Analysis of Thermal Profiles of Various Power Extraction Limits in a PWR Heated Channel

Odii Christopher Joseph[1], Agyekum Ephraim Bonah[2], Afornu Bright Kwame[3], Ansah Michael Nii Sanka[4]

[1,2,3,4] *National Research Tomsk Polytechnic University, Russia, Tomsk, Lenin Avenue, 30, 634050*

Abstract - *Thermal profiling of reactor channel is an important step taken in the operation of nuclear reactor. This is essential considering that the thermal power limits of nuclear power plants are largely dependent on the thermal behavior of the coolant, hence the thermal profiling of coolant, cladding and fuel pellet along the channel helps to ascertain the power limit in such channel. In most cases, the saturation temperature of the coolant and the melting point of the fuel pellet are the active parameters that determine power extraction in the channel. In this research, we examined three (3) cases of power extraction limit along the channel namely; the power extraction limit when the coolant exit temperature is to remain sub-cooled, when the maximum cladding temperature is to remain below saturation condition and when the fuel maximum temperature is to remain below the melting temperature condition. The model of this research is a prototype PWR with UO_2 fuel, the energy equation of thermal fluid flow neglects the energy term due to the pressure gradient and friction dissipation. The channel was considered as a single channel, hence we neglected mixing due to the presence of many channel. The profiling was analyzed and it was clear that the second condition had the least power extraction. This was due to low temperature at the cladding surface, which results to low outlet temperature at the channel, hence low power extraction. The first and the third conditions has almost the same power extraction. But from technical point of view, it is better to use the first condition as a standard in the operation of PWR, even though the power extraction was a bit higher in the third case. This keeps the temperature outlet away from the critical saturation temperature of the coolant, and also heating above certain temperature and close to the fuel melting point leads to higher temperature difference between the centerline and the surface, this greatly limits power extraction and can also lead to failure of the fuel when the melting temperature is reached. Hence it is advisable to always operate the outlet temperature close to the saturation temperature. This is a safety measure in the operation of LWR.*

Key Words: thermal, extraction, PWR, heated, channel, profile, coolant, cladding, fuel, pellet, height, Nuclear, power, temperature, centreline, nucleate, boiling, NB, departure, DNB, linear, density.

1. INTRODUCTION

The extraction of heat along the channel of a nuclear reactor depends on thermal properties of the coolant and geometrical property of the channel. Properties such as outlet and inlet temperature, flow rate, specific heat capacity of coolant and length of the channel, fundamentally determines the heat extracted from the channel [1]. When the temperature of the heated cladding surface exceeds the saturation temperature of the surrounding coolant, boiling on the surface becomes possible. This is true if the bulk coolant temperature is at or below the local saturation temperature. When the bulk fluid temperature is below the saturation temperature, boiling is referred to as local or sub-cooled boiling [2]. When it is equal to the saturation temperature, it is called bulk boiling. Bubbles formed on the heated cladding material surface depart the surface and are transported by the coolant, such that a condition of two phase flow is said to exist. Depending on the degree of sub-cooling and the length of the heated channel, the bubbles may or may not condense and collapse prior to exiting the channel. In sub-cooled boiling this results in further heating of fluid towards the saturation temperature. In saturated or bulk boiling, bubbles can be transported along the entire length of the heated channel without collapsing.

Steady state operation beyond the point of critical heat flux is only possible for wall temperature controlled systems, where the heat input to the surface can be adjusted to maintain a given wall temperature [3]. In reactor systems, it is power and therefore heat flux which is controlled. In a heat flux controlled system, an increase in the flux beyond the critical points results in departure from nucleate boiling (DNB) with an associated increase in the wall temperature. This increase in the wall temperature causes more of the heated surface to be blanketed by vapor, further increasing the wall temperature. DNB is the dominant critical heat flux mechanism in PWR [4]

1.1 Formulation of Analytical Model

The energy equation of thermal fluid flow neglecting the energy terms due to the pressure gradient and friction dissipation yields

$$GA_{z_f}\frac{dh}{dz_f} = q_f''P_h \qquad (1)$$

But $\quad m' = GA_{z_f}$

$$m'\frac{dh}{dz} = q_f'(z_f) \qquad (2)$$

For a particular flow rate, the coolant enthalpy rise depends on the axial variation of the linear heat generation rate. In nuclear reactors, the local heat generation depends on the distribution of both the neutron flux and fission material. The neutron flux is affected by the moderator density, absorbing materials and local concentration of the fissile and fertile nuclear materials [1]. Thus, a coupled neutronic-thermal hydraulics analysis is necessary for a complete design analysis.

