Prediction of Ultimate Shear Strength of Moderate Deep Concrete Beam Including Size Effect

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Abstract - *This paper investigate the effects of beam parameters like effective depth(d) and compressive strength (fck) on ultimate shear strength of moderate deep concrete beam. Strain distribution in moderate deep concrete beam is not remaining linier. So, prediction of ultimate shear strength of moderate deep concrete beam is less accurate and complex. Many empirical equations and theories are available to predict ultimate shear strength of moderate deep concrete beam. Aim of this research is modification of equation (for ultimate load of concrete beam fails in shear without stirrups) given by Dr. S. N. Patel and Dr. S. K. Damle. Effect of effective depth (d) of concrete beam and compressive strength (f_{ck}) is incorporated in terms of size effect factor in above mentioned equation. Also for finding ultimate load of concrete beam fails in shear with stirrups, an extra term is added in above mentioned equation to incorporate effect of stirrups in ultimate shear strength. Modified equation gives good agreement with experiment test results of previous experiments.*

Key Words: **Ultimate shear strength, Moderate deep concrete beam, size effect, IS: 456-2000, Ultimate load.**

1. INTRODUCTION

Beam is one of the major structural element generally subjected to transverse loading. Many research works have been carried out and reported so far to predict the behaviour or response of beam against applied loading. Due to various types of materials, arrangement of loading, support condition and various types of site and environmental condition, research work is still needed in this vast field. Especially considering the beam mainly fail in shear. There is still lack of knowledge about deep understanding of complete structural response of beam which fails in shear. Generally beam can be classified according to their span to depth ratio and shear span to depth ratio. Normal beam has depth very less compared to its length, while deep beam has its depth equal or less compared to its depth. Moderate deep beam falls in between normal and deep beam.

Generally beams classified with effective span-to-depth ratio L/D less than 2.5 as deep beam and ratio L/D greater than 6.0 as shallow beam. Beams falls between above two criteria are classified as moderate deep beam. ACI-ASCE committee 426 classifies a beam with a shear span-to-depth ratio a/D less than 1.0 as a deep beam and a beam with a/D exceeding 2.5 as an ordinary shallow beam. Any beam in between these two limits is categorized as a moderate deep beam. As per Indian Standard Code IS 456: 2000, A beam shall be deemed deep beam when L/D ratio less than (1) 2.0 for simply supported beam (2) 2.5 for continuous beam.

Shallow beams are characterized by linear strain distribution and most of the applied load is transferred through a fairly uniform compression field. It can be analysed generally by pure bending theory based on assumption that plane sections normal to the axis remain plane after bending. The stress distribution across the section is almost linear. Shallow beams are assumed as one-dimensional linear elements so they resist transverse loading mainly by bending and shear or say mainly by developing flexure and shear stresses. There is a negligible effect of normal pressure on stress distribution. Generally shallow beams fail under pure flexure failure as it has very low flexure strength as compared to its shear strength.

Moderate deep beams differ from shallow beams considerably. There is a significant effect of normal pressure on stress distribution. Flexure strength is comparable to shear strength or almost equal in case of moderate deep beams and hence failure of such type of beams is flexure-shear failure. This type of beams is taken as a transition between normal and deep beams regarding to all its properties related to its structural behaviour under applied loading. As the beam become shorter or deeper the stress distribution becomes non-linear with the tensile stresses concentrating toward the bottom of the beam. The stresses at mid span deviate more and more from those predicted by the simple bending theory.

Deep beams behave entirely different from normal beams. Normal pressure has greater effect on stress distribution and hence stress distribution no longer remains linear.

2. SIZE EFFECT

The size effect is a problem of scaling, which is central to every physical theory. The size effect in solid mechanics is understood as the effect of the characteristic structure size (dimension) D on the nominal strength of structure when geometrically similar structures are compared. Generally, specimens of smaller size are observed to have higher strength. A number of factors influence the strength properties and hence the behaviour of material systems. The strength properties include compressive and tensile strengths, bond and fatigue strengths, and creep and various dimensional changes. Along with these properties, the nature of the material and the geometric configuration of specimens are also important. For geometrically similar RCC beam two main factors are important, which is compressive strength of concrete and depth of beam.

Size effect for shear strength in concrete beam is a phenomenon of a decrease in average shear strength due to increasing depth of the member. It has been confirmed that by increasing the depth of beams, the shear strength decreases.

