ASYMPTOTIC PROPERTIES OF THE DISCRETE STABILITY TIME SERIES WITH MISSED OBSERVATIONS BETWEEN TWO-VECTOR VALUED STOCHASTIC PROCESS

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Abstract - *In this paper, we defined the Expanded finite Fourier transform of the strictly stability* $(r+s)$ *vector valued time series where there are some randomly missed observations, asymptotic moments are derived and the application will be studied .*

Key Words: Discrete time stability processes, Data tapers, Finite Fourier transform, Missing values, Complex Normal Distribution.

1.INTRODUCTION

Many authors, as e.g. Brillinger [1]; Dahlhaus[3]; Ghazal and Farag [4] studied "The estimation of the spectral density, autocovariance function and spectral measure of continuous time stationary processes"; E.A,El-Desokey[9] studied "Some properties of the discrete expanded finite Fourier transform with missed observations"; M.A.Ghazal, G.S. Mokaddis and A.El-Desokey[10],[11] are Studied "The Spectral Analysis of strictly stationary continuous time series" and "Asymptotic Properties of spectral Estimates of Second-Order with Missed Observations". The paper is organized as the following: Section1. Introduction, we develop asymptotic properties of estimates the desired $\underline{\mu}$, $a(u)$ In Section 2, the Asymptotic properties of Expanded finite Fourier transform with missed observations was discussed in section 3, section 4 we will apply our theoretical study in two cases in climate and economy.

2. ASYMPTOTIC PROPERTIES OF ESTIMATES THE DESIRED $\underline{\mu}$, $a(u)$

Consider an $(r+s)$ vector-valued stability series

$$Z(t)=\left[X(t) \quad Y(t)\right]^T , \tag{2.1}$$

$t=0,\pm1,\pm2,.....$ with $X(t)$ - r vector-valued and $Y(t)$ s vector-valued.

We assume the series (2.1) is $(r+s)$ stability vector-valued series with components $\left[X_j(t) \quad Y_i(t)\right]^T$, $j=1,2,...,r$, $i=1,2,.....,s$ all of whose moments exist, we define the means as

$$EX(t)=C_x \;,\; EY(t)=C_y \tag{2.2}$$

The covariances

$$E\{[X(t+u)-C_x][X(t)-C_x]^T\}=C_{xx}(u),$$
$$E\{[X(t+u)-C_x][Y(t)-C_y]^T\}=C_{xy}(u), \tag{2.3}$$
$$E\{[Y(t+u)-C_y][Y(t)-C_y]^T\}=C_{yy}(u) \;,$$

and the second-order spectral densities

$$f_{xx}(\lambda)=(2\pi)^{-1}\sum_{u=-\infty}^{\infty}C_{xx}(u)\,Exp(-i\lambda u) \quad ,$$
$$f_{xy}(\lambda)=(2\pi)^{-1}\sum_{u=-\infty}^{\infty}C_{xy}(u)\,Exp(-i\lambda u) \quad , \tag{2.4}$$
$$f_{yy}(\lambda)=(2\pi)^{-1}\sum_{u=-\infty}^{\infty}C_{yy}(u)\,Exp(-i\lambda u)$$

, *for* $-\infty<\lambda<\infty$.

In this section we consider the problem of determining an s -vector $\underline{\mu}$, and an $s\times r$ filter $\{a(u)\}$, so that

$$\underline{\mu}+\sum_{u=-\infty}^{\infty}a(t-u)X(u) \quad , \tag{2.5}$$

Which is close to $Y(t)$. Suppose we measure closeness by the $S \times S$ Hermitian matrix

$$E\left\{[Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)][Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]^T\right\}, \quad (2.6)$$

Theorem 2.1

Consider an $(r+s)$ vector-valued second-order of stability time series of the form (2.1) with mean (2.2) and autocovariance functions (2.3). Suppose $c_{xx}(u)$, $c_{yy}(u)$ are absolutely summable and suppose $f_{xx}(\lambda)$, $f_{xy}(\lambda)$ and $f_{yx}(\lambda)$ are given by (2.4) and $f_{xx}(\lambda)$ is nonsingular, $-\infty<\lambda<\infty$. Then the, $\underline{\mu}$, and $a(u)$ that minimize (2.6) are given by

$$\underline{\mu}=c_y-\left(\sum_{u=-\infty}^{\infty}a(u)\right)c_x=c_y-A(0)c_x \;, \quad (2.7)$$

and

$$a(u)=(2\pi)^{-1}\int_0^{2\pi}A(\alpha)\,Exp\{iu\alpha\}\,d\alpha \;, \quad (2.8)$$

where

$$A(\lambda)=f_{yx}(\lambda)f_{xx}(\lambda)^{-1} \;, \quad (2.9)$$

the filter $\{a(u)\}$ is absolutely summable. The minimum achieved is

$$\int_0^{2\pi}[f_{yy}(\alpha)-f_{yx}(\alpha)f_{xx}(\alpha)^{-1}f_{xy}(\alpha)]d\alpha. \quad (2.10)$$

where $A(\lambda)$ is the transfer function of the $S\times r$ filter achieving the indicated minimum . we call $A(\lambda)$, the complex regression coefficient of $Y(t)$ on $X(t)$ at frequency λ.

Proof

Let $A(\lambda)$, be the transfer function of $a(u)$ which defined as (2.8). We may write as,

$$E\left\{[Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)][Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]^T\right\}$$

$$=\operatorname{cov}[Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]+E[Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]\times$$

$$\times E[Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]^T$$

$$=E\left\{\left([Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]-E[Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]\right)\times\right.$$

$$\left.\times\left([Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]^T-E[Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]\right)^T\right\}+$$

$$+E[Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]\times E[Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]^T$$

$$=\int_{-\pi}^{\pi}[f_{yy}(\alpha)-f_{yx}(\alpha)f_{xx}^{-1}(\alpha)f_{xy}(\alpha)]d\alpha+$$

$$+\int_{-\pi}^{\pi}[A(\alpha)f_{xx}(\alpha)-f_{yx}(\alpha)]f_{xx}^{-1}(\alpha)\times$$

$$\times[A(\alpha)f_{xx}(\alpha)-f_{yx}(\alpha)]^T d\alpha+[c_y-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)c_x]\times$$

$$\times[c_y-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)c_x]^T$$

(2.8)

$$E\left\{[Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)][Y(t)-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)X(u)]^T\right\}\geq$$

$$\geq\int_{-\pi}^{\pi}[f_{yy}(\alpha)-f_{yx}(\alpha)f_{xx}^{-1}(\alpha)f_{xy}(\alpha)]d\alpha$$

let

$$c_y-\underline{\mu}-\sum_{u=-\infty}^{\infty}a(t-u)c_x=0 \;,$$

(2.10)

then

$$\underline{\mu}=c_y-\sum_{u=-\infty}^{\infty}a(t-u)c_x=c_y-A(0)c_x \;,$$

and

$$A(\alpha)f_{xx}(\alpha)-f_{yx}(\alpha)=0$$

$$\Rightarrow \quad A(\alpha)=f_{yx}(\alpha)\,f_{xx}(\alpha)^{-1} \;,$$

Using (2.7) and (2.8) the minimum achieved

$$\int_0^{2\pi}[f_{yy}(\alpha)-f_{yx}(\alpha)f_{xx}^{-1}(\alpha)f_{xy}(\alpha)]d\alpha.$$

