Recent Development in Modelling and Analysis of Functionally Graded Materials

Hari Krishnan K P[1], Shashi Dharan [2]

[1]Mtech Scholar, Department of Civil Engineering, NSS College of Engineering, Palakkad, Kerala, India
[2]Professer, Department of Civil Engineering, NSS College of Engineering, Palakkad, Kerala, India

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Abstract - *Functionally graded materials belong to a class of advanced materials characterised by the variation in properties as the dimension varies (usually along the thickness). The overall properties of FGM can be varied according to our needs, thus one of the main advantages of such a material is that it can be tailored specifically for serving a particular function that makes it unique from any of the base materials used in its synthesis. FGMs can be used to avoid problems associated with the presence of an interface in a material: stress singularities due to thermal or elastic property mismatch, unwanted reflections or poor adhesion at the interface. The Ceramic-Metal FGMs can be designed to reduce thermal stresses and take advantage of the corrosion and heat resistances of ceramic and the mechanical strength, good machinability , high toughness and bonding capability of metals without severe internal thermal stresses, also exhibit higher fracture resistance parameters resulting in higher toughness due to bridging of cracks in a graded volume fraction.*

Various methods and theories for modelling and analyses of functionally graded materials had been reviewed. The characteristics, applications and advantages of functionally graded materials are also briefly described.

Key Words: Functionally graded materials, Homogenisation, Shear deformation theories, Plates.

1.INTRODUCTION

Functionally graded materials(FGM) as their name suggest are those materials that are synthesised for a specific purpose by the gradual mixing of two or more different materials so that the properties of each of the base materials can be used according to the external environment. Unlike laminated composite materials where there exist a boundary or interface between two constituent materials, there is no such interface in the case of FGM. The variation of properties is as smooth as possible thus avoiding the phenomena of stress concentration which could lead to the development or propagation of fractures, thus avoiding delamination.

They are also defined as high performance, microscopically anisotropic materials engineered with great precision in gradients of composition and structure to adapt to various specific purposes and to have definite properties in preferred orientation. The desired mechanical properties of FGMs i.e. Poisson's ratio, Young's modulus, shear modulus and material density can be obtained in a preferred direction through the variation of volume fractions of the constituent materials spatially.

The concept of FGM was first introduced in Japan in 1984 during a space plane project. Where a combination of materials was required that would form a thermal barrier capable of withstanding a surface temperature of about 2000 K and a temperature gradient of the order 1000 K across a 10-mm section. In recent years, this concept has become more popular in different European countries, particularly in Germany. A transregional collaborative research canter (SFB Transregio) is funded since 2006 in order to use maximum potential of grading mono materials, such as aluminium, steel and polypropylene, by using thermo-mechanically coupled manufacturing processes.

The basic structural units of FGMs are elements represented by maxel. The term maxel was coined in 2005 by Rajeev Dwivedi and Radovan Kovacevic at Research Centre for Advanced Manufacturing (RCAM). The attributes of maxel include the location and volume fraction of individual material components. A maxel is also used to represent various methods of the additive manufacturing (such as stereo-lithography, selective laser sintering, fused deposition modelling, etc.) to describe the physical voxel (as derived from the words 'volume' and 'pixel'), that defines the build resolution of either a rapid manufacturing or rapid prototyping process, or the total resolution of a design produced by such fabrication methods.

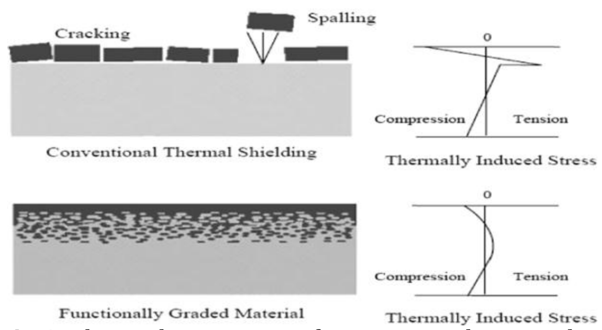

Fig.1. Thermal protection of conventional material and FGM

Fig.2. Schematic representation of a FGM plate.