For purpose of simplicity we apply the following assumption:

The variation of the linear heat density is sinusoidal

$$q_f'(z_f) = q_o'\cos\left(\frac{\pi z_f}{H'}\right) \quad (3)$$

Assuming that we use the maximum power density

$$q_{f\max}''' = \frac{0.75\alpha P_t}{H r_o^2 N n} \quad (4)$$

The power density distribution throughout the reactor can be written as

$$q_f'''(z_f, r) = q_{f\max}''' J_0\left(2.405\frac{r}{R'}\right)\cos\left(\frac{\pi z_f}{H'}\right) \quad (5)$$

Where

$$q_f'''(z_f) = q_{f_o}''' \cos\left(\frac{\pi z_f}{H'}\right) \quad (6)$$

And

$$q_{f_o}''' = q_{f\max}''' J_0\left(2.405\frac{r}{R'}\right) \quad (7)$$

Rearranging (6)

$$q_f'''(z_f) = q_{f_o}''' \cos\left(\frac{2\pi H}{H'}\frac{z_f}{H}\right) = q_{f_o}''' \cos\left(2\beta\frac{z_f}{H}\right) \quad (8)$$

The average power density is:

$$q_{fav}'''(z_f) = \frac{\int_{-\frac{1}{2}H}^{\frac{1}{2}H} q_{f_o}''' \cos\left(2\beta\frac{z_f}{H}\right)dz_f}{\int_{-\frac{1}{2}H}^{\frac{1}{2}H} dz_f} = q_{f_o}''' \frac{\sin\beta}{\beta} \quad (9)$$

The linear power distribution along the channel is

$$q_f'(z_f) = (n\pi r_0^2) q_{f_o}''' \cos\left(2\beta\frac{z_f}{H}\right) = q_{f_o}' \cos\left(2\beta\frac{z_f}{H}\right) \quad (10)$$

The integration of (10) along the channel yields the total fuel channel power.

$$P_{channel} = \int_{-\frac{1}{2}H}^{\frac{1}{2}H} (n\pi r_0^2) q_{f_o}''' \cos\left(2\beta\frac{z_f}{H}\right)dz_f = q_{f_o}''' H\frac{\sin\beta}{\beta} = Hq'_{av} \quad (11)$$

1.2 Analysis of coolant thermal distributions along the channel

Assuming steady state

$$m' \int_{h_{in}}^{h} dh = \int_{-\frac{1}{2}H}^{z_f} q_{f_o}''' \cos\left(2\beta\frac{z_f}{H}\right)dz_f \quad (12)$$

$$h_{cool}(z_f) = h_{in} + \frac{q_{f_o}'}{m'}\frac{H}{2\beta}\sin\beta \cdot \left(\frac{\sin\left(2\beta\frac{z_f}{H}\right)}{\sin(\beta)} + 1\right) = h_{in} + \frac{P_{channel}}{2m'}\left(\frac{\sin\left(2\beta\frac{z_f}{H}\right)}{\sin(\beta)} + 1\right) \quad (13)$$

Applying

$$h_{cool}(z_f) - h_{in} = c_p\left(t_{cool}(z_f) - t_{in}\right) \quad (14)$$

$$t_{cool}(z_f) = t_{in} + \frac{P_{channel}}{2m' \cdot c_p}\left(\frac{\sin\left(2\beta\frac{z_f}{H}\right)}{\sin(\beta)} + 1\right) \quad (15)$$

Also

$$P_{channel} = m'(h_{exit} - h_{in}) = m' \cdot c_p(t_{exit} - t_{in}) \quad (16)$$

$$t_{exit} = t_{in} + H\frac{q_{f_o}'}{m' c_p}\frac{\sin\beta}{\beta} \quad (17)$$

$$x_{cool}(z_f) = \frac{h(z_f) - h_f}{h_g - h_f} = \frac{h_{in} - h_f}{h_g - h_f} + \frac{P_{channel}}{2m'(h_g - h_f)}\left(\frac{\sin\left(2\beta\frac{z_f}{H}\right)}{\sin(\beta)} + 1\right) \quad (18)$$