3. ASYMPTOTIC PROPERTIES OF EXPANDED FINITE FOURIER TRANSFORM WITH MISSED OBSERVATIONS

let $h_a^{(T)}(\lambda)$ be the discrete expanded finite Fourier transform which is defined as

$$h_a^{(T)}(\lambda)=\left[2\pi\sum_{t=0}^{T-1}\left(d_a^{(T)}(t)\right)^2\right]^{-1/2}\sum_{t=0}^{T-1}d_a^{(T)}(t)\,\psi_a(t)\exp\{-i\lambda t\},\ -\infty<\lambda<\infty \quad (3.1)$$

where

$$\psi_a(t)=B_a(t)Z_a(t)\ ,\quad a=1,2,\ldots\ldots,\min(r,s)\ , \quad (3.2)$$

, $X_a(t),Y_a(t)$ are the observations on the stability stochastic processes, $B_a(t)$ is Bernoulli sequence of random variable which is stochastically independent of $X_a(t),Y_a(t)$ which satisfies

$$B_a(t)=\begin{cases}1 & ,\ \text{if } X_a(t),Y_a(t) \text{ are observed ;}\\ 0 & ,\ \textit{otherwise.}\end{cases} \quad (3.3)$$

Let $B_a(t)$ be an independent and identically distributed random variables with

$$P[B_a(t)=1]=p_a\ ,\qquad P[B_a(t)=0]=q_a\ , \quad (3.4)$$

where $p_a+q_a=1$.

The data window function $d_a^{(T)}(t)=d_a^{(T)}(\frac{t}{T})$, $t\in(0,T)$ is bounded has bounded variation and vanishes for all t outside the interval $[0,T]$.

Assumption

Let $d_a^{(T)}(t), t\in R$, $a=\overline{1,r}$ has bounded variation and vanishes for $t>T-1, t<0$ then,

$$G_{a_1,\ldots,a_k}(\lambda)=\sum_{t=0}^{T-1}\left[\prod_{j=1}^{k}d_{a_j}^{(T)}(t)\right]\exp\{-i\lambda t\}\ ,$$

For $-\infty<\lambda<\infty$ and $\boldsymbol{a_1,\ldots,a_k=1,2,\ldots,r}$ .The following theorem will give the asymptotic properties of $\psi_a(t)$ which is defined as (3.2).

Theorem 3.1

Let $\psi_a(t)=B_a(t)Z_a(t)$, $a=1,2,\ldots\ldots,\min(r,s)$ are missed observations on the stable stochastic processes, $X_a(t),Y_a(t)$, $a=1,2,\ldots\ldots,\min(r,s)$ and $B_a(t)$ is Bernoulli sequence of random variables which satisfies equations(3.1),(3.4) , Then,

$$E\{\psi_a(t)\}=0\ , \quad (3.5)$$

$$Cov\{\psi_{a_1}(t_1),\psi_{a_2}(t_2)\}=p_{a_1a_2}\begin{bmatrix}c_{xx}(u) & c_{xy}(u)\\ c_{yx}(u) & A(\alpha)c_{xx}(u)A(\alpha)^T\end{bmatrix}, \quad (3.6)$$

$$Cov\{\psi_{a_1}(t_1),\psi_{a_2}(t_2)\}\overset{(3.1)}{=}$$

$$=p_{a_1a_2}\begin{bmatrix}\int_{-\infty}^{\infty}f_{a_1a_2}(v)\exp\{ivu\}dv & \int_{-\infty}^{\infty}f_{a_1a_2}(v)\exp\{ivu\}dvA(\alpha)^T\\ A(\alpha)\int_{-\infty}^{\infty}f_{a_1a_2}(v)\exp\{ivu\}dv & A(\alpha)\int_{-\infty}^{\infty}f_{a_1a_2}(v)\exp\{ivu\}dvA(\alpha)^T\end{bmatrix}, \quad (3.7)$$

Proof

Since $X(t)$ is a strictly stability series and $B_a(t)$ is independent of $Z_a(t)$ then (3.5) comes directly.

$$Cov\{\psi_{a_1}(t_1),\psi_{a_2}(t_2)\}=$$

$$=Cov\{B_{a_1}(t)Z_{a_1}(t),B_{a_2}(t)Z_{a_2}(t)\}$$

$$=Cov\left\{\begin{bmatrix}B_{a_1}(t_1)X_{a_1}(t_1)\\ B_{a_1}(t_1)Y_{a_1}(t_1)\end{bmatrix},\begin{bmatrix}B_{a_2}(t_2)X_{a_2}(t_2)\\ B_{a_2}(t_2)Y_{a_2}(t_2)\end{bmatrix}^T\right\}$$

$$=E\begin{bmatrix}B_{a_1}(t_1)X_{a_1}(t_1)B_{a_2}(t_2)X_{a_2}(t_2) & B_{a_1}(t_1)X_{a_1}(t_1)B_{a_2}(t_2)Y_{a_2}(t_2)\\ B_{a_1}(t_1)Y_{a_1}(t_1)B_{a_2}(t_2)X_{a_2}(t_2) & B_{a_1}(t_1)Y_{a_1}(t_1)B_{a_2}(t_2)Y_{a_2}(t_2)\end{bmatrix}.$$

$$=\left\{\begin{matrix}E[B_{a_1}(t_1)B_{a_2}(t_2)]Cov[X_{a_1}(t_1),X_{a_2}(t_2)] & E[B_{a_1}(t_1)B_{a_2}(t_2)]Cov[X_{a_1}(t_1),Y_{a_2}(t_2)]\\ E[B_{a_1}(t_1)B_{a_2}(t_2)]Cov[Y_{a_1}(t_1),X_{a_2}(t_2)] & E[B_{a_1}(t_1)B_{a_2}(t_2)]Cov[Y_{a_1}(t_1),Y_{a_2}(t_2)]\end{matrix}\right\}$$

$$=\left\{\begin{matrix}p_{a_1a_2}Cov[X_{a_1}(t_1),X_{a_2}(t_2)] & p_{a_1a_2}Cov[X_{a_1}(t_1),\mu+A(\alpha)X_{a_2}(t_2)]\\ p_{a_1a_2}Cov[\mu+A(\alpha)X_{a_1}(t_1),X_{a_2}(t_2)] & p_{a_1a_2}Cov[\mu+A(\alpha)X_{a_1}(t_1),\mu+A(\alpha)X_{a_2}(t_2)]\end{matrix}\right\}$$

$$=\left\{\begin{matrix}p_{a_1a_2}C_{X_{a_1}X_{a_2}}(t_1-t_2) & p_{a_1a_2}C_{X_{a_1}X_{a_2}}(t_1-t_2)A(\alpha)^T\\ p_{a_1a_2}A(\alpha)C_{X_{a_1}X_{a_2}}(t_1-t_2) & p_{a_1a_2}A(\alpha)C_{X_{a_1}X_{a_2}}(t_1-t_2)A(\alpha)^T\end{matrix}\right\}$$

$$=p_{a_1a_2}\begin{bmatrix}C_{a_1a_2}(t_1-t_2) & C_{a_1a_2}(t_1-t_2)A(\alpha)^T\\ A(\alpha)C_{a_1a_2}(t_1-t_2) & A(\alpha)C_{a_1a_2}(t_1-t_2)A(\alpha)^T\end{bmatrix}$$

from the stability and the independence then,

$$Cov\{\psi_{a_1}(t_1),\psi_{a_2}(t_2)\}=p_{a_1a_2}\begin{bmatrix}c_{a_1a_2}(u) & c_{a_1a_2}(u)A(\alpha)^T\\ A(\alpha)c_{a_1a_2}(u) & A(\alpha)c_{a_1a_2}(u)A(\alpha)^T\end{bmatrix},$$

and

$$Cov\{\psi_{a_1}(t_1),\psi_{a_2}(t_2)\}=$$

$$= p_{a_1a_2}\begin{bmatrix} \int_{-\infty}^{\infty} f_{a_1a_2}(v)\exp\{ivu\}dv & \int_{-\infty}^{\infty} f_{a_1a_2}(v)\exp\{ivu\}dvA(\alpha)^T \\ A(\alpha)\int_{-\infty}^{\infty} f_{a_1a_2}(v)\exp\{ivu\}dv & A(\alpha)\int_{-\infty}^{\infty} f_{a_1a_2}(v)\exp\{ivu\}dvA(\alpha)^T \end{bmatrix}.$$

