# Sensitivity Analysis of Bread Production at Anifowose Bakery Industry, Offa, Nigeria

**Okeniyi Okeyemi, Maroof**

*Principal Lecturer, Mathematics and Statistics Department, the Federal Polytechnic Offa, Offa, Kwara State.*

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**Abstract:** *A system could be any of activities within an organization, from production to a sales operation. In all cases limited resources are allocated for achievement of basic objectives. The objective might be to maximize profit or minimize cost. This paper makes use of data from Anifowose Bakery Industry Offa, Kwara State. Which includes machine hours for cutting, number of laborers', quality of water used, oven (heat) temperature and bags of flours used for production of bread. Linear Programming Sensitivity Analysis was employed, it was discovered that when the price of both basic and non-basic products changed the objectives function also changed and the total profit increased. It was established that the use of electrical oven improved the production and required less labour, time and maximize profit.*

***Key Words:*** Linear Programming, Problem Sensitivity Analysis, Production.

## 1. INTRODUCTION

A relationship between two or more variables could be direct and precisely proportional. For instance a 5% change in the number of production hours of operation will cause 5% change in output. After a linear programming problem has been solved the step taken to study whether current optimal solution is still optimal under the changed circumstances or how far the input parameter values can vary without causing changes in the optimal solution constitutes a sensitivity analysis (post optimal analysis).

The solution from a model is tested to find out whether it yields better performance than the alternative which is usually the one in current use. The test may be prospectively or retrospective against future performance. If it is neither testing is feasible, Onyeizugbe, C. U. (2011)[1] reported that sensitivity is a measure of the extent to which estimates used in the solution would have to be in error before the proposal solution performs less satisfactory than the alternative decision.

### 1.1 Aim and Objectives of the paper

The objectives of manufacturers are to maximize profit and minimize production cost. Hence, the aims of this paper are:

- To examine how changes affect the optimality of solution and how to handle changes.
- To determine changes in the objective function (i.e. profit of different size of bread as the result of changes in variable)

### 1.2 Sources of Data

Data collection is an activity aimed at getting information to satisfy some decision objective. Data used in this paper was collected from Anifowose Bakery Industry opposite railway station, Offa. The data includes machines hour for cutting, number of labours, quantity of water used, oven (heat) temperature and bags of flour used for production of breads.

## 2. LITERATURE REVIEW

Linear programming model refers to finding optimal solution to certain problem in which the solution to must satisfy a given set of constraint. Mathematically, a linear programming is a deterministic technique, which includes the allocation of scarce resources in an optimal way so as to maximize profit or minimize cost.

Onyeizugbe (2011)[1], optimization of problem was discoverved in 1930 by an economist while developing the method for the optimal allocation of resources during the Second World War. The United Air Force sights an effective procedure allocating resources and they turned to linear programming.

Taha, H. A. (2001)[2] defines linear programming as a mathematical technique for finding the best use of organization resources. The term linear is used to decribe a relationship between two or more variables, a relationship which is directly or precisely proportional, in a linear relationship work hours and out-put, for example a 10 percent change in the number of productive hours used in some operations will cause a 10 percent change in output "Taha, H. A." continued that mathematically, these relationships are in form of $a_{11}x_{11} + a_{12}x_{12} + a_{13}x_{13} + \ldots\ldots\ldots\ldots + a_{1n}x_{1n} = bi$ where $a_{ij}$ and bi are known coefficient and the xj's are known variables. The term programming refers to the use of certain mathematical techniques to get the possible solution to a problem involving limited resources.

## 3. RESEARCH METHODOLOGY

### 3.1 Transformation of Managerial problem to Linear Program

To transform a managerial problem to a linear program, the following procedures are to be followed:

i. Determine the quantity to be optimized and express it as LINEAR function doing so serve to define input variable. This gives the objective function.
ii. Identify all stipulated requirements restriction and limitations and express them mathematically. These requirements constitute the constraints.
iii. Express any hidden conditions. Such conditions are not stipulated explicitly in the problem but are apparent from the physical situation being modeled. Generally, they involve non-negative or integer.