2. Cladding Temperature Analysis

The temperature of the outer surface of the cladding is obtained by Newton's law of cooling.

$$t_{clad} - t_{cool}(z_f) = \frac{q'}{h_{clad} P_h} \quad (19)$$

$$t_{clad}(z_f) = t_{cool}(z_f) + \frac{q'}{h_{clad} P_h} \quad (20)$$

$$t_{clad}(z_f) = \frac{(t_{exit} + t_{in})}{2} + \frac{(t_{exit} - t_{in})}{2\sin\beta}\left(\sin\left(2\beta\frac{z_f}{H}\right) + \frac{1}{\gamma}\cos\left(2\beta\frac{z_f}{H}\right)\right) \quad (21)$$

Where

$$\gamma = \frac{h_{clad} P_h H}{2\beta m^{'} c_p}$$

To calculate the maximum temperature of the cladding is calculated thus:

$$\frac{dt_{clad}}{dz_f} = \cos\left(2\beta\frac{z_f}{H}\right) - \frac{1}{\gamma}\sin\left(2\beta\frac{z_f}{H}\right) = 0 \quad (22)$$

$$\gamma = \tan\left(2\beta\frac{z_f}{H}\right)$$

$$z_{\max.clad} = \frac{H}{2\beta}\tan^{-1}(\gamma)$$

Therefore

$$t_{\max.cl} = \frac{(t_{exit} + t_{in})}{2} + \frac{(t_{exit} - t_{in})}{2\sin\beta}\begin{pmatrix}\sin(\arctan\gamma) + \\ + \frac{1}{\gamma}\cos(\arctan\gamma)\end{pmatrix} \quad (23)$$

It should be noted that the maximum cladding temperature should be less than the maximum allowable temperature with a reasonable safety margin. The maximum allowable temperature for Zircaloy-4 Cladding is 380℃-400℃

Analysis of Pellet Surface Temperature Distribution:

$$t_{sf} = t_{clad} + \frac{q^{'}}{2\pi k_c}\ln\left(\frac{r_c}{r_o}\right) + \frac{q^{'}}{2\pi r_o h_{conv.}} \quad (24)$$

Taking

$$\frac{1}{\gamma^{'}} = \frac{1}{\gamma} + \frac{\beta}{\pi k_c}\frac{P_{channel}}{(t_{exit} - t_{in})H}\ln\left(\frac{r_c}{r_o}\right) + \frac{\beta}{\pi r_o h_{conv.}}\frac{P_{channel}}{(t_{exit} - t_{in})H} \quad (25)$$

We have

$$t_{sf}(z_f) = \frac{t_{exit} + t_{in}}{2} + \frac{(t_{exit} - t_{in})}{2\sin\beta}\left(\sin\left(2\beta\frac{z_f}{H}\right) + \frac{1}{\gamma^{'}}\cos\left(2\beta\frac{z_f}{H}\right)\right) \quad (26)$$

The location of the maximum temperature of the surface is easily found by differentiating (29) with respect to z.

$$\frac{dt_{sf}}{dz_f} = \cos\left(2\beta\frac{z_f}{H}\right) - \frac{1}{\gamma^{'}}\sin\left(2\beta\frac{z_f}{H}\right) = 0 \quad (27)$$

$$z_{\max} = \frac{H}{2\beta}\arctan(\gamma^{'})$$

$$t_{sf\max} = \frac{t_{exit} + t_{in}}{2} + \frac{(t_{exit} - t_{in})}{2\sin\beta}\left(\sin\Phi + \frac{1}{\tan\Phi}\cos\Phi\right) \quad (28)$$

$$t_{sf\max} = \frac{t_{exit} + t_{in}}{2} + \frac{(t_{exit} - t_{in})}{2\sin\beta}\left(\frac{1}{\sin\Phi}\right) \quad (29)$$

But

$$\frac{1}{\sin\Phi} = \sqrt{1 + \frac{1}{\tan^2\Phi}} = \sqrt{1 + \left(\frac{1}{\gamma^{'}}\right)^2} \quad (30)$$

Therefore, the maximum temperature distribution of the fuel surface is

$$t_{sf\max} = \frac{t_{exit} + t_{in}}{2} + \frac{(t_{exit} - t_{in})}{2\sin\beta}\sqrt{1 + \left(\frac{1}{\gamma^{'}}\right)^2} \quad (31)$$

It is clear from the above equation that the maximum value of the surface temperature of the fuel decreases with increasing heat transfer coefficient and gap conductance.

