

Modeling of Centrifugal Pendulum Vibration Absorber(CPVA) in Matlab

Rajneesh Septa¹, Ravi Verma²

¹Rajneesh Septa: UG Student, Dept. of Mechanical Engineering, Medicaps University, Madhya Pradesh, India

²Ravi Verma: Assistant Professor , Dept. of Mechanical Engineering, Medicaps University, Madhya Pradesh, India

Abstract- Centrifugal Pendulum Vibration Absorbers (CPVAs) is useful for engines since it reduces undesirable torsional vibration without decreasing performance because it does not depend on frequency but order of applied torque. The dynamic of CPVA in Matlab to present the rotor angle, rotor angular speed, absorber angle and absorber angular speed with respect to time and also show the position of absorber attachment point on rotor and absorber centre of mass and this can be Model with the help of applied external Torque. CPVAs use is to increasing powertrains of cars, in DMF (Dual Mass Flywheel). Today's engines tend to become smaller and smaller and fewer cylinder while power output should increase ,so the DMF plays an important role to reduce vibration upto 70 to 80% but ever increasing vibration create a need of using CPVAs.

Key Words: Torsional Vibration, Centrifugal Pendulum Vibration Absorbers, Dual Mass Flywheel, Matlab(Matrix Laboratory)

1.INTRODUCTION

There are numbers of vibration from the engine but only torsional vibration can be considered in the crankshaft and these vibration induces by inertia in moving parts such as piston and rods and combustion induced gas pressure in cylinder. For four cylinder engine the dominant harmonic is of second order because for every two revolution of the crankshaft two pulses in every revolution i.e. for a four stroke engine the engine will produce vibration with dominant excitation harmonic of half the number of cylinders. For Centrifugal Pendulum Vibration Absorber, it is used Lagrangian Formulation in generalized coordinates.

$$x_1 = x_1(q_1, q_2, t)$$

$$x_2 = x_2(q_1, q_2, t)$$

Where $q_1 = \theta$ is the rotor angle, $q_2 = \varphi$ is the pendulum angle relative to the pendulum attachment point on rotor. The equation for conservative system with holomic constraints (n-coordinates which describe the system) can be expressed as

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \left(\frac{\partial L}{\partial q_j} \right) = 0$$

Where

$$L = T - V$$

is named as Lagrangian. T is the kinetic energy of system,

V is the potential energy of system, and \dot{q}_j and q_j are the generalized velocities and generalized coordinates. And for Simple type CPVA the equation of motion of the second kind are as follows:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \left(\frac{\partial L}{\partial \theta} \right) = 0$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\varphi}} \right) - \left(\frac{\partial L}{\partial \varphi} \right) = 0$$

Notation

t	Time(s)
q_j	Jth generalized coordinate
\dot{q}_j	Jth generalized velocity
θ	CPVA unit rotor angle
φ	CPVA unit pendulum angle
T	Kinetic Energy
P	Potential Energy
L	Lagrangian, the difference in T and V
ω_n	Steady rotary speed of CPVA rotor
m	Total pendulum mass
J	Moment of inertia of rotor
I	Total moment of inertia pendulum for pendulums
y, x	Coordinate in inertial system in CPVA
R	distance from rotor centre to pendulum attachment point on rotor
r	distance from pendulum attachment point on rotor to pendulum centre of mass
k_g	radius of gyration for pendulum
c_s	viscous damping coefficient for rotor
c_a	viscous damping coefficient for pendulum absorber
T_a	external applied torque on CPVA unit
n	harmonic order
θ_0	small sinusoidal fluctuation of amplitude

2.TRANSFORMED INTO FIRST ORDER SYSTEM OF DIFFERENTIAL EQUATION

We can not insert the second order equation directly into the differential equation solver such as Runge-Kutta method solver ode45 in Matlab. The equation of lagrangian for CPVAs , are usually on this form

$$k_1(\varphi)\ddot{\theta} + k_2(\varphi)\ddot{\varphi} + k_3(\varphi)\dot{\theta}^2 + k_4(\varphi)\dot{\varphi}^2 + k_5(\varphi)\dot{\theta}\dot{\varphi} = 0$$

$$k_6(\varphi)\ddot{\theta} + k_7(\varphi)\ddot{\varphi} + k_8(\varphi)\dot{\theta}^2 + k_9(\varphi)\dot{\varphi}^2 + k_{10}(\varphi)\dot{\theta}\dot{\varphi} = 0$$