Definition: The complex normal distribution: Suppose **X** and **Y** are random vectors in R^k such that **vec[X Y]** is a $2k$-dimensional normal vector. Then we say that the complex random vector $Z = X + iY$ has the complex normal distribution. This distribution can be described with three parameters: $\mu = E(Z)$, $\Gamma = E[(Z-\mu)(\overline{Z}-\overline{\mu})^T]$, $C = E[(Z-\mu)(Z-\mu)^T]$,

where Z^T denotes matrix transpose, and $\overline{Z}$ denotes complex conjugate. Here the parameter μ can be an arbitrary k-dimensional complex vector, the covariance matrix Γ must be Hermitian and non-negative definite; the relation matrix C should be symmetric. Moreover, matrices Γ and C are such that the matrix $\overline{\Gamma} - \overline{C}^T\Gamma^{-1}C$ is also non-negative definite. Matrices Γ and C are related to the covariance matrices of **X** and **Y** via expressions

$$V_{xx} \equiv E[(X-\mu_x)(X-\mu_x)^T] = \frac{1}{2}\operatorname{Re}[\Gamma + C],$$
$$V_{xy} \equiv E[(X-\mu_x)(Y-\mu_y)^T] = \frac{1}{2}\operatorname{Im}[-\Gamma + C],$$

$$V_{yx} \equiv E[(Y-\mu_y)(X-\mu_x)^T] = \frac{1}{2}\operatorname{Im}[\Gamma + C],$$
$$V_{yy} \equiv E[(Y-\mu_y)(Y-\mu_y)^T] = \frac{1}{2}\operatorname{Re}[\Gamma - C],$$

and conversely

$$\Gamma = V_{xx} + V_{yy} + i(V_{yx} - V_{xy}),\ \Gamma = V_{xx} - V_{yy} + i(V_{yx} - V_{xy}).$$

Theorem 3.2

Let $\psi_a(t)$ is missed observations on the stable stochastic process $[X_a(t)\ \ Y_a(t)]^T$, $a = 1,\ldots,\min(r,s)$ and $B_a(t)$ is Bernoulli sequence of random variables which satisfies equations (3.3) and (3.4), Let $h_a^{(T)}(\lambda)$ be defined as (3.1),and $d_a^{(T)}(\lambda)$ satisfies assumption, then $h_a^{(T)}(\lambda)$ will be distributed approximately as,

$$h_a^{(T)}(\lambda) \cong$$

$$N_{r+s}^{c}\left(\underline{0}, p_{a_1a_2}\begin{bmatrix} \int_R f_{a_1a_2}(v)\Omega_{a_1a_2}^{(T)}(\lambda_1 - v, \lambda_2 - v)dv & \int_R f_{a_1a_2}(v)A(\alpha)^T\Omega_{a_1a_2}^{(T)}(\lambda_1 - v, \lambda_2 - v)dv \\ \int_R A(\alpha)f_{a_1a_2}(v)\Omega_{a_1a_2}^{(T)}(\lambda_1 - v, \lambda_2 - v)dv & \int_R A(\alpha)f_{a_1a_2}(v)\Omega_{a_1a_2}^{(T)}(\lambda_1 - v, \lambda_2 - v)dv \end{bmatrix}\right), \tag{3.8}$$

where

$$\Omega_{a_1a_2}^{(T)}(\lambda_1 - v, \lambda_2 - v) = (2\pi)^{-1}\left[G_{a_1a_2}^{(T)}(0)\right]^{-1}\sum_{t_1=0}^{T-1} d_{a_1}^{(T)}(t_1)\sum_{t_2=0}^{T-1} d_{a_2}^{(T)}(t_2)\times$$
$$\times\exp\{-i[(\lambda_1 - v)t_1 - i(\lambda_2 - v)t_2]\} \tag{3.9}$$

Proof

From equations (3.1)and (3.5) we have,

$$E\{h_a(t)\} = 0\ , \tag{3.10}$$

$$Cov\{h_{a_1}^{(T)}(\lambda_1), h_{a_2}^{(T)}(\lambda_2)\} =$$
$$= Cov\left\{\left[2\pi\sum_{t_1=0}^{T-1}\left(d_{a_1}^{(T)}(t_1)\right)^2\right]^{-1/2}\sum_{t_1=0}^{T-1} d_{a_1}^{(T)}(t_1)\ \psi_{a_1}(t_1)\exp\{-i\lambda_1 t_1\},\right.$$
$$\left.,\left[2\pi\sum_{t_2=0}^{T-1}\left(d_{a_2}^{(T)}(t_2)\right)^2\right]^{-1/2}\sum_{t_2=0}^{T-1} d_{a_2}^{(T)}(t_2)\psi_{a_2}(t_2)\exp\{-i\lambda_2 t_2\}\right\}$$

$$= (2\pi)^{-1}\left[G_{a_1a_2}^{(T)}(0)\right]^{-1}\sum_{t_1=0}^{T-1} d_{a_1}^{(T)}(t_1)\exp\{-i\lambda_1 t_1\}\sum_{t_2=0}^{T-1} d_{a_2}^{(T)}(t_2)\exp\{i\lambda_2 t_2\}\times$$
$$\times Cov\{\psi_{a_1}(t_1), \psi_{a_2}(t_2)\}$$
$$= p_{a_1a_2}(2\pi)^{-1}\left[G_{a_1a_2}^{(T)}(0)\right]^{-1}\sum_{t_1=0}^{T-1} d_{a_1}^{(T)}(t_1)\exp\{-i\lambda_1 t_1\}\times$$

$$\times\sum_{t_2=0}^{T-1} d_{a_2}^{(T)}(t_2)\exp\{i\lambda_2 t_2\}\begin{bmatrix} c_{xx}(t_1 - t_2) & c_{xy}(t_1 - t_2) \\ c_{yx}(t_1 - t_2) & A(\alpha)c_{xx}(t_1 - t_2)A(\alpha)^T \end{bmatrix},$$

and

$$Cov\{h_{a_1}^{(T)}(\lambda_1), h_{a_2}^{(T)}(\lambda_2)\} = (2\pi)^{-1}\left[G_{a_1a_2}^{(T)}(0)\right]^{-1}\sum_{t_1=0}^{T-1} d_{a_1}^{(T)}(t_1)\exp\{-i\lambda_1 t_1\}\times$$
$$\times\sum_{t_2=0}^{T-1} d_{a_2}^{(T)}(t_2)\exp\{i\lambda_2 t_2\}$$
$$\times p_{a_1a_2}\begin{bmatrix} \int_{-\infty}^{\infty} f_{a_1a_2}(v)\exp\{iv(t_1 - t_2)\}dv & \int_{-\infty}^{\infty} f_{a_1a_2}(v)\exp\{iv(t_1 - t_2)\}dvA(\alpha)^T \\ A(\alpha)\int_{-\infty}^{\infty} f_{a_1a_2}(v)\exp\{iv(t_1 - t_2)\}dv & A(\alpha)\int_{-\infty}^{\infty} f_{a_1a_2}(v)\exp\{iv(t_1 - t_2)\}dvA(\alpha)^T \end{bmatrix}$$

