3R-1P, 5R, 6R where R and P stands for revolute and prismatic joints.

The results of these 3 methods suggests that the study of IK definitely is of complex nature for increased degrees of freedom. In other words, the results of algebraic method are validated with the outputs from RoboAnalyser and MATLAB.

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