Comparison of MOC and Lax FDE for simulating transients in Pipe Flows

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Abstract - ***The method of characteristic (MOC) approach transforms the water hammer partial differential equations into ordinary differential equations along characteristic lines. The fixed-grid MOC is the most accepted procedure for solving the water hammer equations and has the attributes of being simple to code, efficient, accurate and provides the analysts with full control over the grid selection. Some authors are of the opinion that Lax Finite Difference Explicit method provides more convincing results for solving unsteady transient situations in pipe flow. Here an approach is made to compare the MOC and Lax FDE scheme of discretization for hydraulic transient governing equation, with the help of MATLAB as the programming tool and finally Lax FDE scheme is observed to be more effective.***

Key Words: Hydraulic pipe transients, water hammer, valve, numerical model, discharge, velocity, pressure etc.

1. INTRODUCTION:

The variation in discharge and pressure head can be studied by solving the governing equations for hydraulic transients in a pipe using the Method of Characteristics for discretization of the partial differential equations and also by Lax Finite Difference Explicit method. Due to the non-linearity of the governing equations, various numerical approaches have been developed for pipeline transient calculations, which include the Method of Characteristics (MOC), Finite Difference (FD) and Finite Volume (FV) etc. Among these methods, MOC proved to be the most popular among the water hammer analysts. In fact, out of the 14 commercially available water hammer software packages found on the world wide web, 11 are based on MOC, two are based on implicit FD method [11]. After the Finite Difference Equations (FDE) are obtained, the numerical models are developed using MATLAB. The models are then validated using lab data. Chudhury M.H. [13] advocated comparative effectiveness of Lax FDE method over MOC, which has been analyzed and observed here.

2. Literature Review:

The basic unsteady flow equations along pipe due to closing of the valve near the turbine are non-linear and hence its analytical solution is not possible. Watt C.S.et al (1980)[1] have solved for rise of pressure by MOC for only 1.2 seconds and the transient friction values have not been considered. Goldberg D.E. and Wylie B.(1983)[2] used the interpolations in time, rather than the more widely used spatial interpolations, demonstrates several benefits in the application of the method of characteristics (MOC) to wave problems in hydraulics. Chudhury M.H. and Hussaini M.Y.(1985)[3] solved the water hammer equations by MacCormack, Lambda, and Gabutti explicit FD schemes. Sibetheros I. A. et al. (1991) [4] investigated the method of characteristics (MOC) with spline polynomials for interpolations required in numerical water hammer analysis for a frictionless horizontal pipe. Silva-Arya W.F.and Choudhury M.H.(1997)[5] solved the hyperbolic part of the governing equation by MoC in one dimensional form and the parabolic part of the equation by FD in quasi-two-dimensional form. Pezzinga G. (1999) [6] presented both quasi 2-D and 1-D unsteady flow analysis in pipe and pipe networks using finite difference implicit scheme. Pezzinga G. (2000) [7] also worked to evaluate the unsteady flow resistance by MoC. He used Darcy-Weisback formula for friction and solved for head oscillations up to 4 seconds only. Damping with constant friction factor is presented but not much pronounced, as the solution time was very small. Bergant A. et al (2001) [8] incorporated two unsteady friction models proposed by Zielke W. (1968) [9] and Brunone B. et al.(1991)[10] into MOC water hammer analysis. Zhao M. and Ghidaoui M.S. (2004)[11] formulated, applied and analyzed first and second –order explicit finite volume (FV) Godunov-type schemes for water hammer problems. They have compared both the FV schemes with MoC considering space line interpolation for three test cases with and without friction for Courant numbers 1, 0.5.0.1.They modeled the wall friction using the formula of Brunone B. et al (1991) [10]. It has been found that the First order FV Gadunov scheme produces identical results with MoC considering space line interpolation. They advocated that although different approaches such as FV, MOC, FD and finite element (FE) provide an entirely different framework for conceptualizing and representing the physics of the flow, the schemes that result from different approaches can be similar and even identical. Barr D.I.H. (1980)[12] formulated friction losses, while Chudhury M.H. (1994)[13] advocated comparative effectiveness of Lax FDE method over MOC. Saikia M.D. and Sarma A.K.. (2006)[14] also compared Lax FDE method in their approach and found compatible results.

3. GOVERNING EQUATION :

The basic equations of continuity and momentum in unsteady flow along pipe due to closing of the valve near the turbine may be written as:

Continuity:

$$\frac{\delta H}{\delta t}+\frac{a^2}{gA}\frac{\delta Q}{\delta x}=0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Momentum:

$$\frac{\delta H}{\delta x}+\frac{1}{gA}\frac{\delta Q}{\delta t}+\frac{f}{2gDA^2}Q|Q|=0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where, H= pressure head, A = area of pipe or conduit, a=velocity of pressure wave, Q= discharge, g= acceleration due to gravity, t = time, f =friction factor, D= diameter of pipe or conduit x = distance along the pipe.

