Forward and Inverse Kinematic Analysis of Robotic Manipulators

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Abstract - *Today every small and large industry use the robotic manipulator to complete the various task like as picking and placing, welding process, painting and material handling but to complete these task one of the most important problem is to get the desire position and orientation of the robotic manipulators. There are two method for analyzing the robotic manipulator one is forward kinematic analysis and another is inverse kinematic analysis. This project aim to model the forward and inverse kinematic of 5 DOF and 6 DOF robotic manipulator. A movement flow planning is designed and further evaluate all the DH parameter to calculate the desire position and orientation of the end effector. Forward kinematics is simple to design but for inverse kinematic solution traditional method (iterative, DH notation, transformation) are used. And compare the result with analytical solution and see there are acceptable error. A FK and IK solution of aspect robotic manipulator are successfully modeled.*

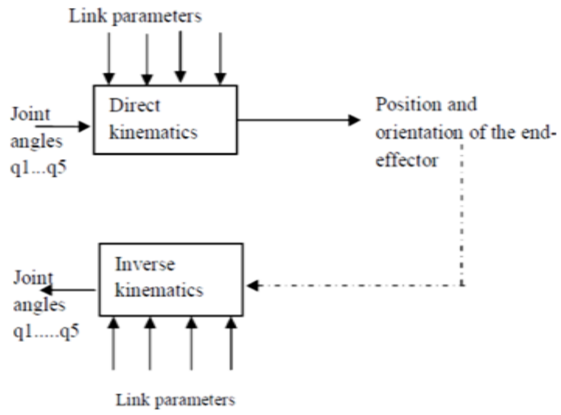
***Key Words*: Forward and inverse kinematics, DOF (degree of freedom), transformation, DH convention, Robotic Arm.**

1. INTRODUCTION

Robot is a machine that collects the information about the environment using some sensors and makes a decision automatically. Today robot are used in various field like as medical, industry, military operation, in space and some dangerous place. Where human don't want to work. But the controlling of robot manipulator has been challenges with higher DOF. Position and orientation analysis of robotic manipulator is an essential step to design and control. In this paper a basic introduction of the position and orientation analysis of a serial manipulator is given. A robot manipulator consist set of links connected together this either serial or parallel manner. The FK analysis is simple to analysis of model and calculate the position using the joint angle. But the challenge in to analyze the IK solution using the position. Complexity of the IK increases with increase the DOF.so to analyze IK in this Paper use the DH convention and transformation type solution.

1.1 Kinematics

Kinematics is the branch of mechanics that deals with the motion of the bodies and system without considering the force. And the robot kinematics applies geometry to the study movement of multi DOF kinematic chains that form the structure of robot manipulator [1].



Robot kinematic studies the relationship between the linkages of robot with the position, orientation and acceleration. And the robotic kinematic analysis are divided into two types

1.1.1 Forward kinematic

This specified the joint parameter and kinematic equation is used to compute the position of end effector from specified value for the each joint parameter. Or Calculation of the position and orientation of the robotic manipulator in terms of the joint variable is called forward kinematic.

1.1.2 Inverse kinematic

This is oppose to the FK. And specified the position of end effectors and kinematics equation is used to compute the joint angle from specified position of end effector.

1.2 DH parameter

This was first introduce by JACQUES DENVIT and RICHARAD S. HARTENBERG. DH convention is used for selecting frame of reference for the robotic arm. In this convention, coordinate frame are attached to the joints between two link to describe the location of each link relative to its previous [2].

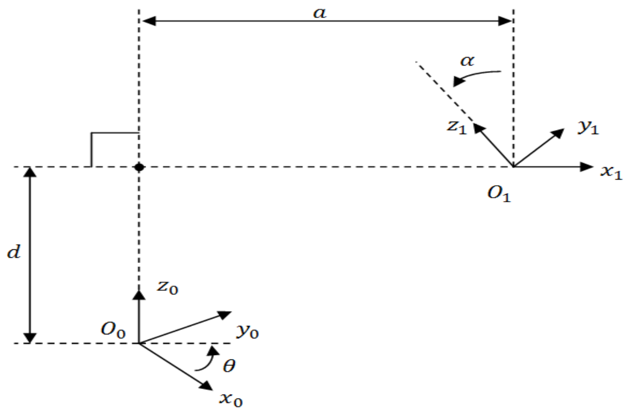


Fig2: Two Coordinate frames system [2]

There are four parameter used in D-H parameter representation. These parameters describe the relative rotation and translation between consecutive frames.

