Quality assessment in image compression by using fast wavelet transformation with 2D haar wavelets

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Abstract - *Image compression is an application or techniques that facilitate to reducing the size of graphics file, without compromising on its quality and also reducing the distortion in digital image processing. Data compression is defined as the process of encoding data that reduces the overall size of data without degrade the value of data. This reduction is possible when the original dataset contains some type of redundancy, where redundant data increased the storing space in storage devices. Digital image compression is an application that studies methods for reducing the total number of bits required to represent an image. This can be achieved by eliminating the various types of redundancy that exist in the pixels values which takes extra spaces to stored the images. The objective of this paper is increased the image quality performance of evaluate a set of wavelets for image compression. Wavelet transformation is one of the best compression technique that improved compression ratio and image quality. Here in this paper we examined the fast wavelet transformation with wavelet family that is Haar wavelet transforms and reconstruct the image by using 2D haar tansformation. The Discrete Wavelet Transform (DWT) analyzes the signals at different frequency bands with different resolutions by decomposing the signal into an approximation and detail information. The study compares Advanced FWT approach in terms of PSNR, Compression Ratios and elapsed time for different Images. Complete analysis is performed at first, second and third level of decomposition using Haar Wavelet. The implementation of the proposed algorithm based on Fast Wavelet Transform. The implementation is done under the Image Processing Toolbox in the MATLAB.*

Key Words: **Discrete Wavelet Transform, Fast Wavelet Transform, Approximation and Detail Coefficients, Haar wavelets.**

1.INTRODUCTION

The objective of image compression is to reduce redundancy of the image, data in order to be able to store or transmit data in an efficient form as an original data. Image compression is categorised in two methods, lossy or lossless. Lossless compression is sometimes preferred for artificial images such as technical drawings, icons or comics where data values are more important, compressed data and original data must be same. This is because lossy methods introduced compression artifacts, especially when used at low bit rates. Lossless compression methods may also be preferred for high value content data, such as medical imagery, or image scans made for archival purposes. In Lossy methods where minor loss of fidelity is acceptable to achieve a substantial reduction in bit rate for good quality of images. Run-length encoding and Huffman encoding are the methods for lossless image compression. Transform coding, where a Fourier related transforms such as DCT or the wavelet transform are applied that followed by quantization and entropy coding can be cited as a method for lossy image compression. In numerical analysis and functional analysis, a discrete wavelet transform (DWT) refers to wavelet transforms for which the wavelets are discretely sampled. A lot of work has been done in the area of wavelet transformation based lossy image compression. However, very little work has been done in lossless image compression using wavelets to improve image quality and data integrity. So the proposed methodology of this paper is to achieve high compression ratio with low mean square error in images using 2D-Haar Wavelet Transform by applying different compression thresholds for the wavelet coefficients. That is, different compression ratios are applied to the wavelet coefficients belonging in the different regions of interest, in which belonging in the different regions of interest, in which either each wavelet domain band of the transformed image. Fast wavelet transform (FWT) is a mathematical algorithm designed to turn a sequence of coefficient based on an orthogonal basis of small finite waves, or wavelets.

The DWT of a signal x is calculated by passing it through a series of filters. First the samples are passed through a low pass filter with impulse g response resulting in a convolution of the two samples:

$$y[n] = (x * g)[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-k]$$

The signal also decomposed simultaneously using a high-pass filter h. The outputs giving the detail coefficients (from the high-pass filter h) and approximation coefficients (from the low-pass g). It is important that the two filters are related to each other and they are known as a quadrature mirror filter (QMF).

However , since half the frequencies of the signals has been removed, half samples can be discarded according to Nyquist's rule. After then, The filter output of the low pass g is subsampled by 2 and further processed by passing it again through a new low pass filter g and high pass filter h with half the cut-off frequency of the previous one, i.e.

$$y_{low}[n]=\sum_{k=-\infty}^{\infty} x[k]g[2n-k]$$

$$y_{low}[n]=\sum_{k=-\infty}^{\infty} x[k]h\,[2n-k]$$

This decomposition has half the time resolution since only half of each filter output characterised the signal. However, each output has half the frequency band of the input, so the frequency resolution has been doubled.