Models tested to failure to determine ultimate load capacity and their use in standards and codes may be subject to size effects which would give non conservative predictions of prototype (and design) strengths. Thus, in addition to efforts in minimizing size effects, there is a need to be able to predict them.

3. ANALYTICAL INVESTIGATION

Equation [2] for ultimate load for beams fails in shear given by Dr. S. N. Patel and Dr. S. K. Damle is:

$$W_u = 2V_u = \frac{3.ft.b.d}{\sqrt{1+0.75(\frac{a}{d})^2}} + \frac{fy}{\sqrt{1+(\frac{a}{d})^2}}.\sum_0^i(\frac{yi}{d}+0.4).Asi.\sin(\alpha_1+\theta) \quad ..... (1)$$

Above equation for predicting ultimate load for beam fails in shear does not give accurate result, because it does not show the size effect in shear strengthfig.1. So size effect factor should be added in above mentioned equation. From past research [3], [4] it is proved that shear strength of concrete beam is directly proportional to $d^{-1/3}$ and $fck^{1/3}$. So we incorporate size effect factor in terms of $d^{-1/3}$ and $fck^{1/3}$ in above equation.

For finding the size effect factor, we have multiply size effect factor Sf with shear component of concrete in above equation. We have done regression analysis of 376 past experimental beam data to find size effect factor Sf. So the new equation obtained is:

$$W_u = 2V_u = \frac{3.ft.b.d}{\sqrt{1+0.75(\frac{a}{d})^2}}.S_f + \frac{fy}{\sqrt{1+(\frac{a}{d})^2}}.\sum_0^i(\frac{yi}{d}+0.4).Asi.\sin(\alpha_1+\theta) \quad ...... (2)$$

S_f = Size effect factor
$= 0.08 + 0.78*(d)^{-1/3}*(fck)^{1/3}$
(Obtained by regression analysis)

If stirrups are provided in moderate deep concrete beam then, in above equation terms corresponding to stirrups should be added. From IS: 456-2000, "Plain and Reinforced Concrete – Code of Practice", we have added the term represent shear resistance due to stirrups. So further for stirrups equation can be given as follow:

$$W_u = 2V_u = \frac{3.ft.b.d}{\sqrt{1+0.75(\frac{a}{d})^2}}.S_f + \frac{fy}{\sqrt{1+(\frac{a}{d})^2}}.\sum_0^i(\frac{yi}{d}+0.4).Asi.\sin(\alpha_1+\theta) + \frac{1.74.fy.Asv.d}{Sv} \quad .... (3)$$

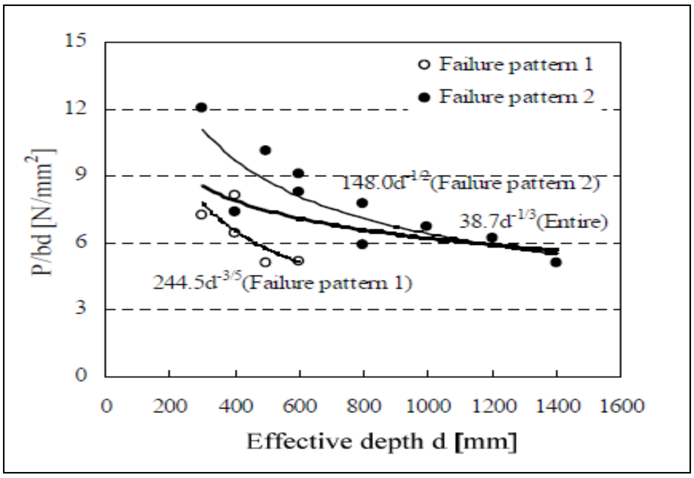

Fig -1: Effect of depth on ultimate nominal shear stress[3]

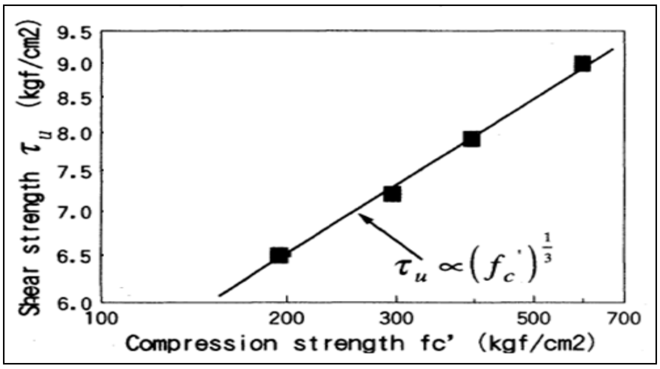

Fig -2: Effect of compressive strength on ultimate nominal shear stress[4]