4. METHOD OF CHARACTERISTIC (MOC):

Method of characteristic (MOC) is the method which is used to solve the governing equation of the flow of fluid through the pipe. In this method the non-linear second order partial differential equation is converted into a second order ordinary differential equation (ODE). The ODE is then discretized to form the algebraic equation, which is then solved numerically using a computer program.
The discretized equations thus obtained are as follows:-

$$H_k^{\,j+1}=\frac{1}{2}\left(H_{k-1}^{j}+H_{k+1}^{j}\right)+\frac{a}{2gA}\left(Q_{k-1}^{j}-Q_{k+1}^{j}\right)+\frac{af\Delta t}{4gDA^2}\left(Q_{k+1}^{j}\left|Q_{k+1}^{j}\right|-Q_{k-1}^{j}\left|Q_{k-1}^{j}\right|\right)$$

$$Q_k^{\,j+1}=\frac{1}{2}\left(Q_{k-1}^{j}-Q_{k+1}^{j}\right)-\frac{gA}{2a}\left(H_{k-1}^{j}+H_{k+1}^{j}\right)-\frac{af\Delta t}{4gDA^2}\left(Q_{k-1}^{j}\left|Q_{k-1}^{j}\right|-Q_{k+1}^{j}\left|Q_{k+1}^{j}\right|\right)$$

5. LAX FINITE DIFFERENCE EXPLICIT METHOD (LAX FDE)

Chaudhury[25] claims that Lax explicit method yields satisfactory results in nonlinear partial difference equation with smaller time step provided initial and boundary conditions are correctly imposed. Although smaller time step apparently would increase the volume of computation time, much iteration needed in implicit method is saved leading to a net decrease in time. Hence, Lax Diffusive method has been considered for comparison.
In Lax finite difference explicit method the equation (1) and (2) have been converted to:

$$H_k^{\,j+1}=\frac{1}{2}\left(H_{k-1}^{j}+H_{k+1}^{j}\right)-\frac{a^2\Delta t}{gA^2}\frac{1}{2\Delta x}\left(Q_{k+1}^{j}-Q_{k-1}^{j}\right)$$

$$Q_k^{\,j+1}=\frac{1}{2}\left(Q_{k-1}^{j}+Q_{k+1}^{j}\right)+\frac{gA\Delta t}{2\Delta x}\left(H_{k+1}^{j}-H_{k-1}^{j}\right)-\frac{f\Delta t}{8gA}\left(\left(Q_{k-1}^{j}+Q_{k+1}^{j}\right).\left|Q_{k-1}^{j}+Q_{k+1}^{j}\right|\right)$$

6. BARR'S FRICTION EQUATION (UNSTEADY/VARIABLE FRICTION EQUATION)

The friction factor f in the above equation is replaced by the following Barr's explicit approximations which covers full range of flow conditions, from laminar to turbulent.

$$\frac{1}{\sqrt{f}}=-2\log_{10}\left[\frac{5.02\log_{10}\left(R_e/4.518\log_{10}\left(R_e/7\right)\right)}{R_e\left(1+R_e^{\,0.52}/29\left(D/k\right)^{0.7}\right)}+\frac{1}{3.7\left(D/k\right)}\right]$$

Where,
f = friction factor
k = sand roughness coefficient
D = Diameter of pipe
R_e = Reynold's number

7. IMPLEMENTATION OF DEVELOPED NUMERICAL MODEL TO THE SIMILAR PROBLEM AS MENTIONED BY SAIKIA M.D. AND SARMA A.K. (2006)[14]

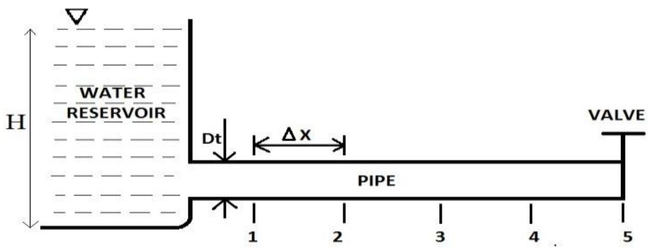

Fig -1: Schematic representation of water hammer situation without surge tank (considering 4 sections of the pipe)

The numerical model is implemented to the data given by Saikia M.D. and Sarma A.K. (2006)[14]. The pipe is divided into 4 sections of equal length, which means there are 5 locations for the calculations. The lab data is given as follows:-
Length of the pipe = 12,000 ft
Discharge = 20 ft^3/sec
Initial Pressure Head at the different locations:
Location 1 (Reservoir end) = 600 ft
Location 2 = 587.5 ft
Location 3 = 565 ft