$$= (2\pi)^{-1}\left[G_{a_1a_2}^{(T)}(0)\right]^{-1}\sum_{t_1=0}^{T-1} d_{a_1}^{(T)}(t_1)\sum_{t_2=0}^{T-1} d_{a_2}^{(T)}(t_2)\times$$

$$\times\, p_{a_1a_2}\exp\{-i\lambda_1 t_1 + i\lambda_2 t_2 + ivt_1 - ivt_2\}\begin{bmatrix}\int_{-\infty}^{\infty} f_{a_1a_2}(v)dv & \int_{-\infty}^{\infty} f_{a_1a_2}(v)A(\alpha)^T dv \\ A(\alpha)\int_{-\infty}^{\infty} f_{a_1a_2}(v)dv & A(\alpha)\int_{-\infty}^{\infty} f_{a_1a_2}(v)A(\alpha)^T dv\end{bmatrix}$$

$$= (2\pi)^{-1}\left[G_{a_1a_2}^{(T)}(0)\right]^{-1}\sum_{t_1=0}^{T-1} d_{a_1}^{(T)}(t_1)\sum_{t_2=0}^{T-1} d_{a_2}^{(T)}(t_2)\times$$

$$\times\, p_{a_1a_2}\exp\{-i[(\lambda_1 - v)t_1 - i(\lambda_2 - v)t_2]\}\begin{bmatrix}\int_{-\infty}^{\infty} f_{a_1a_2}(v)dv & \int_{-\infty}^{\infty} f_{a_1a_2}(v)A(\alpha)^T dv \\ A(\alpha)\int_{-\infty}^{\infty} f_{a_1a_2}(v)dv & A(\alpha)\int_{-\infty}^{\infty} f_{a_1a_2}(v)A(\alpha)^T dv\end{bmatrix}$$

$$= \begin{bmatrix}\beta_1 & \beta_2 \\ \beta_3 & \beta_4\end{bmatrix} \qquad (3.11)$$

where

$$\beta_1 = p_{a_1a_2}\int_{-\infty}^{\infty} f_{a_1a_2}(v)\left\{(2\pi)^{-1}\left[G_{a_1a_2}^{(T)}(0)\right]^{-1}\sum_{t_1=0}^{T-1} d_{a_1}^{(T)}(t_1)\sum_{t_2=0}^{T-1} d_{a_2}^{(T)}(t_2)\times\right.$$

$$\left.\times\exp\{-i[(\lambda_1 - v)t_1 - i(\lambda_2 - v)t_2]\}\right\}dv$$

$$= p_{a_1a_2}\int_R f_{a_1a_2}(v)\Omega_{a_1a_2}^{(T)}(\lambda_1 - v, \lambda_2 - v)dv,$$

similarly

$$\beta_2 = p_{a_1a_2}\int_R f_{a_1a_2}(v)A(\alpha)^T\Omega_{a_1a_2}^{(T)}(\lambda_1 - v, \lambda_2 - v)dv,$$

$$\beta_3 = p_{a_1a_2}\int_R A(\alpha)f_{a_1a_2}(v)\Omega_{a_1a_2}^{(T)}(\lambda_1 - v, \lambda_2 - v)dv,$$

and

$$\beta_4 = p_{a_1a_2}\int_R A(\alpha)f_{a_1a_2}(v)A(\alpha)^T\Omega_{a_1a_2}^{(T)}(\lambda_1 - v, \lambda_2 - v)dv$$

Now from equation (3.10) and (3.11) then equation (3.8) is obtained which complete the proof.

From equation (3.11) we can drive the following corollary by putting $\lambda_1 = \lambda_2 = \lambda$, $\lambda_1, \lambda_2, \lambda \in R.$

Corollary 3.1

let $h_a^{(T)}(\lambda)$, $a = 1,2,\ldots,\min(r,s)$, $\lambda \in R$ be defined as (3.1), then the dispersion of $h_a^{(T)}(\lambda)$ satisfies the following property :

$$Dh_a^{(T)}(\lambda) = p_{aa}\begin{bmatrix}\int_R f_{aa}(\lambda-\gamma)\Omega_{aa}^{(T)}(\gamma)d\gamma & \int_R f_{aa}(\lambda-\gamma)A(\alpha)^T\Omega_{aa}^{(T)}(\gamma)d\gamma \\ \int_R A(\alpha)f_{aa}(\lambda-\gamma)\Omega_{aa}^{(T)}(\gamma)d\gamma & \int_R A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T\Omega_{aa}^{(T)}(\gamma)d\gamma\end{bmatrix}, \qquad (3.12)$$

and

$$\Omega_{aa}^{(T)}(\lambda) = (2\pi)^{-1}\left[G_{aa}^{(T)}(0)\right]^{-1}\left|G_a^{(T)}(\lambda)\right|^2 ,$$

where $G_a^{(T)}(\lambda)$, $a = 1,2,\ldots,\min(r,s)$, $\lambda \in R$ be defined in Assumption .

Proof

From equation (3.11), we get

$$Dh_a^{(T)}(\lambda) = p_{aa}\begin{bmatrix}\int_R f_{aa}(v)\Omega_{aa}^{(T)}(\lambda-v)dv & \int_R f_{aa}(v)A(\alpha)^T\Omega_{aa}^{(T)}(\lambda-v)dv \\ \int_R A(\alpha)f_{aa}(v)\Omega_{aa}^{(T)}(\lambda-v)dv & \int_R A(\alpha)f_{aa}(v)A(\alpha)^T\Omega_{aa}^{(T)}(\lambda-v)dv\end{bmatrix},$$

When $\lambda_1 = \lambda_2 = \lambda, \lambda \in R$ and $a_1 = a_2 = a, a = 1,\ldots,\min(r,s)$.

By putting $\lambda - v = \gamma$,then formula (3.12) is obtained.

Theorem 3.3

For any $\lambda \in R$, the function $\Omega_{aa}^{(T)}(\lambda)$, $a = 1,\ldots,\min(r,s)$ is the kernel that satisfies the following properties:

1. $\int_{-\infty}^{\infty}\Omega_{aa}^{(T)}(\lambda)d\lambda = 1, a = 1,\ldots,\min(r,s),\ \lambda \in R$ (3.13)
2. $\underset{T\to\infty}{Lim}\int_{-\infty}^{-\delta}\Omega_{aa}^{(T)}(\lambda)d\lambda = \underset{T\to\infty}{Lim}\int_{\delta}^{\infty}\Omega_{aa}^{(T)}(\lambda)d\lambda = 0,$. $,\forall\delta > 0, a = 1,\ldots,\min(r,s), \lambda \in R$ (3.14)
3. $\underset{T\to\infty}{Lim}\int_{-\delta}^{\delta}\Omega_{aa}^{(T)}(\lambda)d\lambda = 1,$ $\forall\ a = 1,\ldots,\min(r,s), \delta > 0,\quad \lambda \in R.$ (3.15)