### 3.2 General Linear Programming Problem

The general programming problem is in form of optimize

$Z = C_1X_1 + C_2X_2 + C_3X_3 + .... + C_nX_n$ ............... (i)
$a_{11}x_{11} + a_{12}x_{12} + a_{13}x_{13} + .... + a_{1n}x_{1n} * b_1$
$a_{21}x_1 + a_{21}x_2 + a_{23}x_3 + .... + a_{2n}x_n * b_2$ ............... (ii)
-- - -- - - - --- -- ------ ----
$a_{m1}x_1 + a_{m1}x_2 + a_{m3}x_3 + ....... + a_{mn}x_n * b_m$
$x_1x_2$ ............ $x_n \geq 0$........ (iii)

where * means = ≤, ≥ and (M ≤ n)
Equation (i) is the objective function
Equation (ii) is the constraint
Equation (iii) is the non-negative conditions

### 3.3 Algorithm to solve linear programming program

**Step 1:** If the problem is of minimization, convert it to maximization by multiplying the objective function Z by -1

**Step II:** See that all bi's are positive, if a constraint has negative bi, multiply it by -1 to make the bi to be positive.

**Step III:** Convert all the inequalities by adding slack variables, artificial variable or by subtracting surplus variables.

**Step IV:** Find the starting basic feasible solution

**Step V:** Construct the starting simplex table as follows

**Table 1 – Starting Simplex**

| BV | CB | $X_1$ | $X_2$ | $X_n$ | $X_{n+1}$ | $X_{n+2}$ | $X_{m+n}$ | $X_b$ |
|---|---|---|---|---|---|---|---|---|
| $X_{n+1}$ | 0 | $a_{11}$ | $a_{12}$ | $a_{1n}$ | 1 | 0 | 0 | $b_1$ |
| $X_{n+2}$ | 0 | $a_{12}$ | $a_{22}$ | $a_{2n}$ | 0 | 1 | 0 | $b_2$ |
| $X_{m+n}$ | 0 | $a_{m1}$ | $a_{m2}$ | $a_{mn}$ | 0 | 0 | | $b_m$ |
| | $Z_j$ | $Z_1$ | $Z_n$ | $Z_n$ | 0 | 0 | 0 | $Z = Z_0$ |
| | $C_j$ | $C_1$ | $C_2$ | $C_n$ | 0 | 0 | 0 | |
| $Z_j - C_j$ | | $D_1$ | $D_2$ | $D_n$ | $D_{n+1}$ | $D_{n+2}$ | $D_{m+n}$ | |

Where $a_{ij} = y_{ij}$ and $x_j$ is the role of the starting BFS and $C_j$ is the row of coefficient of the variable in the objective function.

**Step VI:** Testing for optimality of BFS by computing $D_j = Z_j - C_j$, if $Z_j - C_j \geq$, the solution optimal, otherwise we proceed to

**Step VII:** To improve on the BFS, we find the INCOMING VECTOR entering the basic matrix and the OUTGOING VECTOR to be removed from the basic matrix. The variable that corresponds to the most negative $Z_j - C_j$ is the INCOMING VECTOR while the variable that corresponds to the minimum ration $b_i/a_{ij}$ for a particular j and $a_{ij} > 0, 1 = 1, 2, .................., M$ is the OUTGOING VECTOR.

**Step VIII:** THE KEY ELEMENT or the pivot element is determined by considering intersection between the arrows that corresponding to both incoming and outgoing vectors. The key element is used to generate the next table. In the next table, pivot element will be replaced by UNITY while other elements of the pivot column will be replaced by zero. To calculate, we use the relation New Row = Former element in the old row (intersectional element of the old row) x (corresponding element of replacing row). In this way, we get improved BFS.