Link length (a_i): the distance between the axis z_0 and z_1, and this distance measure along the x_1 axis. (Trans, x, a_i)

Link offset (d_i): distance from origin O_0 to the intersection of the x_1 axis with z_0 measured along the z_0 axis (Trans, z, a_i).

Joint angle (Θ_i): angle from x_0 and x_1 measured in plane normal to z_0 (ROT, z, Θ_i).

Link twist (α_i): angle between Z_0 and Z_1, measured in plane normal to X_1 axis (ROT, x, α_i).

2. Literature Review

In robotic kinematic analysis forward kinematic is simple to obtain but Obtaining the inverse kinematics solution has been one of the main concerns in robot kinematics research. The complexity of the solutions increases with higher DOF due to robot geometry, non-linear equations (i.e. trigonometric equations occurring when transforming between Cartesian and joint spaces) and singularity problems. Obtaining the inverse kinematics solution requires the solution of nonlinear equations having transcendental functions. To solving the IK equation many researcher used method like algebraic [3], geometric [4], and iterative [5] for complex manipulators, these methods are time consuming and produce highly complex mathematical formulation. Which can't solve easily. Calderon et al. [6] proposed a hybrid approach to inverse kinematics and control and a resolve motion rate control method are experimented to evaluate their performances in terms of accuracy and time response in trajectory tracking. Xu et al. [7] proposed an analytical solution for a 5-DOF manipulator to follow a given trajectory while keeping the orientation of one axis in the end-effector frame by considering the singular position problem. Gan et al. [8] derived a complete analytical inverse kinematics (IK) model, which is able to control the P2Arm to any given position and orientation, in its reachable space, so that the P2Arm gripper mounted on a mobile robot can be controlled to move to any reachable position in an unknown environment. Here in this project use the D-H convention used to solve forward kinematic equation than by using these equations find the value of IK.

3. Kinematic Modelling of 5 DOF and 6 6DOF robotic manipulators

Take the simple 5r robotic manipulator which have revolute joint only and similar way for 6DOF use 6r robotic manipulator

3.1 Modelling of 5 DOF

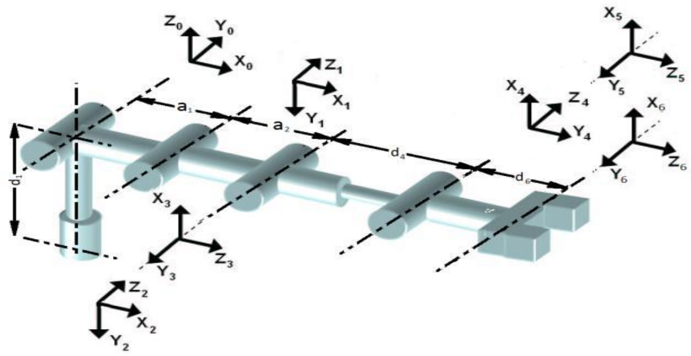


Fig 4: Coordinate frame for the 5-DOF Redundant manipulator

Coordinate frame assignment for 5 degree of freedom shown above. They are establish using the D-H conversation for each joint coordinate. And frame 5 is auxiliary frame attach to end effector and the frame 6 have the same direction as frame 5 and show no rotation for end effectors. The D-H parameter for 5 DOF is given

Table 1: D-H convention for 5 degree of freedom

frame	Joint offset in m	Theta (degree)	Link length in m	Twist angle (Degree)	Initial joint value In m or deg.	Final joint value In m or degree
0-1	D1= 0.13	variable	A1 = o.7	-90	-365	365
1-2	0	variable	A2=1.6	0	-150	30
2-3	0	variable	0	-90	60	-210
3-4	D4= 0.14	variable	0	90	0	360
4-5	0	variable	0	-90	60	-90
5-EE	D6= 0.16	variable	0	0	0	0

In above table initial joint value and the final joint value denote the limit of the value of theta for orientation. These are maximum value at that manipulator rotate.

3.2 Modelling of 6 DOF

In the 6 DOF one joint added at the end effector and all the configuration is same as 5 DOF because adding of one joint one DOF increase. Here all the joint are revolute type and have two type of joint one is that which have the limited rotation and another is which have rotation 360 or many more like base and end effector joint both are second type joint which have no limitation of angle.