Analysis of the Centerline fuel temperature

The maximum centerline is located at

$$z_{clf\max} = \frac{H}{2\beta}\arctan\gamma^{''} \quad (32)$$

And the value of the maximum is:

$$t_{clf\max} = \frac{t_{exit} + t_{in}}{2} + \frac{(t_{exit} - t_{in})}{2\sin\beta}\sqrt{1 + \left(\frac{1}{\gamma^{''}}\right)^2} \quad (33)$$

2.1 Numerical computation of thermal profile distribution for each of the three (3) cases under study

Note: in this calculation, the reactor's extrapolated height and radius due to the reflector, is not considered. Hence the extrapolated height is equal to the channel height.

$$\frac{H}{H'} = 1, \beta = \frac{\pi}{2}\frac{H}{H'} = \frac{\pi}{2}$$

CASE 1: The coolant exit temperature is to remain sub-cooled

Using the Thermal parameters of the coolant and the fuel and Geometrical parameters of the fuel channel used for this study in Appendix, we have the following:

$$q_{f_1}^{'}(z_f) = 42.74\cos\left(\pi\frac{z_f}{3.658}\right) \quad (34)$$

$$h_{1cool}(z_f) = 1300.5 + 148.53\left(\sin\left(\pi\frac{z_f}{3.658}\right) + 1\right)$$

$$t_{1cool}\left(z_f\right)=318.97+25.87\sin\left(\pi\frac{z_f}{3.658}\right)$$

$$x_{1cool}\left(z_f\right)=-0.00412+0.102\left(\sin\left(\pi\frac{z_f}{3.658}\right)+1\right)$$

$$t_{1clad}\left(z_f\right)=318.97+25.87\left(\sin\left(\pi\frac{z_f}{3.658}\right)+1.628\cos\left(\pi\frac{z_f}{3.658}\right)\right)$$

$$t_{1sf}\left(z_f\right)=318.97+25.87\left(\sin\left(\pi\frac{z_f}{3.658}\right)+15.1\cos\left(\pi\frac{z_f}{3.658}\right)\right)$$

$$t_{1clf}\left(z_f\right)=318.97+25.87\left(\sin\left(\pi\frac{z_f}{3.658}\right)+76\cos\left(\pi\frac{z_f}{3.658}\right)\right)$$

Case 2: The maximum clad temperature is to remain below the saturation temperature of the coolant

$$q_{f_2}{}'\left(z_f\right)=29.364\cos\left(\pi\frac{z_f}{3.658}\right)$$

$$h_{2cool}\left(z_f\right)=1300.5+102\left(\sin\left(\pi\frac{z_f}{3.658}\right)+1\right)$$

$$t_{2cool}\left(z_f\right)=310.88+17.78\sin\left(\pi\frac{z_f}{3.658}\right)$$

$$x_{2cool}\left(z_f\right)=-0.00412+0.07\left(\sin\left(\pi\frac{z_f}{3.658}\right)+1\right)$$

$$t_{2clad}\left(z_f\right)=310.88+17.78\left(\sin\left(\pi\frac{z_f}{3.658}\right)+1.628\cos\left(\pi\frac{z_f}{3.658}\right)\right)$$

$$t_{2sf}\left(z_f\right)=310.88+17.78\left(\sin\left(\pi\frac{z_f}{3.658}\right)+15.1\cos\left(\pi\frac{z_f}{3.658}\right)\right)$$

$$t_{2clf}\left(z_f\right)=310.88+17.78\left(\sin\left(\pi\frac{z_f}{3.658}\right)+76\cos\left(\pi\frac{z_f}{3.658}\right)\right)$$