**Step IX:** Test this new BFS for optimality as in step vi if it is not optimal, repeat the process till optimal is obtained.

### 3.4 Sensitivity Analysis

After a linear programming problem has been solved, it is useful to study the effect of discrete changes in the parameters of the problem on the current optimal solution.

i. Changes in the cost coefficient (Ci)
ii. Changes in the right-hand side constants (bi)
iii. Adding new existing or variables
iv. Changing existing column
v. Adding new constraint

## 4. RESULT AND DISCUSSION

Date collected on different sizes of bread

i. 14cm x 9cm size (i.e ₦30 bread)
ii. 15.5cm x 9cm size (i.e. ₦40 bread)
iii. 21cm x 10cm size (i.e. ₦70 bread)
iv. 30cm x 11cm size (i.e. ₦30 bread)

The table below gives data on; machine hour of cutting, numbers of labours, over temperature, quantity of water and bags of flour used on different sizes of bread.

**Table 2**

| **Resources** | **14cm x 9cm sizes (₦30 bread)** | **15.5cm x 90cm size (₦70 bread)** | **21cm x 10cm size (₦70 bread)** | **30cm x 11cm size (₦130 bread)** | **Total Available Resource** |
|---|---|---|---|---|---|
| Machine hours Cutting | 1 ½ | ¾ | 1 | 2/3 | 3 |
| Numbers of Labour | 4 | 2 | 4 | 2 | 12 |
| Quantity of Water | 120litres | 60 litres | 120 litres | 60 litres | 360 |
| Oven (heat) temperature | $150^0$C | $125^0$C | $125^0$C | $120^0$C | $200^0$C |
| Bags of flour used | 4 | 2 | 4 | 2 | 24 |

The decision variable will be:
Let the number of unit of product A be $x_1$
Let the number of unit of product B be $x_2$
Let the number of unit of product C be $x_3$
Let the number of unit of product D be $x_4$

MaxZ + $6x_1 + 9x_2 + 12x_3 + 14x_4$

S.T. = $1\frac{1}{2}x_1 + \frac{3}{4}x_2 + x3 + \frac{3}{4}x_4 \leq 3$

$4x_1 + 2x_2 + 4x_3 + 2x_4 \leq 12$

$120x_1 + 60x_2 + 120x_3 + 60x_4 \leq 360$

$150x_1 + 120x_2 + 125x_3 + 120x_4 \leq 200$

$4x_1 + 2x_2 + 4x_3 + 2x_4 \leq 24$

$x_1x_2x_3x_4 \geq 0$

Then the problem will become:

MaxZ + $6x_1 + 9x_2 + 12x_3 + 14x_4$

S.T. = $1\frac{1}{2}x_1 + \frac{3}{4}x_2 + x_3 + \frac{2}{3}x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 = 3$

$4x_1 + 2x_2 + 4x_3 + 2x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 = 24$

$120x_1 + 60x_2 + 120x_3 + 60x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 = 360$

$150x_1 + 120x_2 + 125x_3 + 120x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 = 2$

$4x_1 + 2x_2 + 4x_3 + 2x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 = 3$

$x_1x_2x_3x_4$ ............................................................ $x_9 \geq 0$

| **BV** | **CB** | **$X_1$** | **$X_2$** | **$X_3$** | **$X_4$** | **$X_5$** | **$X_6$** | **$X_7$** | **$X_8$** | **$X_9$** | **$X_B$** | **M.R** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $X_5$ | 0 | 3/2 | ¾ | 1 | 2/3 | 1 | 0 | 0 | 0 | 0 | 3 | 4.5 |
| $X_6$ | 0 | 4 | 2 | 4 | 2 | 0 | 1 | 0 | 0 | 0 | 12 | 6 |
| $X_7$ | 0 | 120 | 60 | 120 | 60 | 0 | 0 | 1 | 0 | 0 | 360 | 6 |
| $X_8$ | 0 | 150 | 120 | 125 | 120 | 0 | 0 | 0 | 1 | 0 | 200 | 1.60 |
| $X_9$ | 0 | 4 | 2 | 4 | 2 | 0 | 0 | 0 | 0 | 1 | 24 | 12 |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Z=0 | |
| | $C_j$ | 6 | 9 | 12 | 14 | 0 | 0 | 0 | 0 | 0 | | |
| $Z_j$- $C_j$ | | -6 | -9 | -12 | -14 | 0 | 0 | 0 | 0 | 0 | | |