With the sub sampling (disambiguation needed) operator↓.

$$(y \downarrow k)[n] = y[kn]$$

The above summation can be written more concisely.

$$y_{low}=(x * g) \downarrow 2$$

$$y_{high}=(x * h) \downarrow 2$$

However computing a complete its operation $x * g$ with subsequent down sampling would waste computation time.

This decomposition is repeated to further increase the frequency resolution and the approximation coefficients decomposed with high pass and low pass filters and then down-sampled. This processed is represented as a binary tree with nodes representing a sub-space with a different time-frequency localisations. The tree is known as a filter bank.

At each level of filter bank the signal is decomposed into low and high frequencies. Due to the decomposition process the input signal must be a multiple of 2^n where n is the number of levels used in filter bank.

2. Fast wavelet transform

The **Fast Wavelet Transform** is a mathematical algorithm that designed to turn a waveform or signals in the time domain into a sequence of coefficients based on an orthogonal basis of small finite waves, or wavelets. The transform can be easily extended to the multidimensional signals, such as images, where the time domain is replaced with the space domain.

It has theoretical foundation the device of a finitely generated, orthogonal multi resolution analysis (MRA). In the terms of given there, one selects a sampling scale J with sampling rate of 2^J per unit.

Interval, and projects the given signal f onto the space V_j; in theory by computing the scalar products

$s_n^{(j)}=2^j\langle f(t),\phi(2^jt-n)\rangle$

Where ϕ is the scaling function of the chosen wavelet transform; in practically by any suitable sampling procedure under the condition that the signal is highly over sampled, so

$$P_j[f](x)=\sum_{n\in\mathbb{Z}} s_n^{(j)}\phi(2^jx-n)$$

is the orthogonal projection or at least some good approximation of the original signal in $\mathcal{V}j\mathcal{V}j$.

The MRA is characterised by its scaling sequence such as:

$a=(a_{-N},\ldots,a_0,\ldots,a_N)$ Or as a Z-transform, $a(z)=\sum_{n=-N}^{N}a_nz^{-n}$

And its wavelet sequence is:

b=$(b_{-N},\ldots,b_0,\ldots,b_N)$ or $b(z)=\sum_{n=-N}^{N}b_nz^{-n}$

(some coefficients might be zero). Those allowed to compute the wavelet coefficients $d_n^{(k)}$, at least some range as *k=M,...,J-1*, without having to approximate the integrals in the corresponding scalar products. compute those coefficients from the first approximation $s^{(j)}$, Instead, one can directly, with the help of convolution and decimation operators,.

Forward DWT

One computes recursively, starting with the coefficient sequence $s^{(j)}$ and counting down from *k=J-1* to some *M<J*,