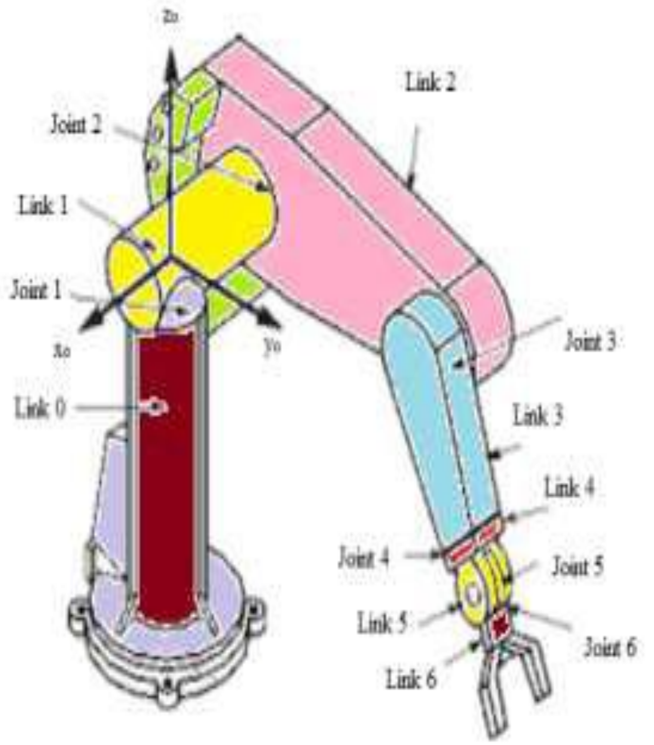


Fig 5: Puma 560 6DOF robot with frame assignment

Table 2: D-H convention for 6 degree of freedom

frame	Joint offset in m	Theta (degree)	Link length in m	Twist angle (Degree)	Initial joint value In m or deg.	Final joint value In m or degree
0-1	0	variable	0	-90	-185	185
1-2	D2= 0.5	variable	A2= 0.7	0	-155	35
2-3	D3= .0948	variable	A3 = 0.948	90	-130	154
3-4	D4= 0.68	variable	0	-90	-350	350
4-5	0	variable	0	90	-130	130
5-EE	D6= 0.853	variable	0	0	-350	350

4. Kinematic analysis of manipulators

By using the D-H conversation easily calculate the homogeneous transformation matrix and by using this FK equation found. The D-H conventions uses a product of four basic transformation to represent the homogeneous transformation and denoted by $\mathbf{A_i}$.

The A matrix is a homogenous 4x4 transformation matrix which describe the position of a point on an object and the orientation of the object in a three dimensional space. The homogeneous transformation matrix from one frame to the next frame can be derived by the using D-H parameters.

In D-H convention, each homogeneous transformation matrix *Ai* is represented as a product of four basic transformations as follows.

$$\mathbf{A_i} = (ROT, z, \theta_i)(Trans, z, a_i)(Trans, x, a_i)(ROT, x, \alpha_i).$$

A_i =

$$\begin{bmatrix} C\theta_{n+1} & -S\theta_{n+1}C\alpha_{n+1} & S\theta_{n+1}S\alpha_{n+1} & a_{n+1}C\theta_{n+1} \\ S\theta_{n+1} & C\theta_{n+1}C\alpha_{n+1} & -C\theta_{n+1}S\alpha_{n+1} & a_{n+1}S\theta_{n+1} \\ 0 & S\alpha_{n+1} & C\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

4.1 Forward kinematic analysis:

By using equation 1st calculate the transformation matrix for each joint and this equation have 3 fix component and one is variable that's one is theta and have one 3*3 rotational matrix that show the orientation of the end effector and have one 1*3 type matrix which show the position of the end effector.

$$A_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & a_1C\theta_1 \\ S\theta_1 & 0 & -C\theta_1 & a_1S\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$A_3 = \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & 0 \\ S\theta_3 & 0 & -C\theta_3 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$A_4 = \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$A_5 = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$A_6 = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

At the $\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p \\ 0 & 0 & 0 & 1 \end{bmatrix} =$ base of the robot, it can be started with the first joint and then transform to the second joint, then to the third until to the arm-end of the robot, and eventually to the end effectors. The total transformation between the base of the robot and the hand is

$${}^{0}T_6 = A_1A_2A_3A_4A_5A_6$$

By multiplying these equation we can get the final equation for forward kinematics and compare the value with equation one and finally get the kinematic equation