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Validity of above modified equation is checked for another 54 beam data [5] fails in shear. And compare its result with different codal provisions and formulas. Summary is as follows:

Table -1

				STM-Eurocode 2	STM-Collins	Eurocode 2	BS 8110	Original equation	Modified equation	STM-Eurocode 2	STM-Collins	Eurocode 2	BS 8110	Original equation	Modified equation
Sr. no	D: mm	b: mm	P test (kN)	Pcalc	Pcalc	Pcalc	Pcalc	Pcalc	Pcalc	Ptest/ Pcalc	Ptest/ Pcalc	Ptes/ Pcalc	Ptes/ Pcalc	Ptest/ Pcalc	Ptes/ Pcalc
1	200	250	281	227.61	162.98	151.74	154.55	428.308	297.01	1.23	1.72	1.85	1.82	0.65607	0.95
2	200	100	137	143.85	130.15	101.38	104.12	284.081	208.49	0.95	1.05	1.35	1.32	0.48226	0.66
3	200	100	201	142.71	130.65	100.50	104.52	284.081	208.49	1.41	1.54	2.00	1.92	0.70754	0.96
4	200	100	145	143.55	114.55	87.00	85.55	238.125	162.53	1.01	1.27	1.67	1.69	0.60892	0.89
5	200	100	161	191.59	141.68	80.50	88.55	241.641	181.32	0.84	1.14	2.00	1.82	0.66628	0.89
6	200	100	135	89.10	82.35	83.70	86.40	240.148	171.85	1.52	1.64	1.61	1.56	0.56215	0.79
7	200	100	165	89.10	82.50	84.15	85.80	240.148	171.85	1.85	2.00	1.96	1.92	0.68708	0.96
8	200	100	178	89.00	81.88	83.66	85.44	240.148	171.85	2.00	2.17	2.13	2.08	0.74121	1.04
9	200	100	180	70.20	68.40	79.20	81.00	229.204	162.94	2.56	2.63	2.27	2.22	0.78533	1.10
10	200	100	134	69.68	68.34	79.06	81.74	229.204	162.94	1.92	1.96	1.69	1.64	0.58463	0.82
11	200	100	133	70.49	67.83	78.47	81.13	229.204	162.94	1.89	1.96	1.69	1.64	0.58027	0.82
12	305	152	302	154.02	87.58	102.68	99.66	328.459	195.87	1.96	3.45	2.94	3.03	0.91945	1.54
13	305	152	169	155.48	81.12	89.57	86.19	303.496	170.91	1.09	2.08	1.89	1.96	0.55684	0.99
14	305	152	249	156.87	87.15	102.09	99.60	328.459	195.87	1.59	2.86	2.44	2.50	0.75809	1.27
15	320	190	531	276.12	228.33	217.71	212.40	648.059	441.04	1.92	2.33	2.44	2.50	0.81937	1.20
16	356	102	232	160.08	134.56	118.32	113.68	366.205	241.07	1.45	1.72	1.96	2.04	0.63353	0.96
17	356	102	298	166.88	140.06	122.18	116.22	371.175	244.18	1.79	2.13	2.44	2.56	0.80286	1.22
18	356	102	148	111.00	69.56	78.44	74.00	251.077	154.91	1.33	2.13	1.89	2.00	0.58946	0.96
19	400	150	560	576.80	464.80	280.00	268.80	859.47	560.14	0.97	1.20	2.00	2.08	0.65156	1.00
20	400	150	440	330.00	277.20	220.00	211.20	701.732	444.15	1.33	1.59	2.00	2.08	0.62702	0.99
21	400	150	310	275.90	210.80	182.90	176.70	583.517	377.68	1.12	1.47	1.69	1.75	0.53126	0.82
22	400	150	580	388.60	295.80	208.80	203.00	672.029	442.77	1.49	1.96	2.78	2.86	0.86306	1.31
23	400	150	490	411.60	289.10	186.20	176.40	589.297	381.68	1.19	1.69	2.63	2.78	0.8315	1.28
24	457	203	524	351.08	298.68	267.24	256.76	887.755	556.48	1.49	1.75	1.96	2.04	0.59025	0.94
25	457	203	626	419.42	350.56	287.96	269.18	945.703	591.21	1.49	1.79	2.17	2.33	0.66194	1.06
26	457	203	577	403.90	340.43	282.73	265.42	928.983	578.80	1.43	1.69	2.04	2.17	0.62111	1.00
27	457	203	581	406.70	342.79	284.69	267.26	928.983	578.80	1.43	1.69	2.04	2.17	0.62542	1.00
28	457	203	582	384.12	325.92	279.36	261.90	919.5	573.77	1.52	1.79	2.08	2.22	0.63295	1.01
29	457	203	608	395.20	334.40	279.68	267.52	919.5	573.77	1.54	1.82	2.17	2.27	0.66123	1.06
30	457	203	448	371.84	286.72	232.96	219.52	777.487	436.39	1.20	1.56	1.92	2.04	0.57622	1.03
31	457	203	537	424.23	322.20	247.02	230.91	811.538	457.05	1.27	1.67	2.17	2.33	0.66171	1.17
32	457	203	448	403.20	309.12	241.92	228.48	802.272	452.09	1.11	1.45	1.85	1.96	0.55841	0.99
33	457	203	537	402.75	311.46	241.65	230.91	802.272	452.09	1.33	1.72	2.22	2.33	0.66935	1.19
34	457	203	445	360.45	253.65	200.25	191.35	689.269	352.99	1.23	1.75	2.22	2.33	0.64561	1.26
35	457	203	448	421.12	286.72	210.56	201.60	735.139	380.65	1.06	1.56	2.13	2.22	0.60941	1.18
36	457	203	359	405.67	276.43	208.22	197.45	724.009	373.82	0.88	1.30	1.72	1.82	0.49585	0.96
37	457	203	377	365.69	256.36	199.81	192.27	689.269	352.99	1.03	1.47	1.89	1.96	0.54696	1.07
38	457	203	242	237.16	193.60	162.14	152.46	573.528	299.19	1.02	1.25	1.49	1.59	0.42195	0.81
39	457	203	188	236.88	195.52	161.68	154.16	573.528	299.19	0.79	0.96	1.16	1.22	0.3278	0.63
40	457	203	256	235.52	194.56	161.28	153.60	573.528	299.19	1.09	1.32	1.59	1.67	0.44636	0.86
41	457	203	349	296.65	296.65	212.89	202.42	715.118	388.10	1.18	1.18	1.64	1.72	0.48803	0.90
42	457	203	334	297.26	287.24	210.42	197.06	704.107	381.43	1.12	1.16	1.59	1.69	0.47436	0.88
43	500	140	680	408.00	326.40	224.40	231.20	838.681	546.59	1.67	2.08	3.03	2.94	0.8108	1.24
44	500	110	370	495.80	421.80	222.00	229.40	828.376	556.71	0.75	0.88	1.67	1.61	0.44666	0.66
45	610	178	592	420.32	361.12	319.68	331.52	1280.75	883.51	1.41	1.64	1.85	1.79	0.46223	0.67
46	610	178	605	477.95	411.40	332.75	344.85	1328.94	908.56	1.27	1.47	1.82	1.75	0.45525	0.67