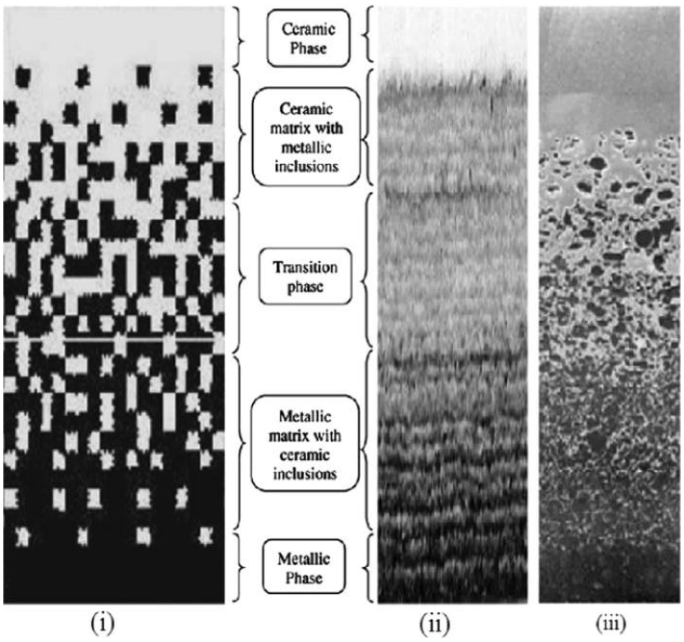

Fig.3. Gradation of microstructure with metal–ceramic constituents*:* (i) smoothly graded microstructure, (ii) enlarged view and (iii) ceramic–metal FGM

2. APPLICATIONS

Some of the applications of functionally graded materials are highlighted below:

1. ***Aerospace:*** Functionally graded materials can withstand very high thermal gradient, this makes it suitable for use in structures and rocket engine component, space plane body etc. If only the processing technique is improved, FGM are promising and can be used in wider areas of aerospace.
2. ***Medicine:*** Living tissues like teeth and bones are examples of functionally graded material from nature, to replace these tissues, a compatible material is needed that will serve the purpose of the original tissue. The ideal material for this application is functionally graded material. FGM has find wide range of application in dental and orthopaedic applications for teeth and bone replacement
3. ***Defense:*** One of the most important characteristics of functionally graded material is the ability to inhibit crack propagation. This property makes it ideal material to be used in defence application, as a penetration resistant materials used for armour plates and bullet-proof vests.
4. ***Energy****:* FGM are used in energy conversion devices. They can be used in making thermal barriers and are used as protective coating on turbine blades in gas turbine engine.
5. ***Optoelectronics****:* FGM also finds its application in optoelectronics as graded refractive index materials and in discs' magnetic storage media.

Other areas of application are: cutting tool insert coating, nuclear reactor components, automobile engine components, turbine blade, heat exchanger, Tribology, fire retardant doors, sensors etc. The list is endless and more application is springing up as the cost of production, processing technology and properties of FGM improve.

3. ADVANTAGES

1. FGMs are composite materials obtained from the mixtures of two or more constituent phases with continuously varying composition along any dimension.
2. Reduction of in-plane stresses and transverse through thickness stresses.
3. Improved thermal properties.
4. Superior fracture toughness
5. Reduction of stress intensity factors.

FGMs are considered to be potential substitute to traditional laminated composite materials as they can mitigate some disadvantage associated with laminates, namely the delamination.

4. MATERIAL PROPERTIES GRADATION

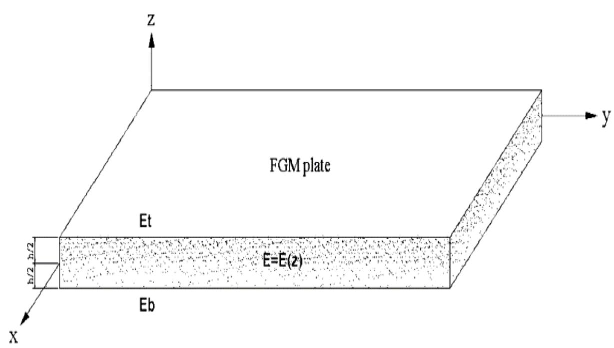

Fig.4. FGM geometry

The material properties gradation in FGM is assumed to follow power law function, exponential function etc.