$$n_x = C_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] + S_1(S_4C_5C_6 + C_4S_6) \quad (8)$$
$$n_y = S_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] - C_1(S_4C_5C_6 + C_4S_6) \quad (9)$$
$$n_z = S_{23}(C_4C_5C_6 - S_4S_6) + C_{23}S_5C_6 \quad (10)$$
$$o_x = C_1[-C_{23}(C_4C_5C_6 + S_4C_6) + S_{23}S_5S_6] + S_1(-S_4S_5S_6 + C_4C_6) \quad (11)$$
$$o_y = S_1[-C_{23}(C_4C_5C_6 + S_4C_6) + S_{23}S_5S_6] - C_1(-S_4C_5S_6 + C_4S_6) \quad (12)$$
$$o_z = -S_{23}(C_4C_5C_6 + S_4C_6) - C_{23}S_5S_6 \quad (13)$$
$$a_x = C_1[C_{23}C_4S_5 + S_{23}C_5] - C_1S_4S_5 \quad (14)$$
$$a_y = S_1[C_{23}C_4S_5 + S_{23}C_5] - C_1S_4S_5 \quad (15)$$
$$a_z = S_{23}C_4S_5 - C_{23}C_5 \quad (16)$$
$$p_x = C_1(C_2a_2 + a_1) + S_1d_3 \quad (17)$$
$$p_y = S_1(C_2a_2 + a_1) - C_1d_3 \quad (18)$$
$$p_z = S_2a_2 \quad (19)$$

4.2 Inverse kinematic analysis:

Inverse kinematic analysis is the opposite of the forward kinematic analysis. The corresponding variables of each joint could found with the given location requirement of the end of the manipulator in the given references coordinates system.

Inverse kinematic analysis is done by multiplying each inverse matrix of T matrices on the left side of above equation and then equalizing the corresponding elements of the equal matrices of both ends [9].

To solve the angle use equation 1st to calculate the value of each angle by using the end effector position

$$A_1^{-1} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_2A_3A_4A_5A_6$$

$$\begin{bmatrix} n_xC_1 + n_yS_1 & o_xC_1 + o_yS_1 & a_xC_1 + a_yS_1 & P_xC_1 + P_yS_1 - a_1(C_1C_1 + S_1S_1) \\ n_z & o_z & a_z & P_z \\ n_xS_1 - n_yC_1 & o_xS_1 - o_yC_1 & a_xS_1 - a_yC_1 & P_xS_1 - P_yC_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{23}(C_4C_5C_6 - S_4C_6) - S_{23}S_5C_6 & C_{23}(-C_4C_5C_6 - S_4C_6) - S_{23}S_5S_6 & C_{23}C_4S_5 + S_{23}C_5 & a_2C_2 \\ S_{23}(C_4C_5C_6 - S_4S_6) - C_{23}S_5C_6 & S_{23}(-C_4C_5C_6 - S_4S_6) - C_{23}S_5S_6 & S_{23}C_4S_5 - C_{23}C_5 & a_2S_2 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_5S_4 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (20)$$

On comparing equation (20) and equation (1)

$$P_z = a_2S_2$$
$$S_2 = P_z/a_2$$
$$\theta_2 = sin^{-1}(P_z/a_2) \quad (21)$$
$$P_xS_1 - P_yC_1 = d_3$$
$$S_1 = \frac{d_3 + P_yC_1}{P_x}$$
$$P_xC_1 + P_yS_1 - a_1 = a_2C_2$$

By the value of S_1 and a_2C_2, calculate the value of C_1

$$C_1 = \frac{P_x(a_2C_2 + a_1) - P_yd_3}{P_x^2 + P_y^2}$$

$$\theta_1 = cos^{-1}\left(\frac{P_x(a_2C_2 + a_1) - P_yd_3}{P_x^2 + P_y^2}\right)$$

Now similarly get the values of all theta in terms of position and the orientation.

$$\theta_1 = cos^{-1}\left(\frac{P_x(a_2C_2 + a_1) - P_yd_3}{P_x^2 + P_y^2}\right)$$

$$\theta_2 = sin^{-1}(P_z/a_2)$$

$$\theta_{23} = cos^{-1}\sqrt{\frac{P_z^2 - a_2^2 + [P_xC_1 + P_yS_1 - a_1]^2}{2P_z^2}}$$

$$\theta_3 = \theta_{23} - \theta_2$$

$$\theta_4 = tan^{-1}\left[\frac{a_xS_1 - a_yC_1}{a_xC_1C_{23} + a_yS_1S_{23} + a_zS_{23}}\right]$$

$$\theta_5 = -tan^{-1}\left[\frac{a_zC_1C_{23}C_4 + a_yS_1S_{23}C_4 + a_2S_{23}C_4 + a_zS_1S_4 - a_yC_1S_4 = S_5}{-a_zS_{23} - a_yS_1S_{23} + a_zC_{23}}\right]$$

$$\theta_6 = tan^{-1}\left[\frac{o_zC_{23} - o_xC_1S_{23} - o_yS_1S_{23}}{n_xC_1S_{23} + n_yS_1S_{23} - n_zC_{23}}\right]$$

5. Results

5.1 Result for 5 degree of freedom

By using the above equation solve the position and orientation for 20 iteration by using the given angle if the position of the end effector are given than use the inverse kinematic equation to get the angle of each joint

Table 3: all possible angle for 5 DOF

Above table shows the all possible joint values for 5 DOF robotic manipulator. And used to calculate the value of position and orientation of end effector using forward kinematic. Similarly for the inverse kinematic analysis in which the value of position of the end effector is known and using these value calculate the joint value for each link. The all possible joint values of end effector are given in above table.