47	610	178	694	499.68	423.34	340.06	347.00	1344.23	916.73	1.39	1.64	2.04	2.00	0.51628	0.76
48	610	178	712	526.88	448.56	348.88	356.00	1359.18	924.81	1.35	1.59	2.04	2.00	0.52385	0.77
49	1000	250	735	573.30	521.85	588.00	529.20	2255.09	1312.34	1.28	1.41	1.25	1.39	0.32593	0.56
50	1000	250	778	567.94	521.26	583.50	521.26	2255.09	1312.34	1.37	1.49	1.33	1.49	0.345	0.59
51	1000	250	752	511.36	473.76	564.00	503.84	2169.5	1276.72	1.47	1.59	1.33	1.49	0.34662	0.59
52	1000	140	940	733.20	592.20	366.60	357.20	1564.04	941.44	1.28	1.59	2.56	2.63	0.60101	1.00
53	1400	140	1380	1048	855.60	496.80	483.00	2257.84	1307.29	1.32	1.61	2.78	2.86	0.6112	1.06
54	1750	140	940	1240	968.20	601.60	564.00	2794.56	1584.13	0.76	0.97	1.56	1.67	0.33637	0.59
									AVERAGE	1.35	1.68	1.98	2.03	0.59522	0.96
									COV	0.13	0.21	0.17	0.18	0.01953	0.05