Location 4 = 547.5 ft
Location 5 (Valve end) = 530 ft
Diameter of pipe = 2 ft
Area of valve opening = 3.1416 ft^2

Surface roughness coefficient = 0.007093 ft

Kinematic Viscosity = 0.000001 ft^2/sec
Coefficient of discharge = 0.90
Velocity of pressure wave = 3000 ft/sec

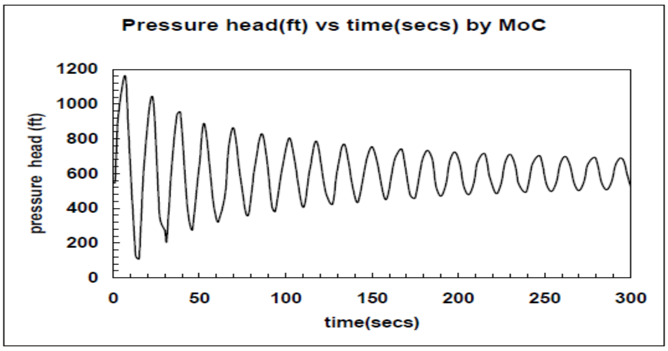

Fig -2: Pressure Head vs time at pipe position, x=5 (from Numerical Model by Saikia M.D. and Sarma A.K.)

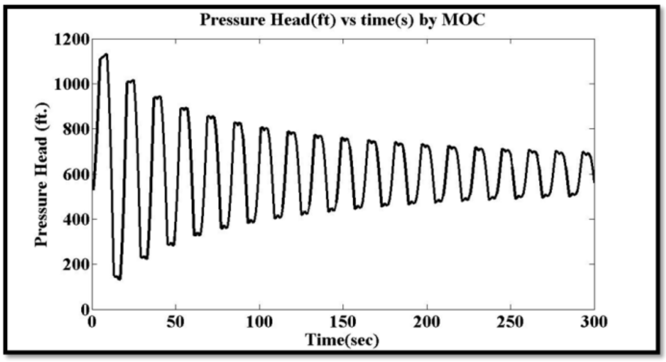

Fig -3: Pressure head vs time at pipe position, x =5 (Developed Numerical Model by MOC and using data from Saikia M.D. and Sarma A.K.)

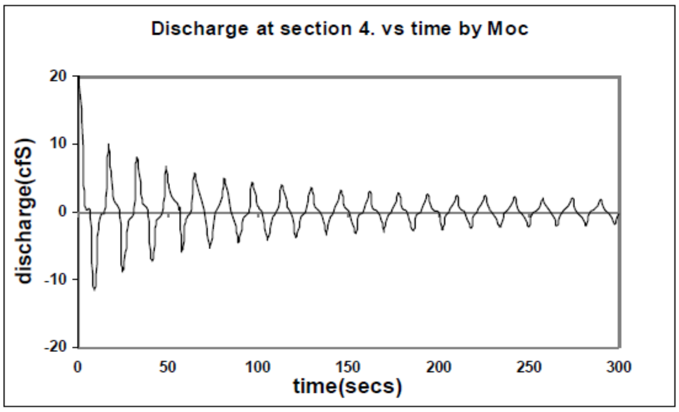

Fig -4: Discharge vs time at pipe position, x =4 (from Numerical Model by Saikia M.D. and Sarma A.K.)

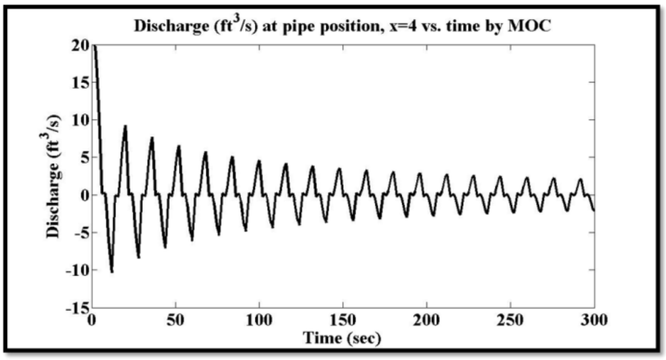

Fig -5: Discharge v/s time at pipe position. x =4 (Developed Numerical Model by MOC method and using data from Saikia M.D. and Sarma A.K.)

From the above analysis it is found that the developed numerical model with MOC using Barr's friction equation is in excellent agreement with the results obtained by Saikia M. D. and and Sarma A. K (2006)[14]. Therefore our algorithm can be used to compare different numerical models to solve hydraulic transient in pipe flow without surge tank.
For the comparision between the two numerical methods viz. MOC and Lax FDE we have taken the hydraulic trainsient case with the input parameters from the quoted reference, Saikia M.D. and Sarma A. K.(2006)[14].
Therefore the output data using MOC as a numerical scheme with variable friction is plotted as shown in below.