1. ***Exponential law:*** This law is generally adopted when we deal with the fracture mechanics problems. According to this law the material property in P(z) in a specific direction is given by,

$$P(z) = P_t e^{\left(\frac{1}{h}\right)\left(\ln \frac{P_b}{P_t}\right)\left(z+\frac{h}{2}\right)}$$

2. ***Power law:*** It is observed from the open literature that this particular power law behavior is most used by many researchers. If FGM plate of uniform thickness 'h' is used for the analysis then according to this law, the material properties P(z) in a specific direction (along 'z') can be determined by,

$$P(z) = (P_t - P_b)V_f + P_b$$

It is noted that material properties are dependent on the volume fraction 'V_f' of FGM which follows the power-law as,

$$V_f = (z/h + 1/2)^n$$

where 'n' is the volume fraction exponent. Suffix 't' and 'b' re-presents the top and bottom surface of the plate respectively. The power law exponent 'n' can vary from '0' to '∞' that show the transition of material from fully ceramic to metallic phase, respectively.

3. ***Sigmoid law:*** Power-law function and exponential function are commonly used to describe the gradation of material properties of FGMs but in both functions, the stress concentrations appear in one of the interfaces in which the material is continuous but changing rapidly. To overcome this, Chung and Chi, in their work suggested the use of another law called sigmoid law which is the combination of two power-law functions. This law in not independent law, it consists of two symmetric FGM layers having power-law distribution. They also suggested that by the use of a sigmoid law the stress intensity factors of a cracked body can be reduced to a certain extend. According to this law, the two power-law functions are defined by,

$$f_1(z) = 1 - (0.5)\left(\frac{\frac{h}{2} - z}{\frac{h}{2}}\right)^p \quad 0 \le z \le \frac{h}{2}$$

$$f_2(z) = (0.5)\left(\frac{\frac{h}{2} + z}{\frac{h}{2}}\right)^p \quad -\frac{h}{2} \le z \le 0$$

5. EFFECTIVE MATERIAL PROPERTIES (HOMOGENISATION) OF FGM

The effective properties of macroscopic homogeneous composite materials can be derived from the microscopic heterogeneous material structures using homogenization techniques. Several models like rules of mixture (Voigt Scheme), Hashin–Shtrikman type bounds, Mori–Tanaka type models, and self-consistent schemes are available in literature for determination of the bounds of the effective properties. Voigt scheme and Mori–Tanaka schemes are generally adopted in analysis of functionally graded material plate and structure by most researchers.

Various methods to determine the effective properties of the plate are:

1. ***Rule of mixture:***

$$P(z) = (P_t - P_b)V_f + P_b$$

2. ***Mori-Tanaka scheme:***

This method method works well for composites with regions of the graded microstructure have a clearly defined continuous matrix and a discontinuous particulate phase. The matrix phase is assumed to be reinforced by spherical particles of a particulate phase. The subscript 'e' denoted the effective value of a particular material property where as 'c' and 'm' denoted that of ceramic and metallic constituents respectively. The Bulk modulus(K) and Shear modulus (μ) is calculated as given below:

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + V_m \frac{K_c - K_m}{K_m + 4/3\mu_m}}$$

$$\frac{\mu_e - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + V_m \frac{\mu_c - \mu_m}{\mu_m + f_1}}$$

where $f_1 = \frac{\mu_m(9K_m + 8\mu_m)}{6(K_m + 2\mu_m)}$

Young's modulus (E), Poisson's ratio (υ) can be calculated from them as given below.

$$E = \frac{9K_e\upsilon_e}{3K_e + \upsilon_e}, \quad \upsilon = \frac{3K_e - 2\upsilon_e}{2(3K_e + \upsilon_e)}$$