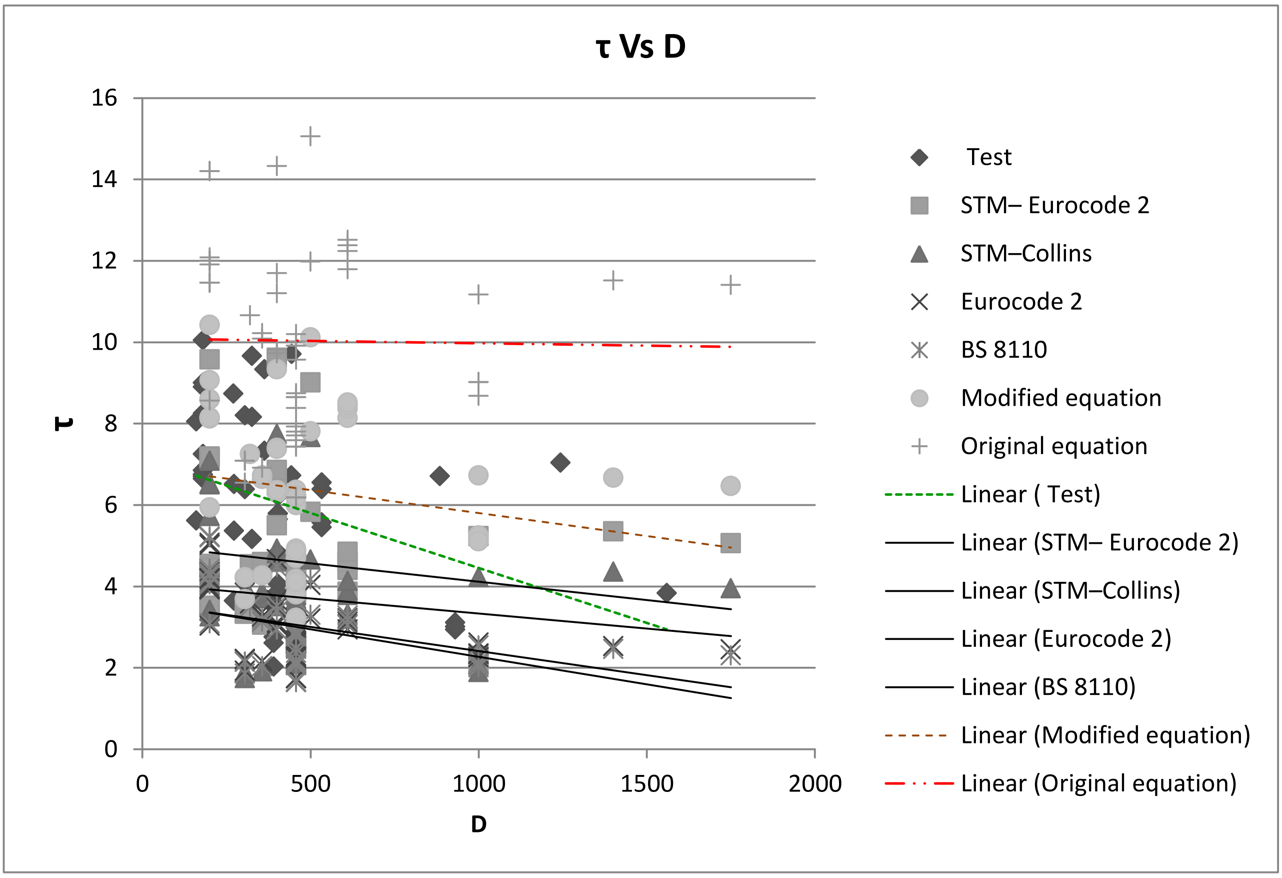

Fig -3: Chart of Nominal ultimate shear stress Vs Total depth

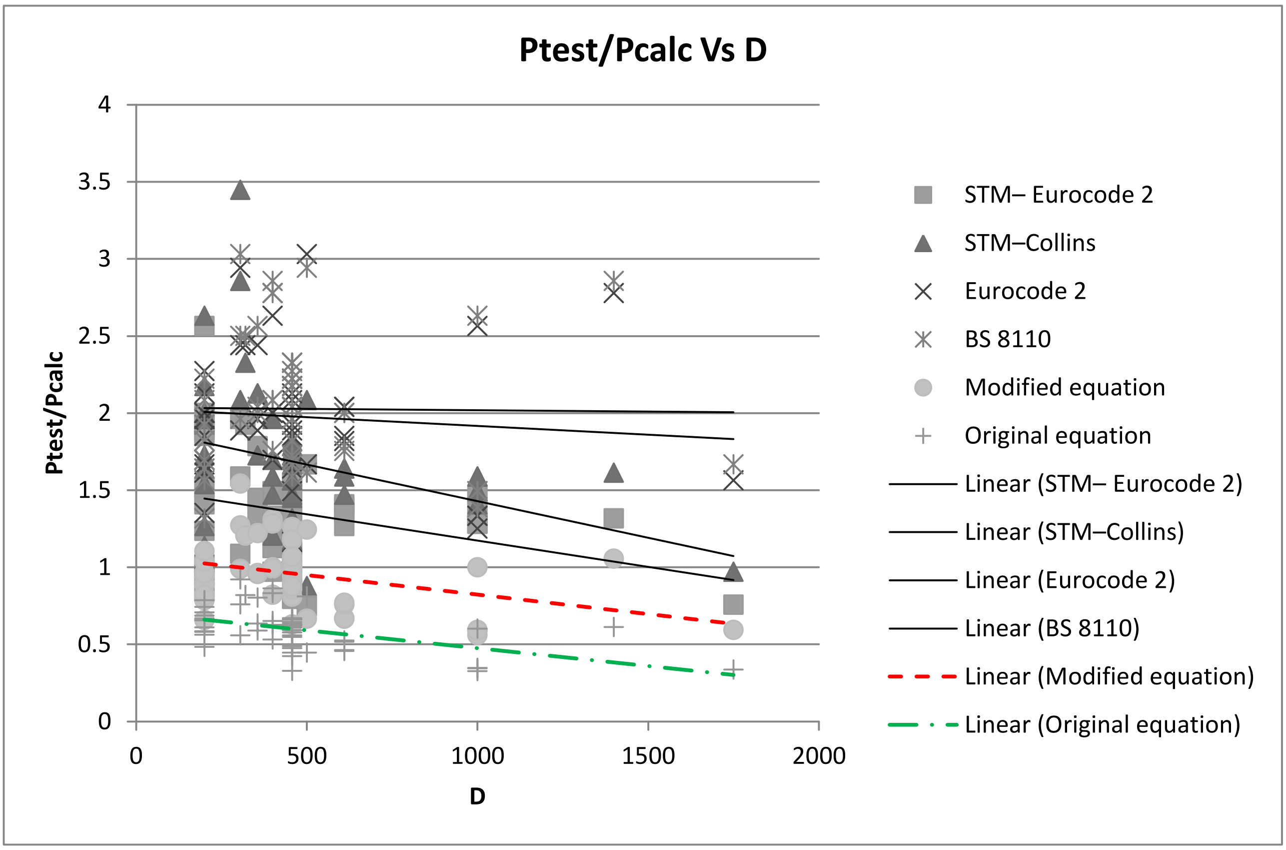

Fig -4: Chart of Pcal/Ptest Vs Total Depth

4. CONCLUSIONS

1. Modified equation for finding ultimate load for moderate deep concrete beam fails in shear give good agreement with experimental result. From table.1 ratio of experimental load to predicted load is Pexp/Pcal=0.96 ≈1 with least COV= 0.05.
2. From fig.3 it is concluded that nominal shear stress (τ) is decrease with increase in depth. It is clear that original equation does not show any size effect. Whether modified equations capture size effect and gives result nearer to experimental result.
3. From above figures and table, it is concluded that modified eqution gives accurate result than provisions given in BS 8110, Eurocode 2, STM-collins, STM-Eurocode 2.

5. NOTATIONS

W_u = Ultimate load

P_{calc}= Calculated ultimate load

P_{test} = Experimental ultimate load

f_{ck} = Characteristic compressive strength

f_t = Cylinder splitting strength of concrete

b = Width of beam

d = Effective depth of beam

a = Shear span

f_y = Yield strength of reinforcement

y_i =Depth of reinforcement layer from top of beam

A_{sv} = Total area of the legs of stirrups within a distance S_v

S_v = Stirrups spacing

τ = Nominal ultimate shear stress of beam (V_u/bd)

$$\alpha_1 = \frac{3}{\sqrt{1+0.75\left(\frac{a}{d}\right)^2}}, \quad \theta = \tan^{-1}\left(\frac{d}{a}\right)$$

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