Introduced new variable to reduce into a first order

$$y(t) = \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \\ \varphi(t) \\ \dot{\varphi}(t) \end{bmatrix}$$

Its time derivative is

$$\frac{d}{dt}(y(t)) = \dot{y}(t) = \begin{bmatrix} \dot{\theta}(t) \\ \ddot{\theta}(t) \\ \dot{\varphi}(t) \\ \ddot{\varphi}(t) \end{bmatrix}$$

Now equation of system formulated as

$$A \dot{y}(t) = f$$

With the above equation, the equation become

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & k_1(y(3)) & 0 & k_2(y(3)) \\ 0 & 0 & 1 & 0 \\ 0 & k_6(y(3)) & 0 & k_7(y(3)) \end{bmatrix}$$

$$\dot{y}(t) = \begin{bmatrix} \dot{\theta}(t) \\ \ddot{\theta}(t) \\ \dot{\varphi}(t) \\ \ddot{\varphi}(t) \end{bmatrix}$$

$$f = \begin{bmatrix} y(2) \\ -(k_3(y(3))y(2)^2 + k_4(y(3))y(4)^2 + k_5(y(3))y(2)y(4)) \\ y(2) \\ -(k_3(y(3))y(2)^2 + k_4(y(3))y(4)^2 + k_5(y(3))y(2)y(4)) \end{bmatrix}$$

The formulation has been converted into first order system of differential equation and it can be inserted into Matlab. The second time derivative should be given overtly for simulation in Simdrive. Vector $\dot{y}(t)$

$$\dot{y}(t) = A^{-1}f$$

The acceleration, $\ddot{\theta}$ and $\ddot{\varphi}$, can be extracted in explicit form, from $\dot{y}(t)$. The differential equation are then unlinked and formulated as the first order system, i.e. functions of $\theta, \dot{\theta}, \varphi, \dot{\varphi}$ only.

$$\dot{y}(t) = \begin{bmatrix} \dot{\theta}(t) \\ \ddot{\theta}(t) \\ \dot{\varphi}(t) \\ \ddot{\varphi}(t) \end{bmatrix} = \begin{bmatrix} \dot{\theta}(t) \\ f(\theta, \dot{\theta}, \varphi, \dot{\varphi}, t) \\ \dot{\varphi}(t) \\ g(\theta, \dot{\theta}, \varphi, \dot{\varphi}, t) \end{bmatrix}$$

3. FUNTION OF CPVAs

Here the study of simple type CPVA, whose point mass pendulum of mass m is attached to a rotor of rotational inertia J . Pendulum is allowed to rotate freely about its attachment point, the assumption for this analysis is only restricted to 1-D, Gravitational and potential energy and coriolis effect is very small compared to Kinetic energy and also assume rigid body.

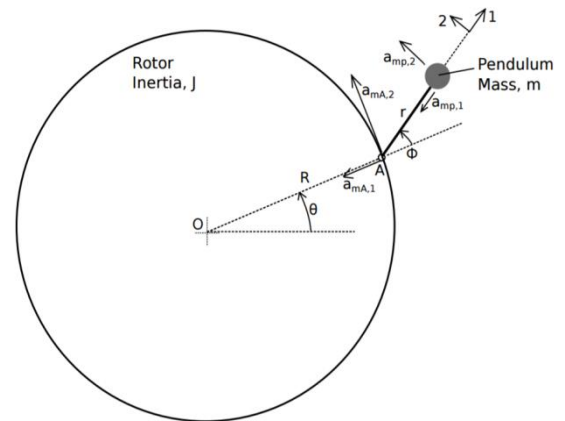


Fig-1: CPVA with point mass pendulum. The pendulum is fixed to the rotor at point A.