3. ***Voigt model:***

Voigt model has been adopted in most analyses of FGM structures. The advantage of Voigt method is that it is easy to calculate and can be considered as the upper and lower bounds for the effective elastic properties of a heterogeneous material. The effective material properties P_f, like Young's modulus E_f, Poisson' ratio ν_f, thermal expansion coefficient α_f, and thermal conductivity K_f may be expressed as,

$$P_f = P_t V_c + P_b V_m$$

where P_t and P_b denoted the temperature-dependent proper ties of the top and bottom surfaces of the plate, respectively. V_m and V_c and are the metal and ceramic volume fractions which can be expressed by,

$$V_c + V_m = 1$$

If volume fraction V_m is assumed to follow a simple power law as

$$V_m = \left(\frac{2Z + h}{2h}\right)^n$$

where 'n' is the volume fraction index and takes only positive values then different effective properties can be given as,

$$E_f(Z, T) = [E_b(T) - E_t(T)]\left(\frac{2Z + h}{2h}\right)^n + E_t(T)$$

$$\alpha_f(Z, T) = [\alpha_b(T) - \alpha_t(T)]\left(\frac{2Z + h}{2h}\right)^n + \alpha_t(T)$$

$$K_f(Z, T) = [K_b(T) - K_t(T)]\left(\frac{2Z + h}{2h}\right)^n + K_t(T)$$

$$\nu_f(Z, T) = [\nu_b(T) - \nu_t(T)]\left(\frac{2Z + h}{2h}\right)^n + \nu_t(T)$$

4. ***Self-consistent schemes:***

This method describes its estimates through the solution of an elastic problem in which an ellipsoidal inclusion is embedded in a matrix possessing the effective material properties of the composites. This method assumes that each reinforcement inclusion is embedded in a continuum material whose effective properties are those of the composite. This method does not distinguish between matrix and reinforcement phases and the same overall moduli are predicted in another composite in which the roles of the phases are interchanged. This makes it particularly suitable for determining the effective moduli in those regions which have an interconnected skeletal microstructure. This is a rigorous analytical method applicable to two-phase isotropic composite materials.

6. PLATE KINEMATICS

The analysis of composite structures is one of the most promising research fields of the last decades. Accurate structural and dynamic analyses are required to design various structural parts of aerospace, mechanical, naval as well as civil constructions to find the behavior of the structural response in real time. Numerous plate theories have been developed by the researchers to analyze the composite plates and shells and out of them, the most commonly used ones are given below.

One of the major classification is the classical plate theory and shear deformation theories. Transverse shear stress components are neglected in the classical plate theory whereas it is included in the shear deformation theories.

Displacement functions used in various plate theories available in literature are given below:

1. ***Classical plate theory***

$$u(x_0 y_0, z) = u_0(x_0, y_0) - z(\partial w_0/\partial x)$$
$$v(x_0 y_0, z) = v_0(x_0, y_0) - z(\partial w_0/\partial y)$$
$$z(x_0 y_0, z) = z_0(x_0, y_0)$$

2. ***First order shear deformation theory***

$$u(x_0, y_0, z) = u_0(x_0, y_0) - z\theta_x$$
$$v(x_0, y_0, z) = v_0(x_0, y_0) - z\theta_y$$
$$w(x_0, y_0, z) = w_0(x_0, y_0)$$

3. ***Third order shear deformation theory(TSDT)***

$$u = u_0 + z\phi_x - z^2\left(\frac{1}{2}\frac{\partial\phi z}{\partial x}\right) - z^3\left[C_1\left(\frac{\partial w_0}{\partial x} + \phi_x\right) + \frac{1}{3}\frac{\partial\phi z}{\partial x}\right]$$
$$v = v_0 + z\phi_y - z^2\left(\frac{1}{2}\frac{\partial\phi z}{\partial x}\right) - z^3\left[C_1\left(\frac{\partial w_0}{\partial y} + \phi_y\right) + \frac{1}{3}\frac{\partial\phi z}{\partial y}\right]$$
$$w = w_0 + z\phi_x + z^2\phi_z$$
$$C_1 = \frac{4}{3h^2},\ u_0 = u(x, y, 0, t),\ v_0 = v(x, y, 0, t),\ w_0 = w(x, y, 0, t)$$

where,
u,v,w denotes the displacement variables.
u_0, v_0, w_0 are the in-plane displacements with respect to a reference plane.
w_0 is the out-of-plane displacements with respect to the reference plane.
ϕ_x, ϕ_y, θ_x, θ_y are the rotation of normal with respect to mid-surface of the plate.