Reduce $\begin{matrix} 2/3 \\ 2 \\ 60 \\ 120 \\ 2 \end{matrix}$ to $\begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{matrix}$ from operation $NR_1 = R_1 - \frac{1}{180} R_4$, $NR_2 \rightarrow R_2 - \frac{1}{60} R_4$

$NR_3 \rightarrow R_3 - \frac{1}{2} R_4$, $NR_4 = \frac{1}{120} R_4$, $NR_5 - \frac{1}{60} R_5$

| **BV** | **CB** | **$X_1$** | **$X_2$** | **$X_3$** | **$X_4$** | **$X_5$** | **$X_6$** | **$X_7$** | **$X_8$** | **$X_9$** | **$X_B$** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $X_5$ | 0 | 2/3 | 1/12 | 11/36 | 0 | 1 | 0 | 0 | -1/180 | 0 | 17/9 |
| $X_6$ | 0 | 2/3 | 0 | 23/12 | 0 | 0 | 1 | 0 | -1/60 | 0 | 26/3 |
| $X_7$ | 0 | 45 | 0 | 115/2 | 0 | 0 | 0 | 1 | -1/2 | 0 | 360 |
| $X_8$ | 14 | 5/4 | 1 | 25/24 | 1 | 0 | 0 | 0 | 0 | 1/12 | 5/3 |
| $X_9$ | 0 | 3/2 | 0 | 23/12 | 0 | 0 | 0 | 0 | -1/60 | 1 | 62/3 |
| | $Z_j$ | 35/2 | 14 | 175/2 | 14 | 0 | 0 | 0 | 0 | 7/60 | Z=70/3 |
| | $C_j$ | 6 | 9 | 12 | 14 | 0 | 0 | 0 | 0 | 0 | |
| $Z_j$- $C_j$ | | 23/2 | 5 | 31/12 | 0 | 0 | 0 | 0 | 0 | 0 | |

The solution is optimal because at the values in the row $Z_j$ - $C_j$ are positive (i.e. $Z_j - C_j \geq 0$).

The optimal table shows that optimal product mix is to produce 2 units of product D for a total profit of 70/3 = ₦23.33k. By performing a sensitivity analysis, it is possible to obtain valuable information regarding the alternative production schedules in the neighbourhood of the optimal solution. This information will be more significant on the product mix itself.

The sensitivity analysis of the current optimal solution can be best obtained by studying how the optimal table changes in the relative profit coefficient of non basic variable $X_1(Z_1 - C_1)$, $X_2(Z_2 - C_2)$, and $X_3(Z_3 - C_3)$ change in the optimal tableau. In the present optimal tableau.

$$CB \longrightarrow (x_5, x_6, x_7, x_8, x_9) = (0,0,0,14,0)$$

Hence $$Z_1 - C_1 = (0, 0, 0, 14, 0)\begin{pmatrix} 2/3 \\ 3/2 \\ 45 \\ 5/4 \\ 3/2 \end{pmatrix} - C_1 \geq 0$$

$$\Rightarrow \frac{70}{4} - C_j \geq 0, \frac{70}{4} \geq 0, C_1 \leq \frac{70}{4} C_1 \leq ₦17.5k$$

It means that as long as the unit profit of product A is less than ₦17.5k, it is not economical to produce product A.