$$s_n^{(k)}:=\frac{1}{2}\sum_{m=-N}^{N}a_m s_{2n+m}^{(k+1)}$$

or $s^{(k)}(z):=(\downarrow 2)(a^*(z).s^{(k+1)}(z))$ And

$$d_n^{(k)}:=\frac{1}{2}\sum_{m=-N}^{N}b_m s_{2n+m}^{(k+1)}$$

or $d^{(k)}(z):=(\downarrow 2)(b^*(z).s^{(k+1)}(z))$

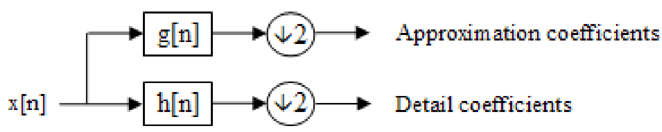

Fig1: block diagram of filter analysis

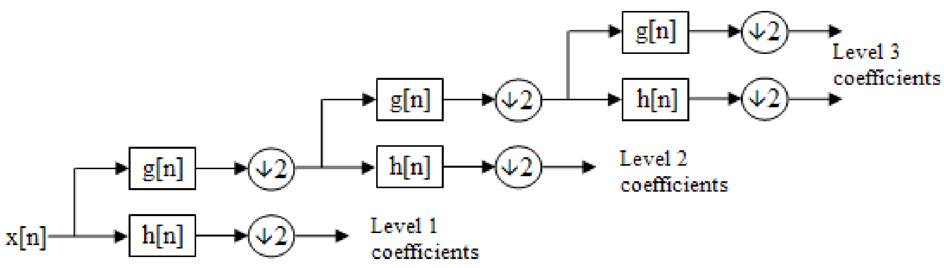

Fig2: 3 level filter bank

where $g=a^*$ and $h=b^*$, for *k=J-1,J-2,...,M* and all $n\in\mathbb{Z}$. In the Z-transform notation:

- The down sampling operator $(\downarrow 2)$ reduces an infinite sequence, given by its Z- transform, which is simply a Laurent series, to the sequence of the coefficients with even indices, $(\downarrow 2)(c(z))=\sum_{k\in\mathbb{Z}}(c_{2k}z^{-k})$.

- The starred Laurent-polynomial $a^*(z)$ denotes the adjoint filter, it has *time-reversed* adjoint coefficients, $a^*(z) = \sum_{n=-N}^{N}(a^*_{-n}z^{-n})$. (The adjoint of a real number being the number itself, of a complex number being its conjugate, of a real matrix the transposed matrix, of a complex matrix its hermitian adjoint).
- Multiplication is form of polynomial multiplication, which is equivalent to the convolution of the coefficient sequences.

It follows that

$$P_k[f](x) = \sum_{n\in\mathbb{Z}} s_n^{(k)}\phi(2^k x - n)$$

This is the orthogonal projection of the original signal *f* or at least of the first approximation$P_j[f](x)$ onto the subspace $\mathcal{V}_k$, that is, with sampling rate of 2^k per unit interval. The difference to the first approximation is given by:

$$P_j[f](x) = P_k[f](x) + D_k[f](x) + \ldots + D_{j-1}[f](x).$$

where the difference or detail signals are computed from the detail coefficients as:

$$D_k[f](x) = \sum_{n\in\mathbb{Z}} d_n^{(k)}\psi(2^k x - n)$$

with $\psi\psi$ denoting the *mother wavelet* of the wavelet transform.

Inverse DWT

Given the coefficient sequence $s^{(M)}$ for some *M<J* and all the difference sequences$d^{(k)}$, *k=M,...,J-1*, one computes recursively,

$$s_n^{(k+1)} := \sum_{k=-N}^{N} a_k s_{2n-k}^{(k)} + \sum_{k=-N}^{N} b_k d_{2n-k}^{(k)}$$

Or

$$s^{k+1}(z) = a(z).(\uparrow 2)\left(s^{(k)}(z)\right) + b(z).(\uparrow 2)\left(d^{(k)}(z)\right)$$

for *k=J-1,J-2,...,M* and all $n \in \mathbb{Z}$. In the Z-transform notation:

The upsampling operator $(\uparrow 2)$ creates zero-filled holes inside a given sequences. That is, every second element of the resulting sequence is an element of the given sequence, every other second element is zero or $(\uparrow 2)(c(z)) = \sum_{n\in\mathbb{Z}} c_n z^{-2n}$. This linear operator is, in the Hilbert space $\ell^2(\mathbb{Z},\mathbb{R})$, the adjoint to the downsampling operator $(\downarrow 2)$.

3. Haar wavelet transform

Haar wavelet compression is very simple and an efficient way to perform both lossless and lossy image compression. It relies on averaging the pixels values and differencing values in an image matrix to produce a matrix which is sparse or nearly sparse. A sparse matrix is a matrix in which a large portion of its entries values are 0. A sparse matrix can be stored in an very efficient manner leading to the smaller file sizes of image. By using haar wavelet compression we concentrate on grayscale images; however, rgb images can be handled by compressing each of the color layers with separately. The basic method is to start with any image A, which can be regarded as an m×n matrix with values 0 to 255. In Matlab, this would be a matrix with an unsigned 8-bit integer values. We then subdivide to image into 8×8 blocks, padding as necessary. This is the 8×8 blocks that we work with. Haar wavelet basis can be used to represent the image by computing a wavelet transform. To do this, first compute average the pixels together, pair wise, is calculated to get the new lower resolution image with pixel values [14, 10, 6, 2]. This single number is used to recover the first two pixels of our original four-pixel image. Similarly, the first detail coefficient is -1, since 14 + (-1) = 13 and 14 - (-1) = 15. Thus, the original image is decomposed into a lower resolution (two-pixel) version and a pair of detail coefficients. Repeating this process recursively on the averages gives the full decomposition shown in table1.