Case 3: The fuel maximum centerline temperature is to remain below the melting temperature of 2400℃, ignoring sintering

$$q_{f_3}{}'\left(z_f\right)=44.43\cos\left(\pi\frac{z_f}{3.658}\right)$$

$$h_{3cool}\left(z_f\right)=1300.5+155\left(\sin\left(\pi\frac{z_f}{3.658}\right)+1\right)$$

$$t_{3cool}\left(z_f\right)=320.1+27\sin\left(\pi\frac{z_f}{3.658}\right)$$

$$x_{3cool}\left(z_f\right)=-0.00412+0.106\left(\sin\left(\pi\frac{z_f}{3.658}\right)+1\right)$$

$$t_{3clad}\left(z_f\right)=320.1+27\left(\sin\left(\pi\frac{z_f}{3.658}\right)+1.628\cos\left(\pi\frac{z_f}{3.658}\right)\right)$$

$$t_{3sf}\left(z_f\right)=320.1+27\left(\sin\left(\pi\frac{z_f}{3.658}\right)+15.1\cos\left(\pi\frac{z_f}{3.658}\right)\right)$$

$$t_{3clf}\left(z_f\right)=320.1+27\left(\sin\left(\pi\frac{z_f}{3.658}\right)+76\cos\left(\pi\frac{z_f}{3.658}\right)\right)$$