Suppose the unit profit of product A is increased to ₦18.00k, then $Z_1$-$C_1$ = 0.5, the current optimal solution is no more optimal. Therefore, the maximization profit can be increased further by product $C_1$

| BV | CB | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ | $X_9$ | $X_B$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $X_5$ | 0 | 9/16 | 0 | 77/32 | -1/12 | 1 | 0 | 0 | -1/180 | -1/1440 | 17/9 |
| $X_6$ | 0 | 3/2 | 0 | 23/12 | 0 | 0 | 1 | 0 | -1/60 | 0 | 26/3 |
| $X_7$ | 0 | 45 | 0 | 115/2 | 0 | 0 | 0 | 1 | -1/2 | 0 | 360 |
| $X_8$ | 15 | 5/4 | 1 | 25/24 | 1 | 0 | 0 | 0 | 0 | 1/120 | 5/3 |
| $X_9$ | 0 | 3/2 | 0 | 23/12 | 0 | 0 | 0 | 0 | -1/60 | 1 | 62/3 |
| | $Z_j$ | 75/4 | 15 | 125/8 | 15 | 0 | 0 | 0 | 0 | 3/24 | 70/3 = 25 |
| | $C_j$ | 6 | 15 | 12 | 14 | 0 | 0 | 0 | 0 | 0 | |
| $Z_j$- $C_j$ | | 51/4 | 0 | 29/8 | 1 | 0 | 0 | 0 | 0 | 3/24 | |

Hence, the new optimal product mix is to produce (5/3), 2 unit of product B with a total profit of ₦25.

## 5. SUMMARY OF RESULTS CONCLUSION

The optimality stage, as long as product A is less than ₦17.5k. it is not economical to produce A. If $C_1$ is increase to ₦18.00k, the new optimal product is to produce (4/3) i.e. 1 of product A to make total profit of ₦24. Also, as long as the unit profit of product B is less than ₦4, it is not economical to produce product B. If $C_2$ is increase to ₦15, the new (optimal) produce mix is to produce 2 unit of product B with a total profit of ₦25.

It is clear that when $C_4$ decreases below a certain level, it may not be profitable to include product D in the optimal product. Assuming $C_4$ is reduced to 12 the new maximum profit is reduced to ₦20.

### 5.1 Conclusion

When the price of both the basic and non-basic products changed, the objective function also changes from maxZ = $6x_1 + 9x_2 + 12x_3 + 14x_4$: $7x_1 + 10x_2 + 11x_3 + 16x_4$. Hence, the total profit increases to ₦26.67k.

Suppose an additional one hour is added to the machine hours the new optimal product mix is $x_1 = x_2 = x_3 = 0$ and 1/5 and the total profit is ₦2.8k with this, it is not advisable to increase the machine hours because the profit will decrease. Also, an additional 1 unit of labour is made available and the available product mix still remain as $x_4 = 1/5$ while $x_1 = x_2 = x_3 = 0$ with a profit of ₦2.8k. Likewise, when an additional one bag of flour is added, the products mix change to $x_4$ 5/24 and maximum profit reduced to ₦2.92k. Hence, it is not advisable to increase the hour, labour and number of bags of flour used. It is profitable to add new activity.

### 5.2 Recommendations

In view of the analysis so far, the following recommendations are just forward:

i. Only product D is effective and profitable out of all the flour products schedule.

ii. The Bakery should embark on electrically heat regular i.e. electrical oven should be used for baking to improve the productivity of their product. Hence, this will require less labour, less time, at the same time maximizes the profit.

iii. The management of the Bakery should facilitate the production scheme through the use of linear programming problem to make optimal use of the activities and employ the services of an operation researcher in order to ensure sufficient and effective management.

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## BIOGRAPHY

Dr. Okeyemi Maroof Okeniyi is a Head of Department of Mathematics and Statistics, Federal Polytechnic, Offa, Kwara State, Nigeria. He holds B.Sc., M.Sc. in Statistics from the University of Ilorin, Nigeria in 1993 and 2001 respectively and Ph.D in Statistics from the Atlantic International University, Honolulu, 2016.