Theorem 3.4

If the spectral density function $f_{aa}(X), a = 1,\ldots,\min(r,s)$, $X \in R$ is bounded continuous at a point $X = \lambda, \lambda \in R$ and the function $\Omega_{aa}^{(T)}(X),\ a = 1,\ldots,\min(r,s),\quad X \in R$ satisfies the properties of theorem 3.3, then,

$$\lim_{T\to\infty} Dh_a^{(T)}(\lambda) = p_{aa}\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix},$$

$$,\ a = 1,\dots,\min(r,s). \qquad (3.16)$$

Proof

To prove formula (3.16), we must prove that

$$\lim_{T\to\infty}\left| Dh_a^{(T)}(\lambda) - p_{aa}\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\right| = 0,$$

Now , from corollary 3.1 we have,

$$\left| Dh_a^{(T)}(\lambda) - p_{aa}\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\right| \le$$

$$\le p_{aa}\int_{-\infty}^{\infty}\left|\begin{bmatrix} f_{aa}(\lambda-\gamma) & f_{aa}(\lambda-\gamma)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T \end{bmatrix} - \begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\right|\Omega_{aa}^{(T)}(\gamma)d\gamma \le$$

$$\le p_{aa}\int_{-\infty}^{-\delta}\left|\begin{bmatrix} f_{aa}(\lambda-\gamma) & f_{aa}(\lambda-\gamma)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T \end{bmatrix} - \begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\right|\Omega_{aa}^{(T)}(\gamma)d\gamma +$$

$$+ p_{aa}\int_{-\delta}^{\delta}\left|\begin{bmatrix} f_{aa}(\lambda-\gamma) & f_{aa}(\lambda-\gamma)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T \end{bmatrix} - \begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\right|\Omega_{aa}^{(T)}(\gamma)d\gamma +$$

$$+ p_{aa}\int_{\delta}^{\infty}\left|\begin{bmatrix} f_{aa}(\lambda-\gamma) & f_{aa}(\lambda-\gamma)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T \end{bmatrix} - \begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\right|\Omega_{aa}^{(T)}(\gamma)d\gamma$$

$$= J_1 + J_2 + J_3.$$

Since $f_{a_1a_2}(\gamma)$ is continuous at a point $\gamma = \lambda$, $a_1, a_2 = 1,\dots,\min(r,s)$, $\lambda \in R$, then we get

$$J_2 = p_{aa}\int_{-\delta}^{\delta}\left|\begin{bmatrix} f_{aa}(\lambda-\gamma) & f_{aa}(\lambda-\gamma)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T \end{bmatrix} - \begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\right|\Omega_{aa}^{(T)}(\gamma)d\gamma$$

$$= p_{aa}\int_{-\delta}^{\delta}\left|\begin{bmatrix} f_{aa}(\lambda-\gamma) - f_{aa}(\lambda) & f_{aa}(\lambda-\gamma)A(\alpha)^T - f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda-\gamma) - A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda-\gamma)A(\alpha)^T - A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\right|\times$$

$$\times\Omega_{aa}^{(T)}(\gamma)d\gamma \le \varepsilon\int_{-\delta}^{\delta}\Omega_{aa}^{(T)}(\gamma)d\gamma$$

$$\le \varepsilon\int_{-\infty}^{\infty}\Omega_{aa}^{(T)}(\gamma)d\gamma$$

Hence, $J_2 \le \varepsilon$. Now J_2 is very small according to any ε is very small, consequently $J_2 = 0$ Suppose that $f_{aa}(\lambda)$ $a = 1,\dots,\min(r,s), \lambda \in R$ is bounded by a constant M, then

$$J_1 \le 2M\int_{-\infty}^{-\delta}\Omega_{aa}^{(T)}(\gamma)d\gamma \xrightarrow[T\to\infty]{} 0,$$

according to property (3.14). similarly $J_3 \xrightarrow[T\to\infty]{} 0$, therefore,

$$\left| Dh_a^{(T)}(\lambda) - p_{aa}\begin{bmatrix} f_{aa}(\lambda) & f_{aa}(\lambda)A(\alpha)^T \\ A(\alpha)f_{aa}(\lambda) & A(\alpha)f_{aa}(\lambda)A(\alpha)^T \end{bmatrix}\right| \xrightarrow[T\to\infty]{} 0.$$

which completes the proof of the theorem.

Lemma 3.1

If the data window function $d_a^{(T)}(t), t \in R$, $a = \overline{1,r}$ is bounded and has bounded variations and equal zero outside the interval $[0, T-1]$; then

$$\sum_{t=0}^{T-1} d_a^{(T)}(t) \sim T\int_0^1 d_a^{(T)}(u)\,du, \qquad (3.17)$$

where,

$$\frac{1}{T}\sum_{t=0}^{T-1} d_a^{(T)}(t) \xrightarrow[T\to\infty]{} \int_0^1 d_a^{(T)}(u)\,du,\ a = \overline{1,r},\ T = 1,2,\dots \quad (3.18)$$

Lemma 3.2

Suppose $d_a^{(T)}(t), t \in R, \quad a = \overline{1,r}$ is bounded by a constant L and satisfying the Lipschitz condition,

$$\sum_{u=0}^{T-1} \left| d_a^{(T)}(t+u) - d_a^{(T)}(t) \right| -$$

$$-\sum_{t=0}^{T-1} d_{a_1}^{(T)}(t)\, d_{a_2}^{(T)}(t) \, \exp\{-i\lambda t\} \leq \varepsilon |u| , \qquad (3.19)$$

then,

$$\left| \sum_{t=0}^{T-1} d_{a_1}^{(T)}(u+t)\, d_{a_2}^{(T)}(t) \, \exp\{-i\lambda t\} - \right| \leq L\varepsilon |u| , \qquad (3.20)$$

for all constant ε, $u = \overline{[-(T-1),(T-1)]}$ and $\lambda \in [-\pi, \pi]$.

Lemma 3.3

For all $\lambda_1, \lambda_2 \in [-\pi, \pi]$, $(\lambda_1 - \lambda_2) \neq (\mathrm{mod}\ 2\pi)$ and $d_a^{(T)}(t),\ t \in R,\ a = 1,\ldots,\min(r,s)$ is bounded by a constant L and satisfying Lipschitz condition (3.19), then,

$$Cov\left\{h_{a_1}^{(T)}(\lambda_1), h_{a_2}^{(T)}(\lambda_2)\right\} \leq \frac{L\varepsilon}{2\pi \sqrt{\sum_{t_1,t_2=0}^{T-1} \left(d_{a_1}^{(T)}(\lambda_1)\right)^2 \left(d_{a_2}^{(T)}(\lambda_2)\right)^2}} \times$$

$$\times \left\{ \frac{1}{Lc|(\lambda_1-\lambda_2)/2|} \sum_{\tau=-T+1)}^{T-1} \left| C_{a_1 a_2}(u) \right| + \sum_{\tau=-T+1}^{T-1} \left| C_{a_1 a_2}(u) \right| [|u|+1] \right\}, \qquad (3.21)$$

for all $a_1, a_2 = 1,\ldots,\min(r,s)$.

Theorem 3.5

For all $\lambda_1, \lambda_2 \in [-\pi, \pi]$, $(\lambda_1 - \lambda_2) \neq (\mathrm{mod}\ 2\pi)$ and $d_a^{(T)}(t), t \in R,\ a = 1,\ldots,\min(r,s)$ is bounded and

$$\sum_{\tau=-\infty}^{\infty} [|u|+1] \left| C_{a_1 a_2}(u) \right| < \infty, \qquad (3.22)$$

then

$$\lim_{T \to \infty} Cov\left\{h_{a_1}^{(T)}(\lambda_1), h_{a_2}^{(T)}(\lambda_2)\right\} = 0, \qquad (3.23)$$

for all $a_1, a_2 = 1,\ldots,\min(r,s)$.