3.0 Graphical Distribution Results for the three cases of study

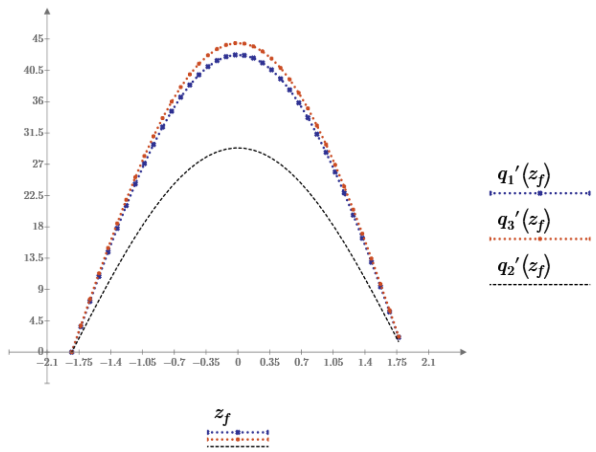

Chart – 1: Linear heat density distribution for the 3 cases

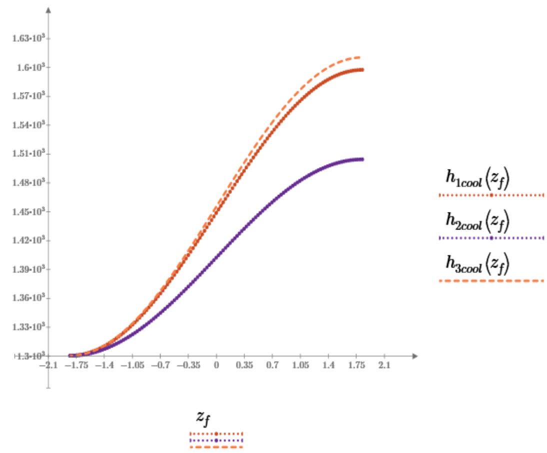

Chart – 2: Coolant enthalpy distribution for the 3 cases

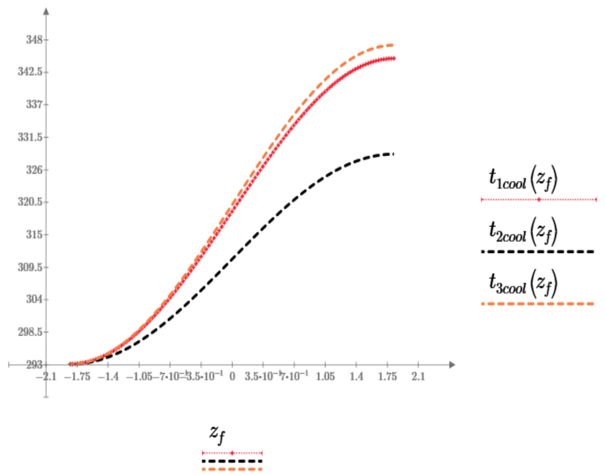

Chart -3: Coolant temperature distribution for the 3 cases

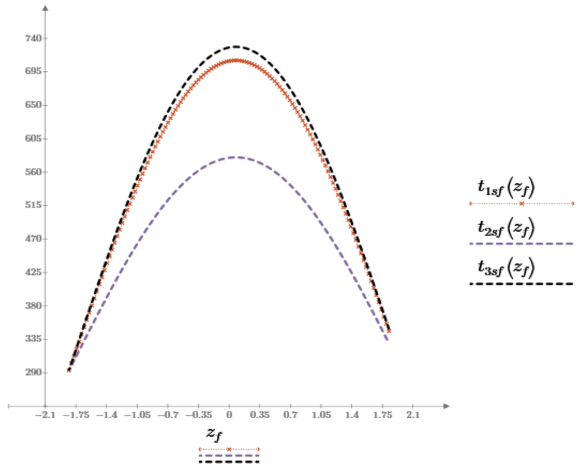

Chart – 6: Fuel surface temperature distribution for the 3 cases

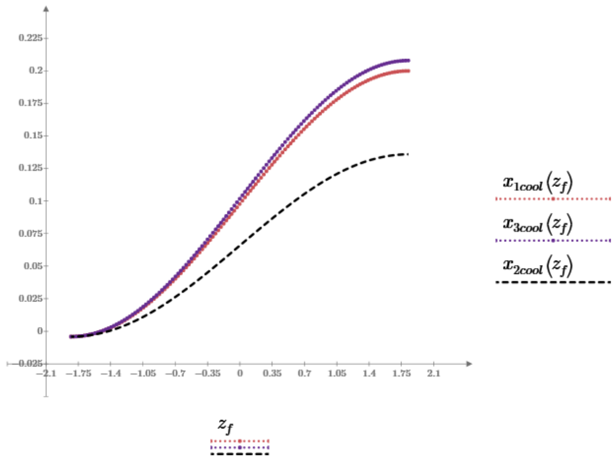

Chart – 4: Coolant dryness factor for the 3 cases

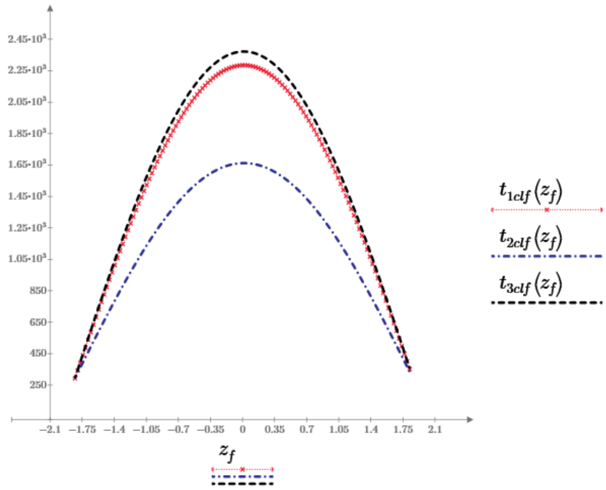

Chart – 7: Centerline temperature distribution for the 3 cases

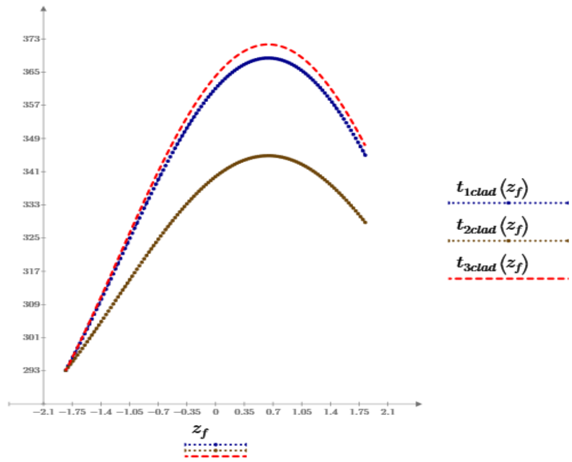

Chart -5: Clad temperature distribution for the 3 cases

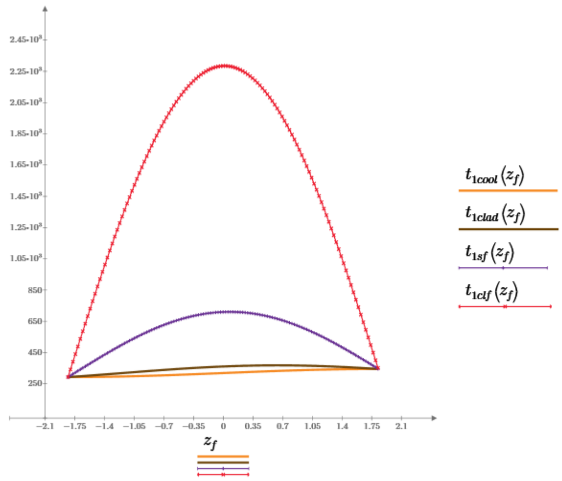

Chart – 8: All the temperature distributions for case 1

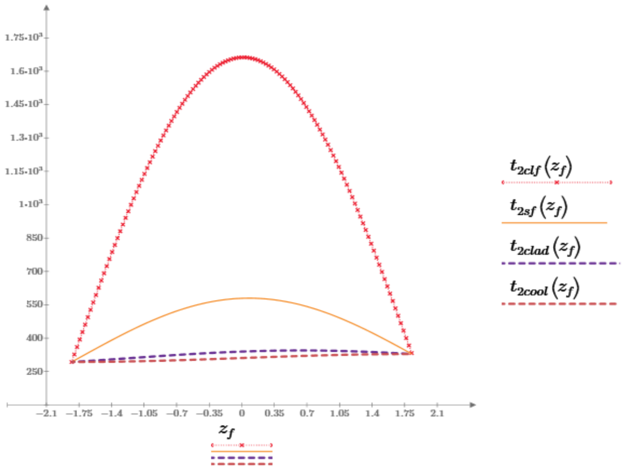

Chart – 9: All the temperature distributions for case 2

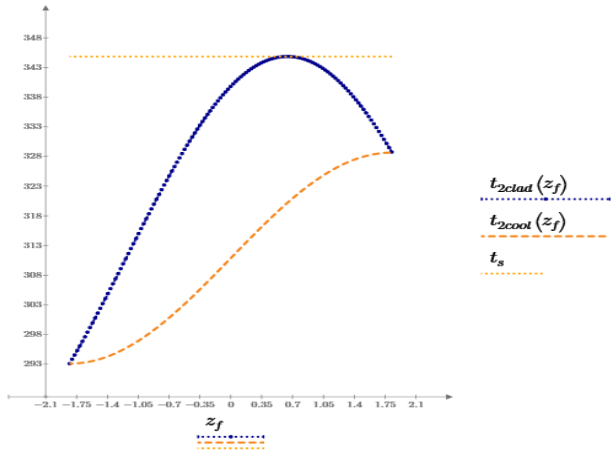

Chart – 12: Coolant, saturation and clad temperature distribution for case 2

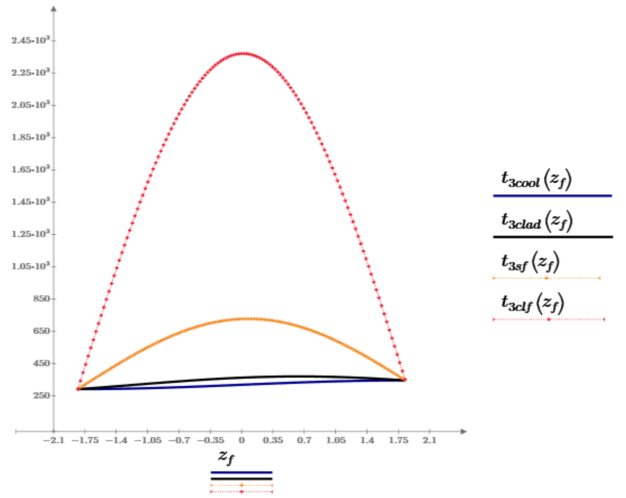

Chart – 10: All the temperature distributions for case 3

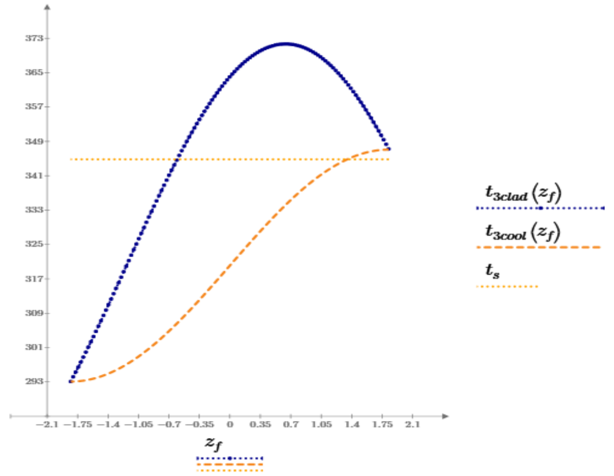

Chart – 13: Coolant, saturation and clad temperature distribution for case 3