The acceleration components are, from figure 1:

$$a_{mA,1} = R\dot{\theta}^2$$

$$a_{mA,2} = R\ddot{\theta}$$

$$a_{mp,1} = r(\dot{\theta} + \dot{\varphi})^2$$

$$a_{mp,2} = r(\ddot{\theta} + \ddot{\varphi})$$

From Geometry the acceleration vector is found to be

$$a_m = [-a_{mp,1} - a_{mA,1} \cos(\varphi) + a_{mA,2} \sin(\varphi), a_{mp,2} + a_{mA,2} \cos(\varphi) + a_{mA,1} \sin(\varphi)]$$

Then

$$a_m = [-r(\dot{\varphi} + \dot{\theta})^2 - R\dot{\theta}^2 \cos(\varphi) + R\ddot{\theta} \sin(\varphi), r(\ddot{\varphi} + \ddot{\theta}) + R\ddot{\theta} \cos(\varphi) + R\dot{\theta}^2 \sin(\varphi)]$$

For moment about A is zero

$$M_A = m[r(\ddot{\theta} + \ddot{\varphi}) + R\ddot{\theta} \cos(\varphi) + R\dot{\theta}^2 \sin(\varphi)]r = 0$$

For pendulum angle, angle are small i.e. $\cos(\varphi) \approx 1$ and $\sin(\varphi) \approx \varphi$ This is the differential equation of pendulum and possible to analyze the CPVA behaviour. The natural frequency of pendulum is:

$$\omega_n = \sqrt{\frac{R}{r}}$$

4. CPVA WITH CIRCULAR PATH

For equation of motion of CPVA has converted into first-order ordinary differential equation, now Kinetic and Potential energy is needed for equation of motion

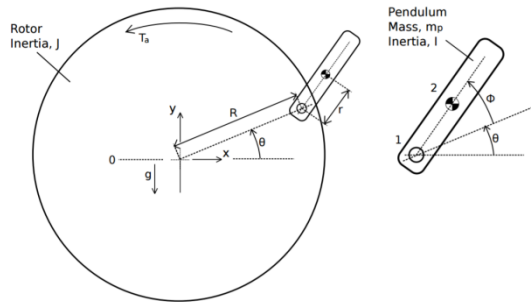


Fig -2: Free body diagram of Simple Type CPVA unit with circular

At first $T = \frac{1}{2}m|v|^2 + \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}I(\dot{\theta} + \dot{\phi})^2$

$$V = mg(R\sin(\theta) + r\sin(\theta + \phi))$$

The coordinates of CPVA, from figure 2, are:

$$\begin{aligned} x_1 &= R\cos(\theta) \\ y_1 &= R\sin(\theta) \\ x_2 &= R\cos(\theta) + r\cos(\theta + \phi) \\ y_2 &= R\sin(\theta) + r\sin(\theta + \phi) \end{aligned}$$

Derivation of equation of x_2 and y_2 , then velocity of the pendulum centre of mass is :

$$\begin{aligned} v &= (\dot{x}_2, \dot{y}_2) \\ |v|^2 &= \dot{x}_2^2 + \dot{y}_2^2 \end{aligned}$$

Moment of inertia is expressed as

$$I = mk_\lambda^2$$

After substitution in Langrangian as $L = T - V$, we get

$$\begin{aligned} L = \frac{1}{2} & (J\dot{\theta}^2 + mk_\lambda^2(\dot{\theta} + \dot{\phi})^2 + m[(R\dot{\theta} \cos(\theta) \\ & + r(\dot{\theta} + \dot{\phi}) \cos(\theta + \phi))^2 \\ & + (R\dot{\theta} \sin(\theta) + r(\dot{\theta} + \dot{\phi}) \sin(\theta + \phi))^2] \\ & - mg(R\sin(\theta) + r\sin(\theta + \phi)) \end{aligned}$$

The Lagrangian equation of motion for simple type CPVA are:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = T_a - c_s \dot{\theta}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\phi}} \right) - \frac{\partial L}{\partial \phi} = -c_a \dot{\phi}$$

After substitution in equation of we get second order differential equation, to reduce equation to a first order system of ODEs, method of section II are used, from expression.

$$k_1 = m(R^2 + r^2 + k_\lambda^2) + J$$

$$k_2 = mRr$$

$$k_3 = m(r^2 + k_\lambda^2)$$

The equation become

$$A\dot{y}(t) = f$$

With,

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & k_1 + k_2 \cos(y(3)) & 0 & k_3 + k_2 \cos(y(3)) \\ 0 & 0 & 1 & 0 \\ 0 & k_3 + k_2 \cos(y(3)) & 0 & k_3 \end{bmatrix}$$

$$f = \begin{bmatrix} y(2) \\ T_a - c_s y(2) + k_2 y(4)(2y(2) \sin(y(3)) + y(4) \sin(y(3))) \\ y(4) \\ -c_a y(4) - (k_2 y(2)^2 \sin(y(3))) \end{bmatrix}$$