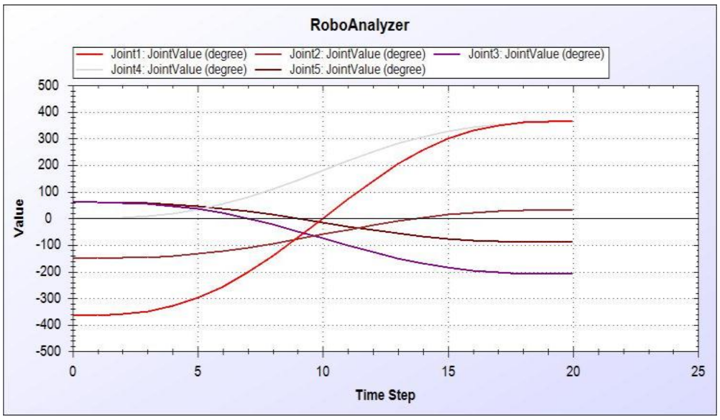


Fig 6: plot for joint value

The below table shows the all possible position of EE at the given joint value and calculate by using FK equation and this value is further used for IK.

End X	End Y	End Z
71.1641	-6.22605	210
71.43058	-5.49949	210.536
73.07748	-0.3708	214.2102
75.71005	14.04008	223.7318
73.0788	43.05237	240.8419
47.10427	86.09795	265.6854
-27.6663	115.2092	296.2026
-134.304	58.38066	327.8614
-140.923	-112.938	354.2492
65.55203	-206.678	368.853
248.9949	-5.97E-15	367.559
81.94976	258.3784	350.5228
-218.21	174.8769	322.1171
-252.509	-109.763	288.9284
-61.2086	-254.887	257.1425
117.7797	-215.28	230.8243
198.0047	-116.649	211.6126
214.6015	-39.7969	199.3397
211.5727	1.073525	192.8173
208.3327	16.03968	190.3564

Table 4: Position value for End effector of 5DOF

jointValue1	jointValue2	jointValue3	jointValue4	jointValue5
-365	-150	60	0	60
-364.403	-149.853	59.77903	0.29463	59.87724
-360.291	-148.839	58.25821	2.322386	59.03234
-349.494	-146.177	54.26494	7.646741	56.81386
-329.497	-141.246	46.86864	17.50848	52.7048
-298.683	-133.648	35.47183	32.70422	46.37324
-256.497	-123.246	19.86864	53.50848	37.7048
-203.494	-110.177	0.264944	79.64674	26.81386
-141.291	-94.8388	-22.7418	110.3224	14.03234
-72.4026	-77.8527	-48.221	144.2946	-0.12276
0	-60	-75	180	-15
72.40255	-42.1473	-101.779	215.7054	-29.8772
141.2907	-25.1612	-127.258	249.6776	-44.0323
203.4941	-9.82337	-150.265	280.3533	-56.8139
256.4967	3.245762	-169.869	306.4915	-67.7048
298.6831	13.64789	-185.472	327.2958	-76.3732
329.4967	21.24576	-196.869	342.4915	-82.7048
349.4941	26.17663	-204.265	352.3533	-86.8139
360.2907	28.83881	-208.258	357.6776	-89.0323
364.4026	29.85268	-209.779	359.7054	-89.8772