Resolution	Averages	Detail Coefficients
8	[13,15,11,9,7,5,1,3]	
4	[14,10,6,2]	[-1,1,1,-1]
2	[12,4]	[2,2]
1	[8]	[4]

Table1: decomposition to lower resolution

Thus this is the basis of one dimensional haar wavelet transforms procedure to compute the detail coefficients of an image matrix data. We used the way to compute the wavelet transform by recursively averaging and differencing coefficients, filter bank. We can reconstruct the image to any resolution values by recursively adding and subtracting the detail coefficients from the lower resolution versions.

3.1. Compression of image with 2D Haar Wavelet Techniques:

It has been shown in previous section how one dimensional image reconstructed with any resolution and also it can be treated as sequences of coefficients. Alternatively, we can also think of images as a piecewise constant functions on the half-open interval [0, 1). To do so, there used the concept of a vector space. A one-pixel image is just as a function that is constant over the entire interval [0, 1). Let $\mathcal{V}_0$ be the vector space of all these functions. A two pixel image has a two constant pieces over the intervals [0, 1/2) and [1/2, 1). We call the space containing all these functions$\mathcal{V}_1$. If we continue in this manner, the space$\mathcal{V}_j$ will include all piecewise-constant functions that defined on the interval [0, 1) with constant pieces over each of 2^j equal subintervals. Now, We think of every one-dimensional image with 2^j pixels as an element, or vector, in $\mathcal{V}_j$. Note that because of these vectors are all functions are defined on the unit interval, every vector in $\mathcal{V}_j$ is also contained in $\mathcal{V}_{j+1}$. For example, we always describe a piecewise constant functions with two intervals as a piecewise-constant function with four intervals, with each interval in the first function corresponding to a pair of intervals in the second intervals. Thus, the spaces $\mathcal{V}_j$ are nested; that is, $\mathcal{V}_0 \subset \mathcal{V}_1 \subset \mathcal{V}_2 \subset$ ...... This nested set of vector spaces $\mathcal{V}_j$ is a necessary ingredient for the mathematical theory of multiresolution analysis (MRA) [1]. It guarantees that every member of $\mathcal{V}_0$ can be represented exactly as a member of higher resolution space $\mathcal{V}_1$. The converse, however, is not true: not every function G(x) in $\mathcal{V}_1$ can be represented exactly in lower resolution space $\mathcal{V}_0$; in general there is some lost detail [2]. Now we define a basis for each vector space $\mathcal{V}_j$. The basic functions for the spaces $\mathcal{V}_j$ are called scaling functions, are usually denoted by the symbol. A simple basis for $\mathcal{V}_j$ is given by the set of scaled and translated box functions [3]:

$$\varphi_i^j(x) = \varphi(2^j x - i) \quad i = 0,1,2, \ldots \ldots 2^j - 1 \text{ where}$$

$$\varphi(x) = \begin{cases} 1 & for\ 0 \leq x < 1 \\ 0 & otherwise \end{cases}$$

The wavelets corresponding to the box basis are known as the Haar wavelets, given by-

$$\psi_i^j(x) = \psi(2^j x - i) \quad i = 0,1,2, \ldots \ldots 2^j - 1 \text{ where}$$

$$\psi(x) = \begin{cases} 1 & for\ 0 \leq x < 1/2 \\ -1 & for\ 1/2 \leq x < 1 \\ 0 & otherwise \end{cases}$$