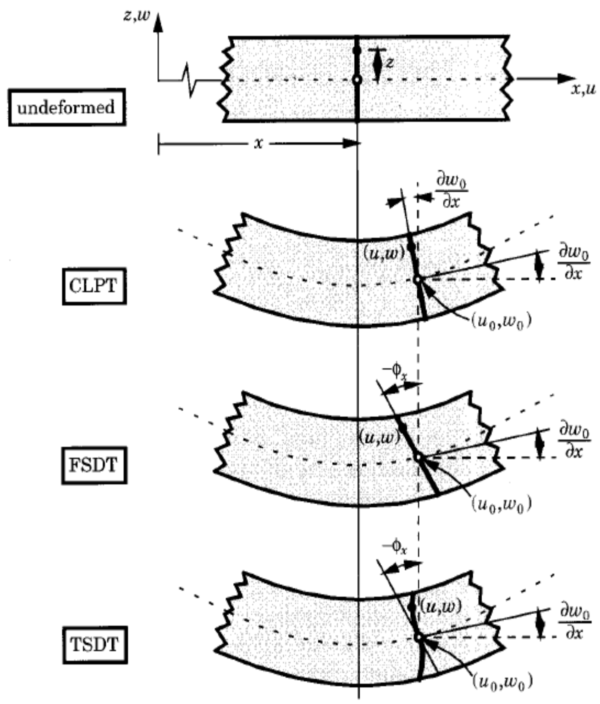

Fig.5. Tranverse shear deformation of a plane according to various plate theories

7. LITERATURE REVIEW

A large number of literature work had been done in the field of analysis as well as processing techniques of functionally graded materials. We can find large amount of works on the flexural, vibrational and buckling behavior of functionally graded plates. As most of the applications are going to be in extreme conditions we can find studies on the thermal characteristics of the material. A brief study on the various works done till date is provided here. Those works based on classical plate theory (CLPT) and First order shear deformation theory (FSDT) has been neglected as they are very approximate and conservative and incapable of accounting for the level of inhomogeneity in FGM. This review is purely on the works done according to the Higher order shear deformation theory (HSDT). Also, those works based on FEM has also been omitted as they involve very tedious and complex calculations and only those works that involves numerical methods has been included.

Pagano (1969 & 1970)**, Srinivas** and **Rao**, and **Srinivas** et al. (1970) developed the exact solutions of simply supported laminated plates by using 3D elasticity theory. The benchmark solutions that they have provided has proved to be very useful in assessing the accuracy of various 2D approximate plate theories by various researchers. Their methods are valid for laminated plates or shells, where the material properties are piecewise constant, but they are not applicable for finding solutions of plate problems with continuous anisotropy of material properties such as with FGMs.