Proof

The proof comes directly from Lemma 3.3 and Lemma 3.1.

4.APPLICATIONS

We will apply our theoretical study in two cases in climate and economy as in the following sections.

4.1.Studying the temperature and solar radiation

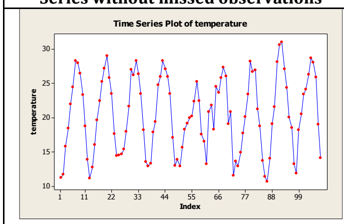
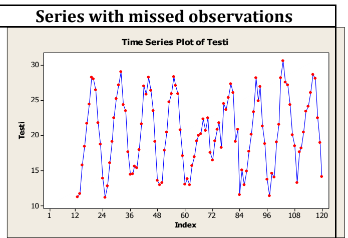
The data manipulated in this research make up a monthly chronic series that represents the average of the monthly temperature and solar radiation in Tripoli in Libya. The data is extracted from the meteorological centre of Tripoli, for the period from January 2005 to December 2013.

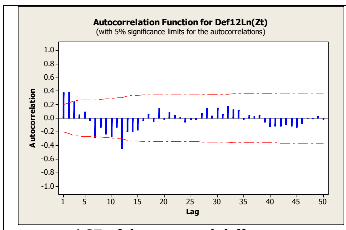
4.1.1.Studying the temperature

In this study we will comparison between our results, model of strictly stability time series (temperature) with some missing observations and the classical results, where all observations are available.

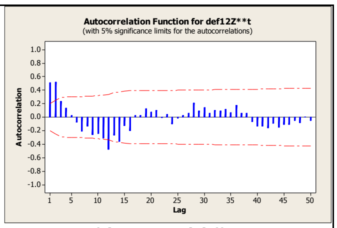
Let $\Phi_a(t) = B_a(t) X_a(t)$, $a = 1,2,\ldots\ldots, r$,where $X_a(t)$,$(t = 0,\pm 1,\ldots..)$ be a strictly stability r-vector valued time series and $B_a(t)$ is Bernoulli sequence of independent random variable of $X_a(t)$ which satisfies equations (3.3) and (3.4), we suppose that the data $X_a(t), t = (1,2,\ldots..,T]$ is the average of the monthly temperature, where all observations are available, $B = 1,\ \Phi_a(t) = X_a(t),$ which is the classical case suppose that there is some missing observations in a random way, i.e., $B = 0$, table 4.1.1 shows the comparison of these results with and without missed observations.

Table-4.1.1: The comparison of the results with and without missed observations

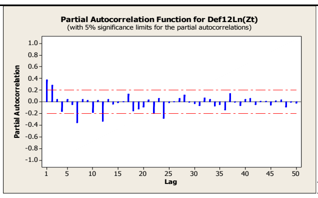
Series without missed observations	Series with missed observations
 The average monthly temperatures	 The average monthly temperatures



ACF of the seasonal difference

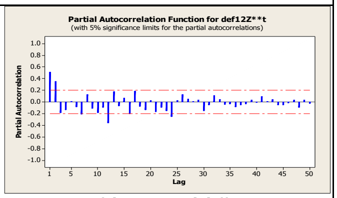


ACF of the seasonal difference



P

ACF of the seasonal difference



PACF of the seasonal difference

ARIMA Model: Temperature

$ARIMA(2,0,0)\times(0,1,1)12$

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
AR 1	0.3446	0.1036	3.33	0.001
AR 2	0.1522	0.1031	1.48	0.143
SMA 12	0.8445	0.0839	10.07	0.000
Constant	0.002439	0.002641	0.92	0.358

Differencing: 0 regular, 1 seasonal of order 12

Number of observations: Original series 108, after differencing 96

Residuals: SS = 1.03233 (back forecasts excluded)

MS = 0.01122 DF = 92

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	15.1	25.0	36.4	41.4
DF	8	20	32	44
P-Value	0.057	0.203	0.269	0.585

ARIMA Model: Temperature

$ARIMA(2,0,0)\times(0,1,2)12$

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
AR 1	0.5700	0.1058	5.39	0.000
AR 2	-0.0155	0.1059	-0.15	0.884
SMA 12	1.5643	0.1035	15.12	0.000
SMA 24	-0.6502	0.1014	-6.41	0.000
Constant	0.0011289	0.000935	1.21	0.230

Differencing: 0 regular, 1 seasonal of order 12

Number of observations: Original series 108, after differencing 96

Residuals: SS = 0.775840 (back forecasts excluded)

MS = 0.008526 DF = 91

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	10.7	20.8	44.1	53.4
DF	7	19	31	43
P-Value	0.153	0.35	0.059	0.133

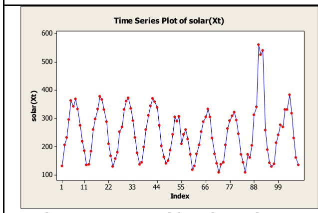
4.1.2.Studying the solar radiation

In this study we will comparison between our results, model of strictly stability time series (Solar Radiation) with some missing observations and the classical results, where all observations are available.

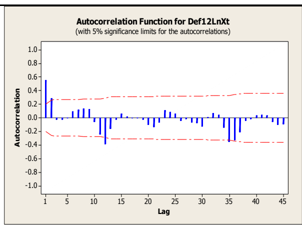
Let $\phi_a(t)=B_a(t)Y_a(t)$ $,a=1,2,\ldots\ldots,s,$ where $Y_a(t)$, $t=0,\pm1,\ldots..)$, be a strictly stability s-vector valued time series and $B_a(t)$ is Bernoulli sequence of random variable which is stochastically independent of $Y_a(t)$ which satisfies equations (3.3) and (3.4), we suppose that the data $Y_a(t)$, $t=(1,2,\ldots,T]$ is the average of the monthly temperature, where all observations are available, $B=1$, $\phi_a(t)=Y_a(t)$, which is the classical case, suppose that there is some missing observations in a random way, i.e., $B=0$, table 4.1.2 shows the comparison of these results with and without missed observations.

Table-4.1.2: The comparison of the results with and without missed observations of the solar radiation

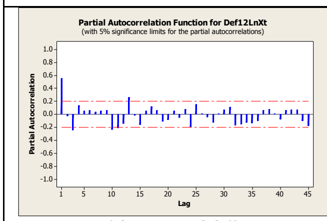
Series without missed observations



The average monthly solar radiation



ACF of the seasonal difference



PACF of the seasonal difference

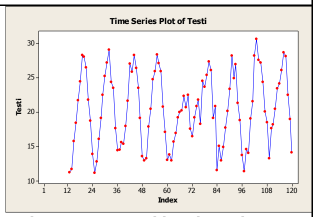
ARIMA Model: solar radiation without missed observations

$ARIMA(3,0,0)\times(0,1,2)12$

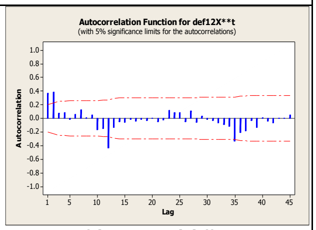
Final Estimates of Parameters

Type	Coef	SE Coef	T	P
AR 1	0.6443	0.1035	6.22	0.000
AR 2	0.1997	0.1218	1.64	0.105