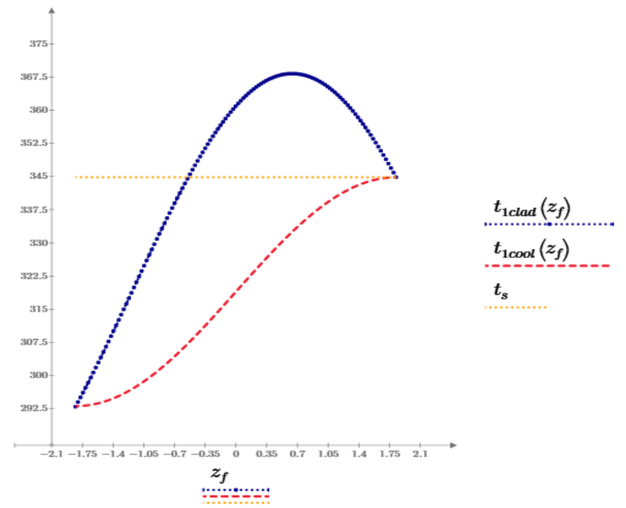

Chart – 11: Coolant, saturation and clad temperature distribution for case 1

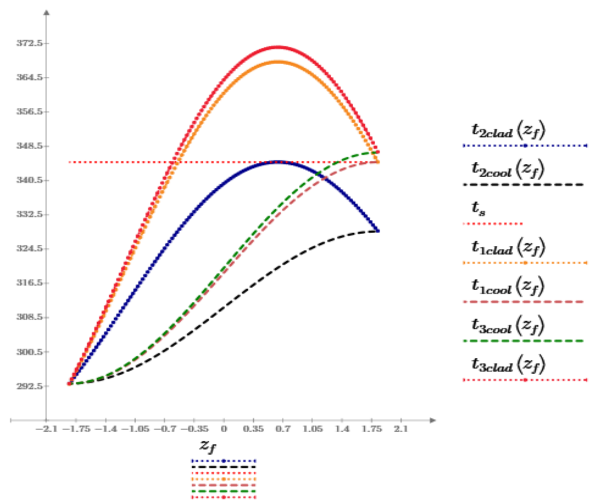

Chart – 14: Coolant, saturation and clad temperature distribution for the 3 cases

4. Analysis of Results

From the distribution results, the three cases were within the thermal design limits for PWR. In the first and third cases, the temperature difference between the wall and the saturation temperature of the coolant is within the region of Nucleate boiling. This region is not dangerous, rather, due to the formation of many bubbles which grows at the wall and detach under the effect of buoyancy, there is creation of agitation and mixing near the wall, this phenomenon enhances heat transfer. As a result, nucleate boiling is a much more effective mechanism than free convection. However, due to the fact that the exit temperature of the coolant in the third case is greater than the saturation temperature, we needed to be careful because this means that the maximum temperature at the wall also increased, and if not monitored could create a temperature difference between it and the saturation temperature that will attain the critical heat flux and hence deviate from Nucleate Boiling, which is the point of Boiling crisis for PWR. Based on this, the safest mode of operation should be operating the exit temperature at or a little below the saturation temperature so that the maximum temperature of the clad wall would still be higher than saturation temperature but Nucleate boiling would occur within safety limit.

In the second case, the temperature difference between the wall temperature and the saturation temperature of the coolant is close to zero, this accounted for low thermal extraction of heat from the reactor. The main reason was because the wall temperature is not sufficiently beyond the saturation temperature to initiate bubble nucleation. In this region the heat from the wall is transferred by single phase free convection.

5. Conclusion

In concluding this work, we noted that

1. The first case had good safety limit and also good thermal extraction of heat

2. The second case had the best safety limit but bad thermal extraction of heat

3. The third case had moderate safety limit and good thermal extraction of heat

The research therefore concluded that from both economical and safety consideration, the first case was the best option in the operation of PWR.

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