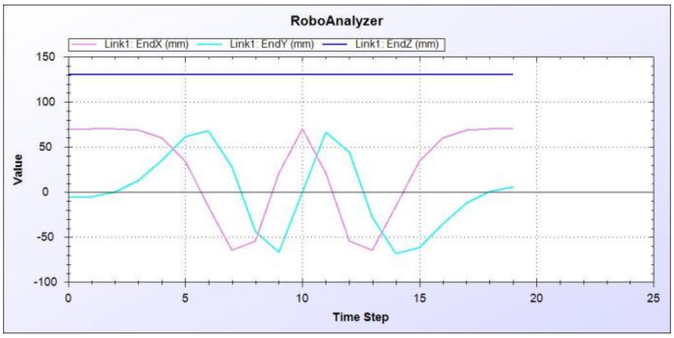


Fig7: plot for link 1 position

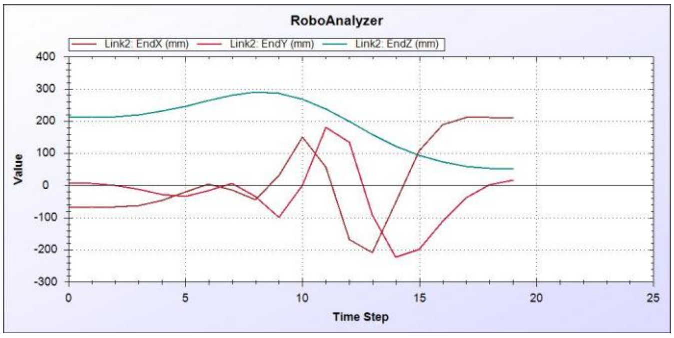


Fig 8: plot for link 2 position

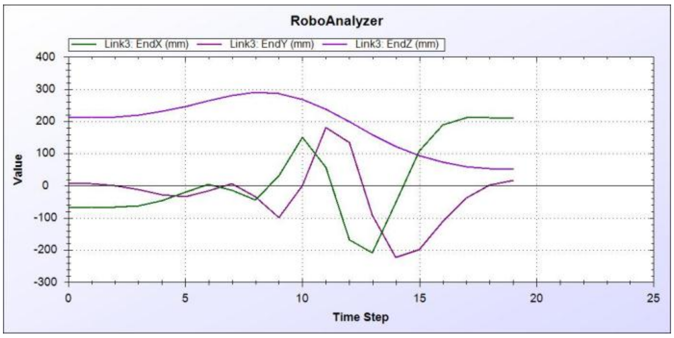


Fig 9: plot for link 3 position

jointValue1	jointValue2	jointValue3	jointValue4	jointValue5	jointValue6
0	0	0	0	0	0
0.049105	0.049105	-0.04911	0.049105	0.049105	0.049105
0.387064	0.387064	-0.38706	0.387064	0.387064	0.387064
1.274457	1.274457	-1.27446	1.274457	1.274457	1.274457
2.918079	2.918079	-2.91808	2.918079	2.918079	2.918079
5.450703	5.450703	-5.4507	5.450703	5.450703	5.450703
8.918079	8.918079	-8.91808	8.918079	8.918079	8.918079
13.27446	13.27446	-13.2745	13.27446	13.27446	13.27446
18.38706	18.38706	-18.3871	18.38706	18.38706	18.38706
24.04911	24.04911	-24.0491	24.04911	24.04911	24.04911
30	30	-30	30	30	30
35.95089	35.95089	-35.9509	35.95089	35.95089	35.95089
41.61294	41.61294	-41.6129	41.61294	41.61294	41.61294
46.72554	46.72554	-46.7255	46.72554	46.72554	46.72554
51.08192	51.08192	-51.0819	51.08192	51.08192	51.08192
54.5493	54.5493	-54.5493	54.5493	54.5493	54.5493
57.08192	57.08192	-57.0819	57.08192	57.08192	57.08192
58.72554	58.72554	-58.7255	58.72554	58.72554	58.72554
59.61294	59.61294	-59.6129	59.61294	59.61294	59.61294
59.95089	59.95089	-59.9509	59.95089	59.95089	59.95089