This matrix equation is appropriate for simulation of CPVA unit in Matlab. The natural frequency of the Simple type CPVA is:

$$\omega_n = \Omega \sqrt{\frac{Rr}{(r^2 + k_\lambda^2)}}$$

And the harmonic tuning order of the CPVA is:

$$\frac{\omega_n}{\Omega} = n = \sqrt{\frac{Rr}{(r^2 + k_\lambda^2)}}$$

5. CONCLUSION

In the Matlab Dynamic Simulation, all five figures satisfy the equation of Simple type CPVA. The first figure shows the Rotor Angle, second figure shows the Rotor angular speed, third Absorber angle, fourth Absorber angular speed and fifth calculate the position of absorber point on rotor and absorber centre of mass with the help of Runge-Kutta differential equation solver.

6. MATLAB CODE FOR DYNAMIC SIMULATION OF CPVA

```
clear all
close all

R=0.17;
r=0.04;
m=1;
J=0.5;
klambda=0.004;
Initspeedtheta=(1000)*2*pi/60;
Initposabsorber=(0)/180*pi;
n=sqrt((r*R)/(r^2+klambda^2));
t0=0; tf=5; y0=[0 Initspeedtheta Initposabsorber 0]';
tspan=[t0 tf];
[t,y]=ode45('dynamicsystem',tspan,y0);
theta=y(:,1);
thetadot=y(:,2);
phi=y(:,3);
```

```

phidot=y(:,4);

figure(1) % Plot rotor angle
plot(t,theta/pi*180), grid
xlabel('time [s]')
ylabel('theta(t), [degrees]')

figure(2) % Plot rotor angular speed
plot(t,thetadot/pi*180), grid
xlabel('time [s]')
ylabel('thetadot(t), [degrees/s]')

figure(3) % Plot absorber angle
plot(t,phi/pi*180), grid
xlabel('time [s]')
ylabel('phi(t), [degrees]')

figure(4) % Plot absorber angular speed
plot(t,phidot/pi*180), grid
xlabel('time [s]')
ylabel('phidot(t), [degrees/s]')

% Calculate positions of absorber attachment point on rotor
x1=R.*sin(theta);
y1=-R.*cos(theta);

% Calculate positions for absorber centre of mass
x2=R.*sin(theta)+r.*sin(theta+phi);
y2=-R.*cos(theta)-r.*cos(theta+phi);

% Plot the above positions
figure(5)
plot(x1,y1)
hold on
plot(x2,y2)
axis([- (R+r)*1.1 (R+r)*1.1 - (R+r)*1.1 (R+r)*1.1])

```

```

k3=m*(r^2+klambda^2);

A=[1 0 0 0;
0 k1+2*k2*cos(y(3)) 0 k3+k2*cos(y(3))
0 0 1 0;
0 k3+k2*cos(y(3)) 0 k3];

f=[y(2);
k2*(2*y(2)*sin(y(3))+y(4)*sin(y(3)))*y(4)+Ta-cs*y(2);
y(4);
-k2*sin(y(3))*(y(2))^2-ca*y(4)];
ydot=A\f;

```

REFERENCES

- [1] A.S. Alsuwaiyan and S.W.Shaw. Performance and Dynamic stability of General-Path Centrifugal Pendulum Vibration Absorber. Journal of sound and Vibration,252(5),791-815,2002.
- [2] A.G. Haddow and S.W.Shaw. Centrifugal Pendulum Vibration Absorbers : An Experiment and theoretical Investigation. Nonlinear Dynamic, 34:293-307,2003.
- [3] T.M. Nester Experiment Investigation of Circular Path Centrifugal Pendulum Vibration Absorbers. Master thesis. Michigan state University,2002.
- [4] A. Wedin,"Reduction of Vibration in Engine using Centrifugal Pendulum Vibration Absorbers," Chalmers university of Techology, Gothenburg,2011.

```

function ydot=dynsystem(t,y)
R=0.17;
r=0.04;
m=1;
J=0.5;
klambda=0.004;
A=60;
a=50;
n=2;
Ta=(A+a*sin((n)*y(1)));
ca=0.1;
cs=0;

k1=m*(R^2+r^2+klambda)+J;
k2=m*R*r;

```