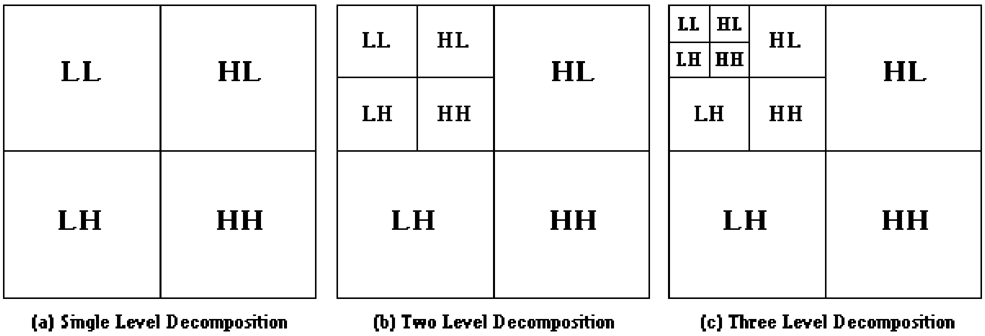

Fig3: Structure of wavelet decomposition

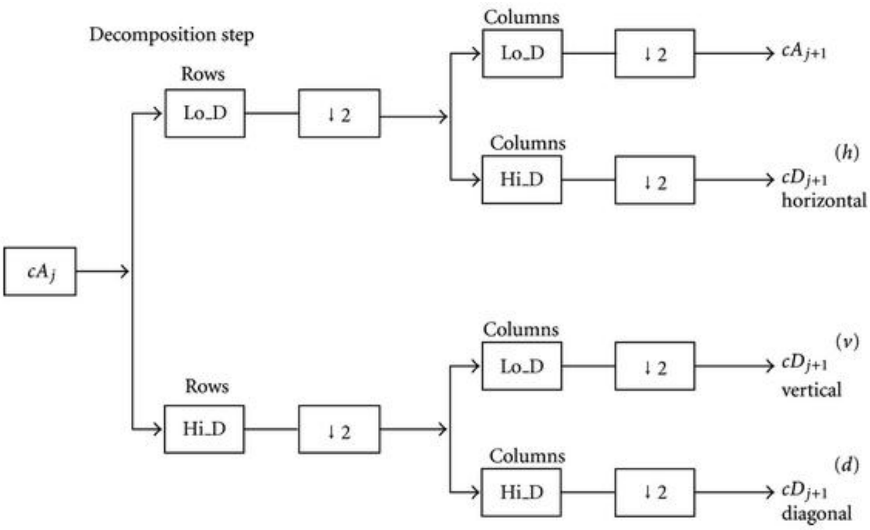

Fig4: Two level 2D wavelet decomposition tree.

Thus, the DWT for an image as a 2D signal will be obtained from 1D DWT. By using these filters in one stage, an image is decomposed into four bands. There exist three types of detail images for each resolution: horizontal (HL), vertical (LH), and diagonal (HH). The operations can be repeated on the low (LL) band using the second stage of identical filter bank. Thus, a typical 2D DWT, used in image compression, generates the hierarchical structure shown in Fig. 4. Now let us see how the 2D Haar wavelet transformation is performed.

The image is comprised of pixels represented by numbers [4]. The number of decompositions levels determines the quality of compressed image and also determines the resolution of the lowest level in wavelet domain. If a larger number of decompositions is used, it will provide more success in resolving important DWT coefficients from less important coefficients and it helps to improved the quality of image.

Fig5: A 8× 8 image.