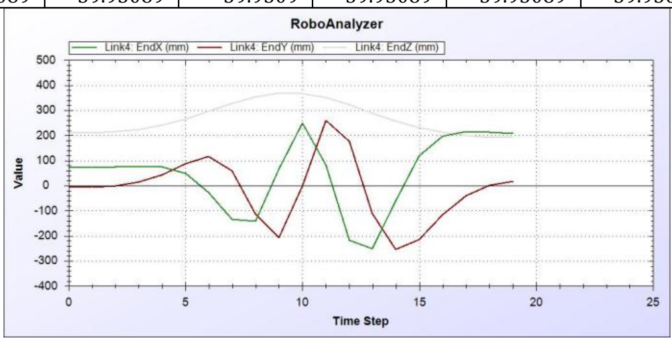


Fig 10: plot for link 4 position

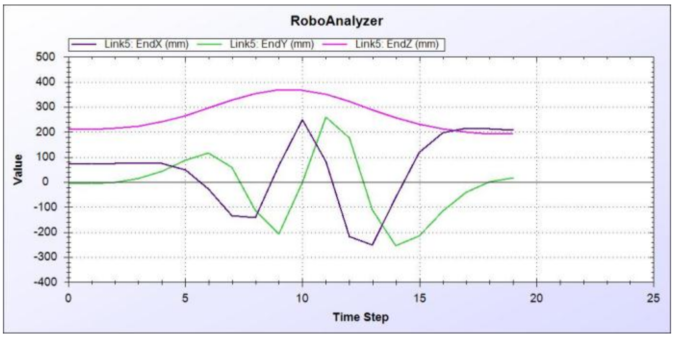


Fig 11: plot for link 5 position

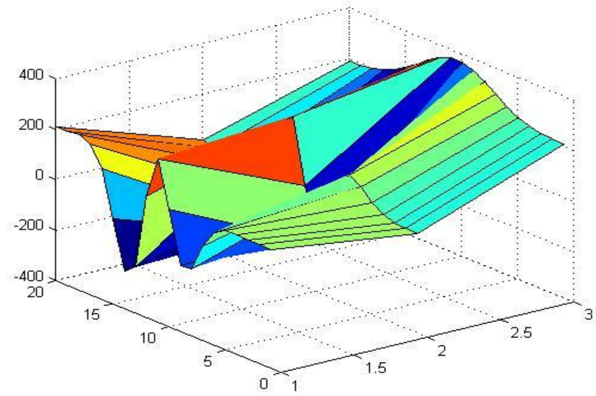


Fig 12: surface plot for position of end effector

5.2 Result for 6 degree of freedom

By using the forward kinematic analysis get all the possible position of end effectors. And these value are further used for calculation of inverse kinematic in which calculate the angle using the position of end effectors. Take all possible joint value as input for forward kinematic analysis and get all the possible position of the end effector. And for the inverse kinematic position values of the end effector are taken as input and by using this calculate the joint value for each joint to get the desire orientation.

Table 5: all possible angle for 6 DOF

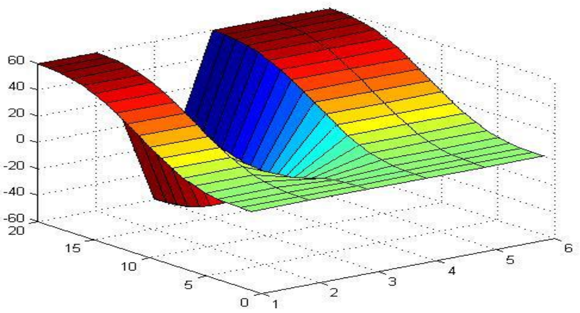


Fig 13: surface plot for joint value

End X	End Y	End Z
526.628	139.692	1100.599
527.0173	140.1437	1100.13
529.6739	143.2708	1096.88
536.4599	151.6324	1088.162
548.2687	167.6693	1071.309
564.3856	193.6511	1043.588
581.9591	231.3008	1002.315
595.9011	280.9468	945.3523
599.6258	340.4005	871.9466
586.8507	404.2318	783.6389
554.0557	464.3222	684.7641
502.4461	512.0784	582.0885
438.1163	541.5359	483.4645
370.0438	551.6382	395.9441
306.9984	546.2927	324.1966
255.0976	532.2926	269.9224
216.9997	516.4974	232.3015
192.4576	503.7352	208.942
179.3259	496.0306	196.7058
174.3527	492.9474	192.118