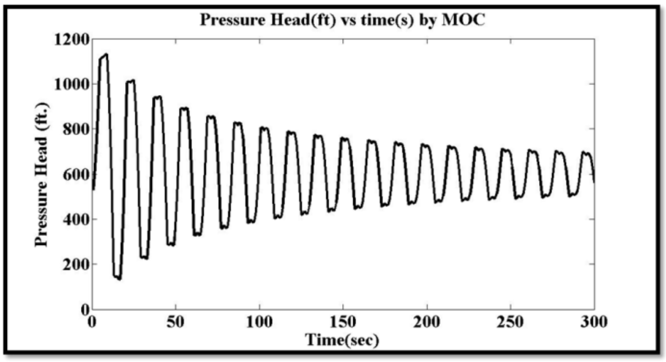

Fig -6: Pressure head vs time at pipe position, x =5 (Developed Numerical Model by MOC method and using data from Saikia M.D. and Sarma A. K.)

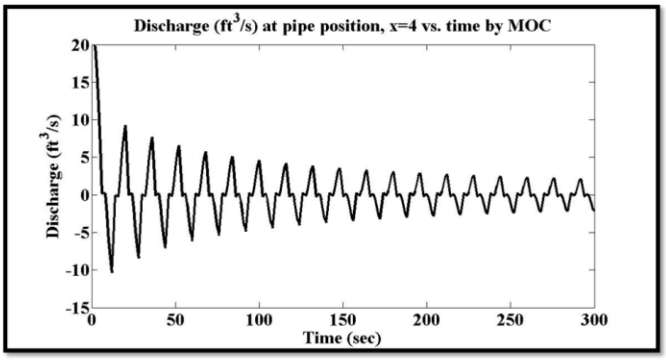

Fig -7: Discharge vs time at pipe position, x =4 (Developed Numerical Model by MOC method and using data from Saikia M.D. and Sarma A.K.)

Now applying the developed numerical model using Lax FDE method to the hydraulic transient case discussed above the following results are obtained and plotted as shown graphically. In this case we have considered Barr's unsteady/variable friction equation to calculate the friction factor.

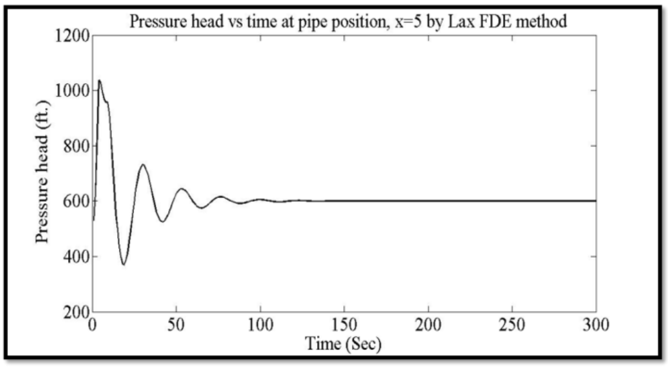

Fig -8: Graph for variation of pressure head vs time (at pipe position, x=5) (by developed numerical model using LAX FDE method and using data from Saikia M.D. and Sarma A.K.)

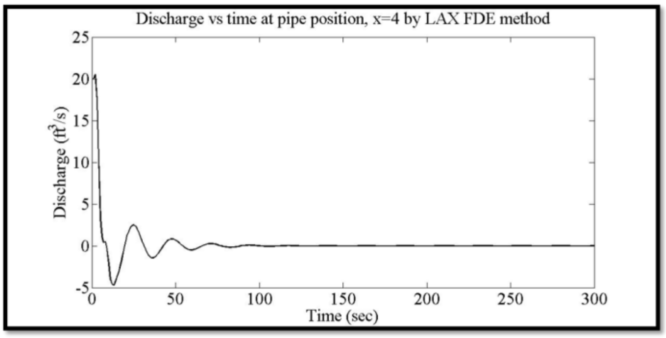

Fig -9: Graph for variation of Discharge vs. time (at pipe position, x= 4) (by developed numerical model using LAX FDE method and using data from Saikia M.D. and Sarma A.K.)

8. CONCLUSIONS

As seen from the above analysis the Lax FDE proves to be more advantageous than MOC for simulating transients in pipe .More over the damping effect of the fluctuations is more evident if we use Lax FDE method compared to MOC method.Hence Lax FDE method is much better numerical method to calculate hydraulic transient fluctuations with surge tank in case of pipe flow.

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