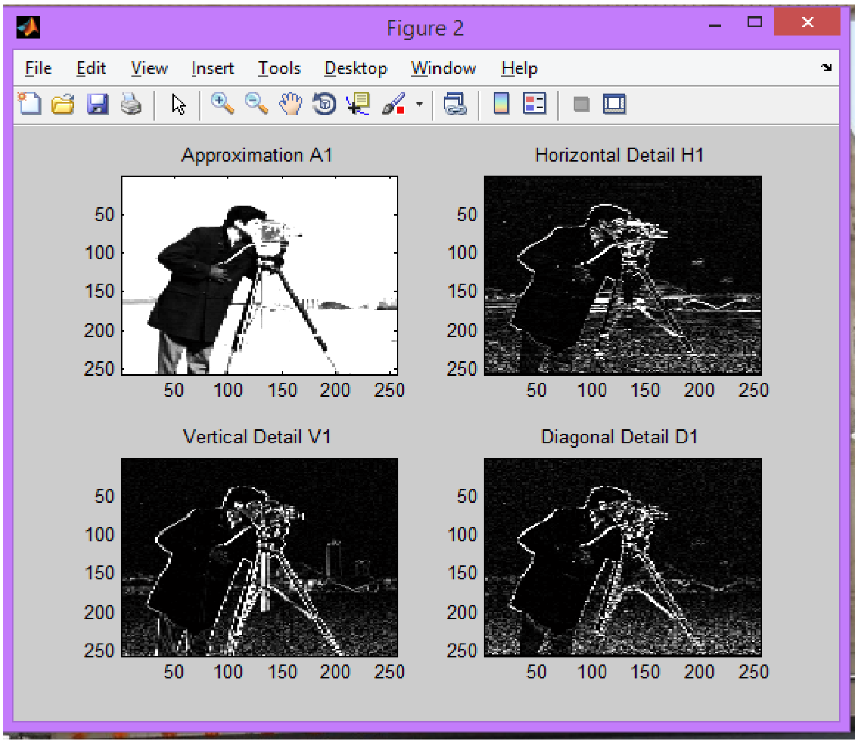

Fig6:1D level-decomposition

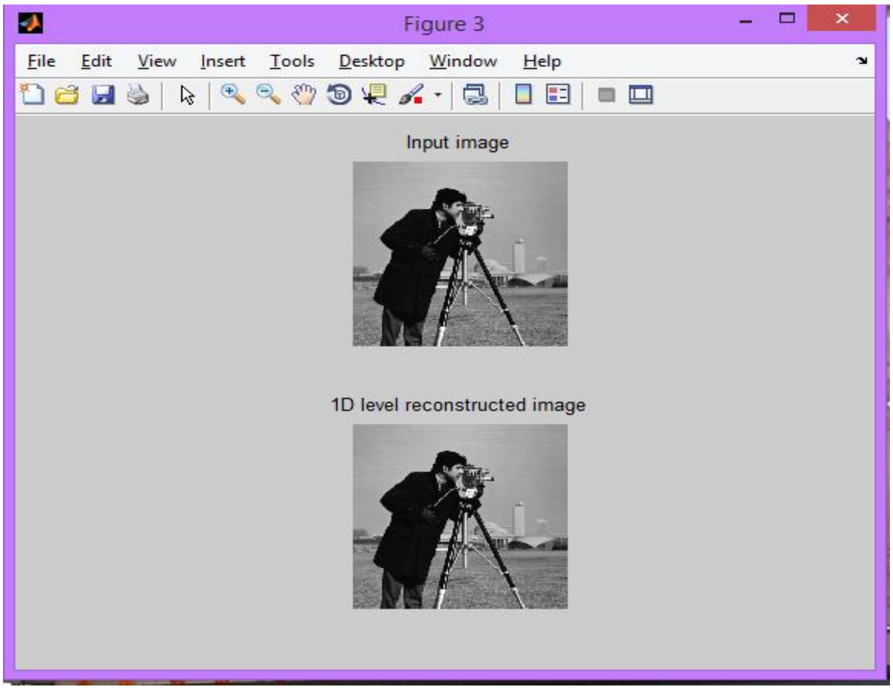

Fig7: 1D level reconstructed image

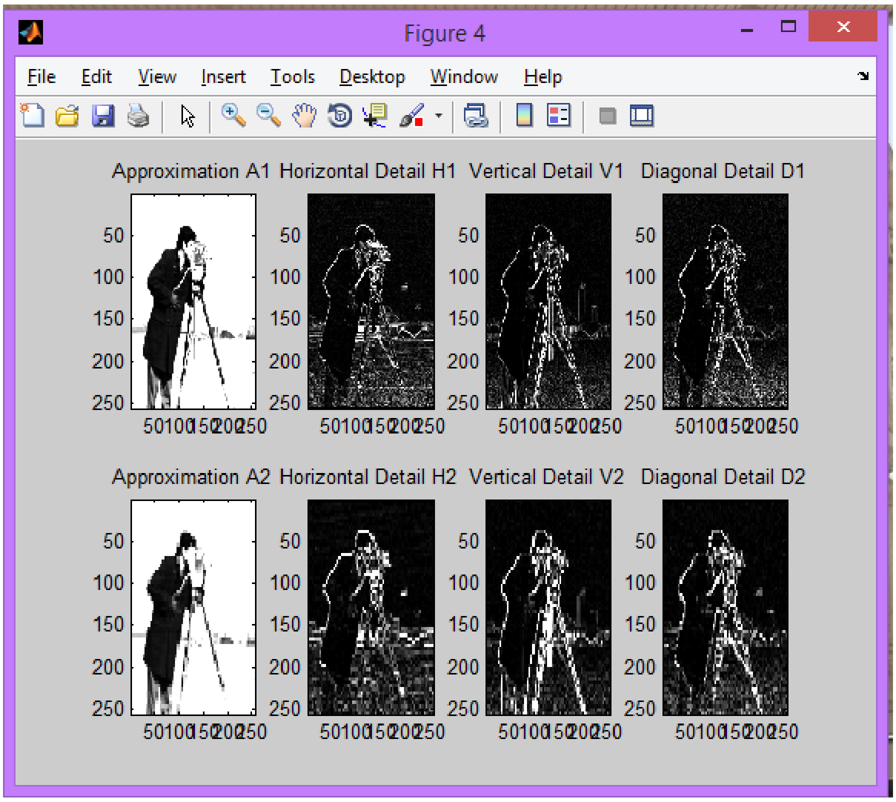

Fig8:2D level decomposition

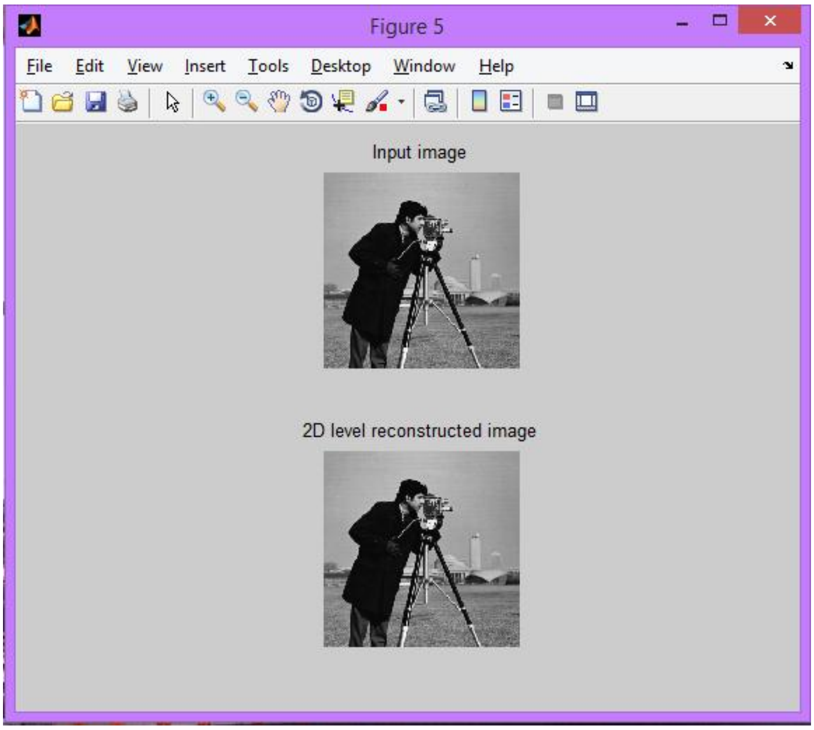

Fig9: 2D level reconstructed image

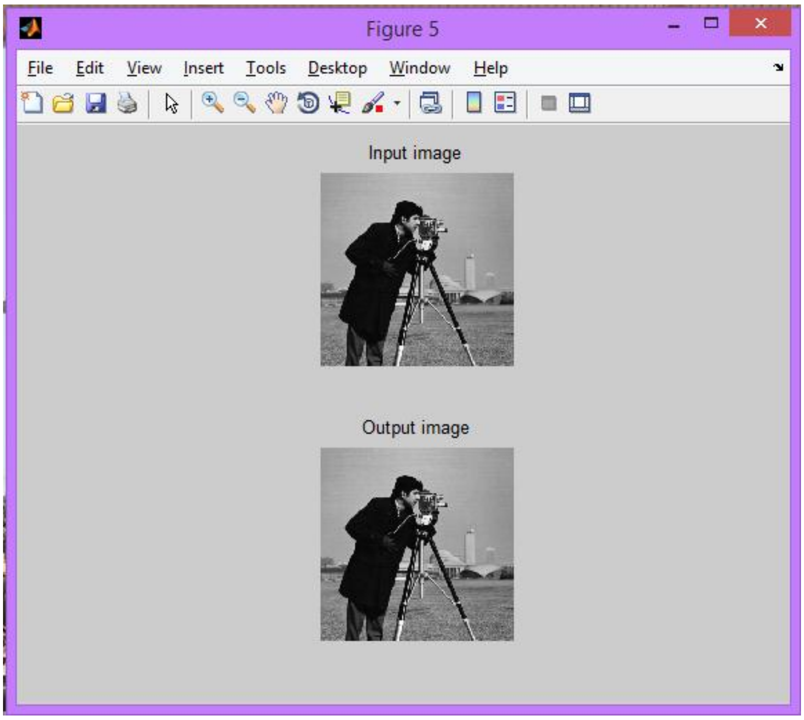

Fig10: input and output image

Fig11: output image

3.2. QUALITY MEASUREMENT

We define the compression ratio (CR) as in percentage of number of bits in original image and compressed image.The compression scores in percentages 26.6602, after implement to 2D haar wavelets. It is noted here that the hard thresholding provides the best CR. The soft thresholding gives a better CR in comparison to universal thresholding method. The PSNR for gray scale image (8 bits/pixel) is defined by-

$$PSNR=20 \times \log_{10}(\frac{255}{\sqrt{MSE}})$$

$$MSE = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}[F'(i,j) - F(i,j)]^2$$

Where F' is approximation of decompressed image and F is original image and M, N are dimensions of the image. These results are widely acceptable in most cases except in medical application where no loss of