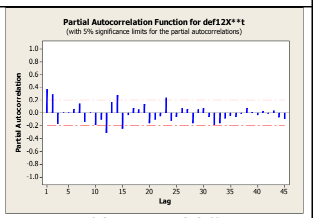
Series with missed observations



The average monthly solar radiation



ACF of the seasonal difference



PACF of the seasonal difference

ARIMA Model: solar radiation with missed observations

$ARIMA(3,0,0)\times(0,1,2)12$

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
AR 1	0.5507	0.0974	5.65	0.000
AR 2	0.4379	0.1044	4.19	0.000

AR 3 -0.3299 0.1004 -3.29 0.001	AR 3 -0.4197 0.0970 -4.33 0.000
SMA 12 0.9057 0.1119 8.09 0.000	SMA 12 1.0427 0.1089 9.58 0.000
SMA 24 -0.0570 0.1637 -0.35 0.729	SMA 24 -0.2015 0.1577 -1.28 0.205
Constant -0.1524 0.8091 -0.19 0.851	Constant -0.0518 0.7478 -0.07 0.945
Differencing: 0 regular, 1 seasonal of order 12	Differencing: 0 regular, 1 seasonal of order 12
Number of observations: Original series 108, after differencing 96	Number of observations: Original series 108, after differencing 96
Residuals: SS = 113197 (back forecasts excluded)	Residuals: SS = 122927 (back forecasts excluded)
MS = 1258 DF = 90	MS = 1366 DF = 90
Modified Box-Pierce (Ljung-Box) Chi-Square statistic	Modified Box-Pierce (Ljung-Box) Chi-Square statistic
Lag 12 24 36 48	Lag 12 24 36 48
Chi-Square 10.0 16.0 35.1 44.7	Chi-Square 8.5 21.3 41.1 51.2
DF 6 18 30 42	DF 6 18 30 42
P-Value 0.123 0.589 0.240 0.358	P-Value 0.206 0.266 0.085 0.15

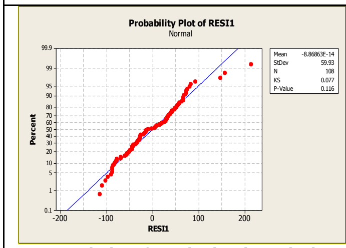
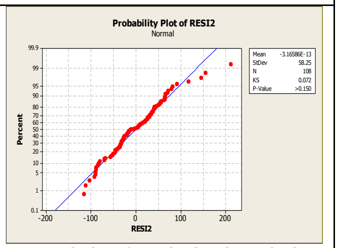
4.1.3. Studying The Regression Between Solar Radiation And Temperature

In this section we adjust the regression model which represents the relationship between Monthly rate of solar radiation in watt /m^2 rate and the average monthly temperature in the period from 2005 to 2013.

In this study we will comparison between our results with some missing observations and the classical results where all observations are available.

Let $Z(t)=[X(t) \quad Y(t)]^T$ where $X(t)$ is the series of average of temperature and $Y(t)$ is the series of the average of solar radiation, first we consider that the observations are available $P=1, \psi(t)=B(t)Z(t)=pZ(t)=Z(t)$, then consider that there are some missing of observations randomly, $P=0$. We used SPSS,MINITAB to investigate our results which is shown in table 4.1.3

Table - 4.1.3: The comparison of the results with and without missed observations of the regression analysis

Without missed observations	With missed observations
The regression equation is	**The regression equation is**
solar radiation = - 10.4 + 12.7 temperature	Solar = - 9.9 + 12.7 Temperature
Predictor Coef SE Coef T P	Predictor Coef SE Coef T P
Constant -10.42 22.36 -0.47 0.642	Constant -9.94 22.71 -0.44 0.663
temperature 12.685 1.055 12.02 0.000	Test. 12.726 1.074 11.84 0.000
S = 60.2159 R-Sq = 57.7% R-Sq(adj) = 57.3%	S = 58.5263 R-Sq = 57.0% R-Sq(adj) = 56.6%
Analysis of Variance	Analysis of Variance
Source DF SS MS F P	Source DF SS MS F P
Regression 1 523989 523989 144.5 0.00	Regression 1 480536 480536 140.3 0.00
Residual Error 106 384352 3626	Residual Error 106 363085 3425
Total 107 908341	Total 107 843621
 Normal-plot of standardized Residuals	 Normal-plot of standardized Residuals

4.1.4.Conclusion

1. Tables 4.1.1 and 4.1.2 shows the study of time series with missed observations and the original time series and we investigated that they have the same results.
2. Table 4.1.3 shows the study of regression model between Monthly average of solar radiation and average monthly temperature with some missed observations which had the same results of the study of the classical regression model.

4.2. Studying the Export and the Gross domestic product

The data manipulated in this research make up chronic series that represents the Export and the Gross domestic product. The data is extracted from the Central Bank of Libya for the period from 1970 to 2012.

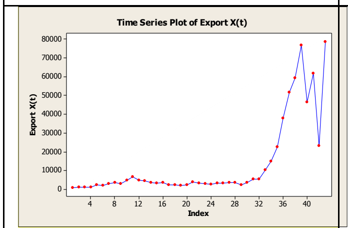
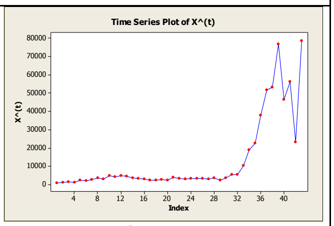
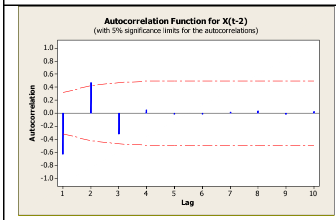
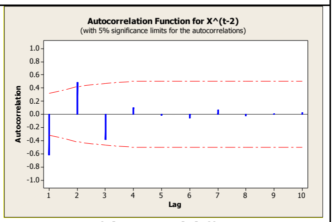
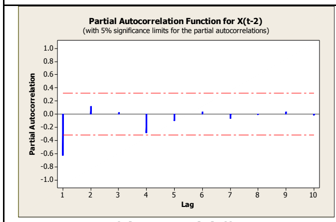
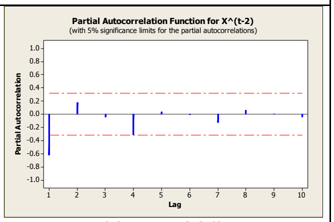
4.2.1. Studying the Export

In this study we will comparison between our results, model of strictly stability time series (Export) with some missing observations and the classical results, where all observations are available.

Let $\Phi_a(t)=B_a(t)X_a(t)\,, a=1,2,......,r$, where $X_a(t),(t=0,\pm1,.....)$ be a strictly stability r-vector valued time series and $B_a(t)$ is

Bernoulli sequence of independent random variable of $X_a(t)$ which satisfies equations (3.3) and (3.4), we suppose that the data $X_a(t), (t=(1,2,......,T]$ is the Export, where all observations are available, $B=1, \Phi_a(t)=X_a(t)$, which is the classical case suppose that there is some missing observations in a random way, i.e., $B=0$, table 4.2.1 shows the comparison of these results with and without missed observations.