Table 6: all possible angle for 6 DOF

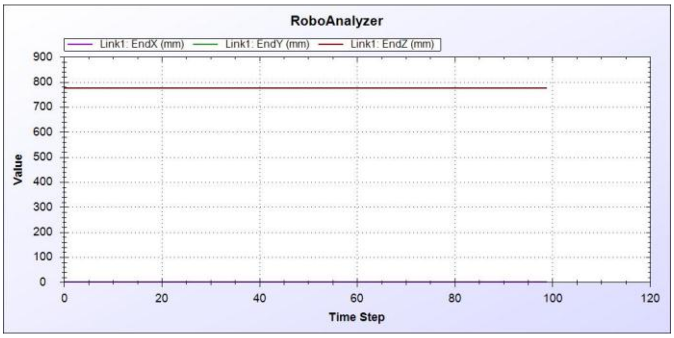


Fig 14: plot for link 4 position

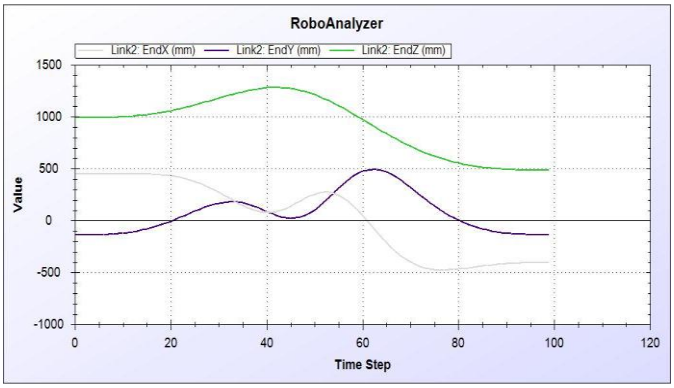


Fig 15: plot for link 4 position

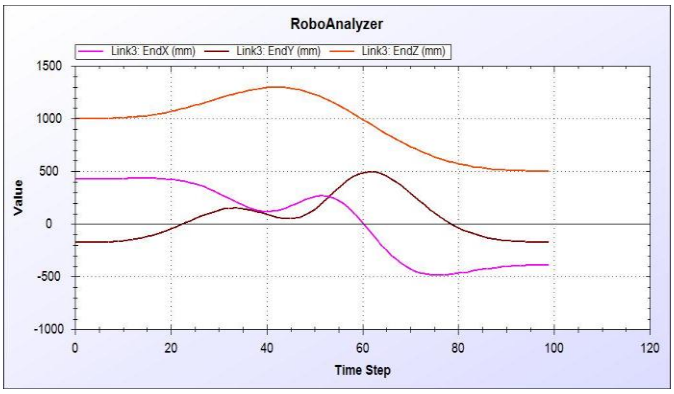


Fig 16: plot for link 4 position

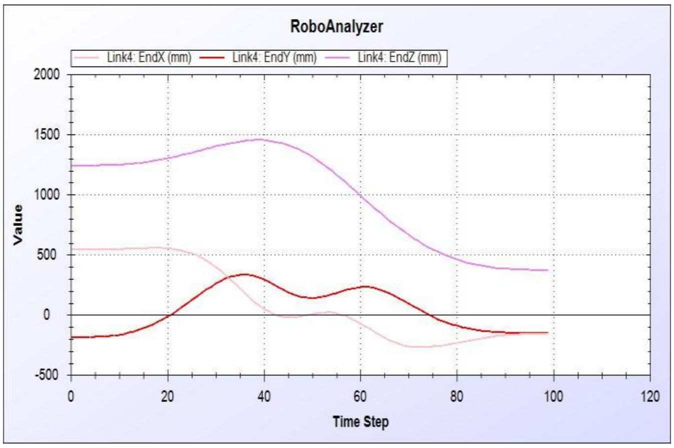


Fig 17: plot for link 4 position

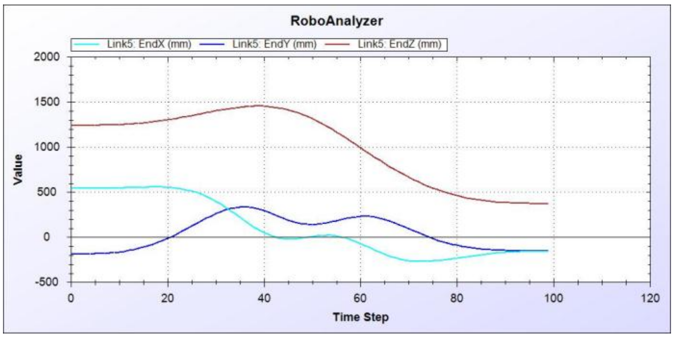


Fig 18: plot for link 4 position

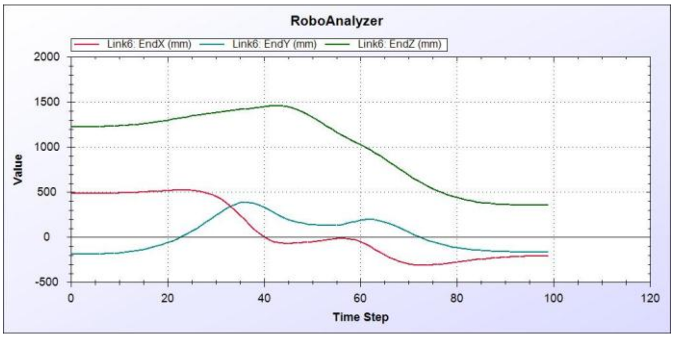


Fig 19: plot for link 4 position

6. Conclusion and future work

To conclude, this paper proposed mathematical approach for solving the forward and inverse kinematic for 5 DOF and 6 DOF (PUMA 560) robotic manipulators. The experimental result is obtained by using robot-analyzer and mat-lab software and compare the result with the result calculate by using the D-H transformation.

This technique can be used in various field to determine the positions and orientations. It can be used for:

- Under water manipulator
- Nuclear, toxic waste disposal and mining robot
- Firefighting, construction and agricultural robot
- Medical application

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