information is to be guaranteed. However, the PSNR is Not adequate as a perceptually meaningful measure of pictures quality, because the reconstruction errors generally do not have the characteristic of signal independent additive noise and the seriousness of the impairments that cannot be measured by a simple power measurement. At present in image compression, the most widely used objective distortion measures are the MSE and the related PSNR. They can be easly computed to represent the deviation of the distorted image from the original image in the pixelwise or bitwise sense. The subjective perceptual quality improvement includes surface smoothness, edge sharpness and continuity, proper background noise level, and so on.

4. CONCLUSION

Image compression helps to decrease the size of image to stored images in appropriate storage space. it helps to reduced the bits redundancy by which image takes large space. Compressed image has a low quality after compression applied. the wavelet transformation is one of the best technique to improved the image quality and also reduced distortion and enhanced the compression performance. A low complex 2D image compression method using Haar wavelets which is the family of wavelet transformation, as the basis functions along with the quality measurement of the compressed images have been presented here . As for the further work, the decomposition level are greatly helps to increase the quality of image 3D haar wavelet is the future work of this techniques which is help to compressed the 3D image with best quality also the tradeoff between the value of the threshold ε and the image quality can be studied and also fixing the correct threshold value is also of great interest. Furthermore, finding out the exact number of transformation level required in case of application several image compression at one time, can be studied. Also, more thorough comparison of various still image quality measurement algorithms may be conducted also decide which one is the best approach, is considered . Though many published algorithms left a few parameters unspecified, here good estimates of them and simple procedures for implementation have been provided so that it is difficult to conclude any decisive advantage of one algorithm over another.

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