Reddy J N (1984) has published a simple higher order theory for simply supported laminated composite plates that would account for parabolic transverse shear strain unlike FSDT in which a constant variation is assumed, the results for deflection when compared with that of FSDT, it was found out that the results where more accurate and closer to the three-dimensional elasticity solutions. He later extended his study to the vibration and buckling characteristics using his new theory in his work (1985) and showed that it yielded more accurate results compared to FSDT. **Reddy J N** (2000) presented a theoretical formulation based on Navier's solutions of rectangular plates, and on third-order shear deformation theory (TSDT) to analyze through-thickness functionally graded plates. A two phase material was assumed to be isotropic, having a distribution that varies through the thickness according to the exponent power law. **Nguen T K**, **Sab K**, **Bonnet G** (2007) calculated the value of shear coefficient (SCF) to be used for FSDT and they found out that the value of SCF depends on the material gradation, ratio of young's modulus of the constituent materials. **Prakash T**, **Singha M K**, and **Ganapati M** (2007) investigated the influence of neutral surface position on the non-linear stability behavior of skew FGM plates by using FSDT and Newton-Raphson technique to solve non-linear governing equations and they found out that the neutral surface shift towards ceramic rich side and the shift increase with increase in gradient index also results obtained from the present formulation based on the neutral surface position, is qualitatively similar to those of mid-surface based formulation. But however, they obtained a much higher out-of-plane deflection and its difference from mid-surface calculations increased with increase in gradient index and non-linearity. **Efraim E** (2011) derived an empirical formula that gives a correlation for natural frequencies of FGM plate with that of the constituent materials even those with different Poisson's ratios and the natural frequencies obtained are compared with results obtained with other numerical methods for thick FGM annular plates. **Birman V** (2012) had shown that there is a strong coupling between micromechanics and heat transfer aspect in-order to obtain accurate results and concluded that once the distribution of temperature and properties has been specified, the structural response of the structure can be analyzed using numerical methods if the temperature variation is limited to thickness direction and by finite element or difference methods if 3-dimensional variation is assumed. **Rasheedat M M** et al. (2012) has given an overview on FGM, describing its peculiarities, applications and processing techniques. **Jha D K** et al. (2012) presented a detailed review on the research works done in various fields of FGMs. In their paper, they showed in details the amount of works done in each field and stressed the need for the development of improved 2D models that would yield much more accuracy with much less computational efforts and cost. **Kennedy D** et al. (2012) has given an equivalent isotropic plate model for the FGM plate based on CLPT. This holds good only for thin FGM plates where transverse shear is negligible. **Tran L V**, **Ferreira A J**

M, **Nguen-Xuan H** (2013) performed an iso-geometric analysis of FGM plates using HSDT and non-uniform rational basis spline (NURBS) formulations, the static, dynamic and buckling analysis were conducted for rectangular and circular plates for various boundary conditions and they found out that the results obtained were in excellent agreement with that of several other published solutions. **Gupta A** and **Talha M** (2015) published a detailed review on the different processing techniques, applications, methods for material properties gradations, methods to determine effective material properties, different theories for analysis and on various research works done on FGM. **Kennedy D** et al. (2015) in his work presented an equivalent layered model for FGM plate. The idea was to replace the original FGM plate with an equivalent isotropic one, thus making the analysis much more simple, he presented a single layer isotropic model and two-layer model based on CLPT by assuming Poison's ratio constant and varying respectively and shown that this can be extended to three-layered and four-layered composite plates if we use HSDT that would be necessary for thick FGM plate. **Bernardo G M S** et al. (2015) studied the structural behavior (static and free vibration analyses) of FGM plates using FSDT and various numerical techniques and gradation laws like power law and exponential law and is compared with each other as well as with published FEM results. The layered and continuous configurations were considered and they highlight the development of a package of different methods and models that enable the selection of those that fit better the needs of the study in terms of accuracy, robustness or computational cost.

8. CONCLUSIONS

The developments in the modelling techniques and analysis of functionally graded materials are reviewed. The following points can be highlighted

1. From the reviewed literature, it was clear that most of the work done is purely analytical or with numerical simulations.
2. Although 3D analytical solutions are considered benchmark results, but since it involves tedious calculations most of the work has been does using approximated 2D theories and compared with already published solutions.
3. Among various plate theories like CLPT, FSDT, TSDT, HSDT etc. FSDT and HSDT are the most extensively used. CLPT is not suitable for moderately thick and thick FGM plates and for FSDT it involves calculation of SCF that makes the analysis little more complex whereas in HSDT there is no such SCF and although it involves higher order terms the level of accuracy increases when compared to CLPT and FSDT.
4. The most important or critical factor involved in the analysis of FGM is the proper gradation as it influences the static, dynamic as well as its buckling behavior. And among various methods used to describe material gradation Voigt's scheme and Mori Tanaka scheme is commonly used.

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