Table -4.2.1: The comparison of the results with and without missed observations

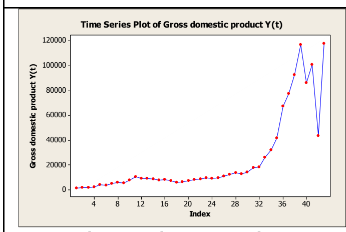
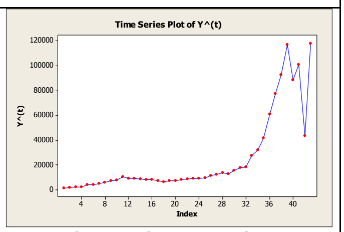
Series without missed observations	Series with missed observations
 The Export	 The Export
 ACF of the second difference	 ACF of the second difference
 PACF of the second difference	 PACF of the second difference
ARIMA Model: Export $ARIMA(1,2,1)$ **Final Estimates of Parameters** Type Coef SE Coef T P AR 1 -1.1606 0.1112 -10.44 0.000 MA 1 0.0554 0.2152 0.26 0.798 Constant -197 1271 -0.15 0.878 Differencing: 2 regular differences Number of observations: Original series 43, after differencing41	**ARIMA Model: Export** $ARIMA(1,2,1)$ **Final Estimates of Parameters** Type Coef SE Coef T P AR 1 -1.1559 0.1081 -10.69 0.000 MA 1 -0.0757 0.2246 -0.34 0.738 Constant 776 1596 0.49 0.630 Differencing: 2 regular differences Number of observations: Original series 43, after differencing 41
Residuals: SS = 2806388520 (back forecasts excluded) MS = 73852329 DF = 38 Modified Box-Pierce (Ljung-Box) Chi-Square statistic Lag 12 24 36 48 Chi-Square 12.8 13.3 14.7 DF 9 21 33 P-Value 0.173 0.899 0.998	Residuals: SS = 3338821601 (back forecasts excluded) MS = 87863726 DF = 38 Modified Box-Pierce (Ljung-Box) Chi-Square statistic Lag 12 24 36 48 Chi-Square 7.6 7.8 8.1 DF 9 21 33 P-Value 0.570 0.996 1.000

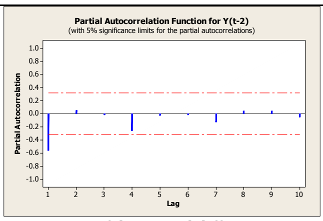
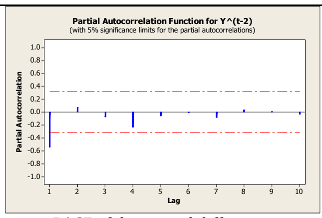
4.2.2. Studying the Gross domestic product

In this study we will comparison between our results, model of strictly stability time series (Gross domestic product) with some missing observations and the classical results, where all observations are available.

Let $\phi_a(t)=B_a(t)Y_a(t)$, $a=1,2,......,s$, where $Y_a(t), (t=0,\pm1,.....)$, be a strictly stability s-vector valued time series and $B_a(t)$ is Bernoulli sequence of random variable which is stochastically independent of $Y_a(t)$ which satisfies equations (3.3) and (3.4), we suppose that the data $Y_a(t)$, $t=(1,2,.....,T]$ is the Gross domestic product, where all observations are available, $B=1$, $\phi_a(t)=Y_a(t)$, which is the classical case, suppose that there is some missing observations in a random way, i.e, $B=0$, table 4.2.2 shows the comparison of these results with and without missed observations.

Table-4.2.2 The comparison of the results with and without missed observations of the Gross domestic product

Series without missed observations	Series with missed observations
 The Gross domestic product	 The Gross domestic product

 PACF of the second difference	 PACF of the second difference
ARIMA Model: Gross domestic product without missed observations $ARIMA(1,2,1)$ **Final Estimates of Parameters** Type Coef SE Coef T P AR 1 -1.1536 0.1637 -7.05 0.000 MA 1 0.2757 0.1961 1.41 0.168 Constant 1 1429 0.00 0.999 Differencing: 2 regular differences Number of observations: Original series 43, after differencing 41 Residuals: SS = 5983868951 (back forecasts excluded) MS = 157470236 DF = 38 Modified Box-Pierce (Ljung-Box) Chi-Square statistic Lag 12 24 36 48 Chi-Square 7.3 7.6 8.6 DF 9 21 33 P-Value 0.608 0.996 1.000	**ARIMA Model: Gross domestic product without missed observations** $ARIMA(1,2,1)$ **Final Estimates of Parameters** Type Coef SE Coef T P AR 1 -1.1266 0.1694 -6.65 0.000 MA 1 0.1373 0.2419 0.57 0.574 Constant 486 1725 0.28 0.780 Differencing: 2 regular differences Number of observations: Original series 43, after differencing 41 Residuals: SS = 6027685315 (back forecasts excluded) MS = 158623298 DF = 38 Modified Box-Pierce (Ljung-Box) Chi-Square statistic Lag 12 24 36 48 Chi-Square 8.3 8.5 8.9 DF 9 21 33 P-Value 0.509 0.993 1.000

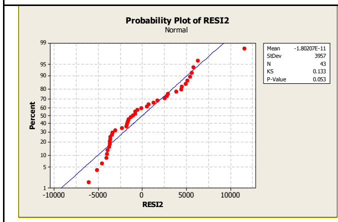
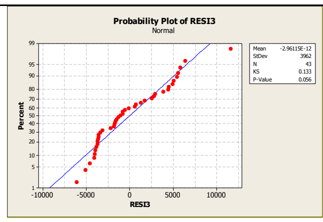
4.2.3. Studying the regression between Gross domestic product and Export

In this section we adjust the regression model which represents the relationship between the Gross domestic product and Export in the period from 1970 to 2012 million Libyan dinars.

In this study we will comparison between our results with some missing observations and the classical results where all observations are available.

Let $Z(t)=[X(t)\quad Y(t)]^T$ where $X(t)$ is the series of the Export average and $Y(t)$ is the series of the Gross domestic product, first we consider that the observations are available $P=1$, $\psi(t)=B(t)Z(t)=pZ(t)=Z(t)$, then consider that there are some missing of observations randomly, $P=0$. We used SPSS,MINITAB to investigate our results which is shown in table 4.2.3

Table-4.2.3: The comparison of the results with and without missed observations of the regression analysis

Without missed observations	**With missed observations**
Regression Analysis: Y(t) versus X(t) The regression equation is Y(t) = 4029 + 1.52 Export X(t) Predictor Coef SE Coef T P Constant 4029.2 724.2 5.56 0.000 Export X(t) 1.52488 0.02872 53.10 0.000 S = 4004.51 R-Sq = 98.6% R-Sq(adj) = 98.5% Analysis of Variance Source DF SS MS Regression 1 45207962941 45207962941 F P 2819.14 0.00 Residual Error 41 657479494 16036085 Total 42 45865442435	Regression Analysis: Y^(t) versus X^(t) The regression equation is Y^(t) = 4013 + 1.55 X^(t) Predictor Coef SE Coef T P Constant 4012.5 807.0 4.97 0.000 Export X^(t) 1.5544 0.03273 47.49 0.000 S = 4449.61 R-Sq = 98.2% R-Sq(adj) = 98.2% Analysis of Variance Source DF SS MS Regression 1 44646419639 44646419639 F P 2254.98 0.00 Residual Error 41 811760984 19799048 Total 42 45458180623
 Normal-plot of standardized Residuals	 Normal-plot of standardized Residuals

4.2.4. Conclusion

1. Tables 4.2.1 and 4.2.2 shows the study of time series with missed observations and the original time series and we investigated that they have the same results.
2. Table 4.2.3 shows the study of regression models between Gross domestic product and Export with some missed observations which had the same results of the study of the classical regression models.

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