

Numerical study of gas effect at high temperature on the supersonic plug and expansion deflexion nozzles design

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Abstract - The aim of this work is to develop a new computational program to determine the effect of using the gas of propulsion of combustion chamber at high temperature on the shape of the two-dimensional Plug Nozzle and Expansion Deflexion Nozzle giving a uniform and parallel flow at the exit section. The selected gases are H₂, O₂, N₂, CO, CO₂, H₂O, NH₃, CH₄ and air. All design parameters depend on the stagnation temperature, the exit Mach number and the used gas. The specific heat at constant pressure varies with the temperature and the selected gas. The gas is still considered as perfect. It is calorically imperfect and thermally perfect below the threshold of dissociation of molecules. A error calculation between the parameters of different gases with air is done in this case for purposes of comparison. Endless forms of nozzles may be found based on the choice of T₀, M_E and the selected gas. For nozzles delivering same exit Mach number with the same stagnation temperature, we can choose the right gas for aerospace manufacturing rockets, missiles and supersonic aircraft.

Key Words: Supersonic flow, Two Dimensional Plug Nozzle, Expansion Deflexion Nozzle, High temperature, Calorically imperfect gas, Trust coefficient, Mass of the nozzle, Specific heat at constant pressure, Gas.

1. INTRODUCTION

Gases play an important role in aerospace propulsion including supersonic jets gear. The use of such gas of propulsion essentially affects the behavior of the supersonic flows, and in particular on all design parameters. The choice of a such gas to propel aerospace vehicles is done on the basis of the need in the design parameters and for construction specifications. For example, the nozzles used for supersonic propulsion of rockets, missiles and supersonic aircraft engines; it is desired to have short lengths and mass of the nozzles to have a reduced weight of the vehicle.

The model with constant C_p is typically used at low temperatures to illustrate the design parameters of various forms of supersonic nozzles [1-8].

In references [9-11], studies at HT on the supersonic 2D plug nozzle nozzles design only for air are presented.

Thermodynamic parameters of a supersonic flow at HT, with application for air in supersonic nozzle are presented in reference [12].

The treated gases are selected from the group of gases found in the literature, having different thermodynamic properties. We focus on the specific heat at constant pressure C_p(T) at HT and the constant R. This function C_p(T) is available depending on the temperature in several references [13-16].

The molecules of these gases are H₂, O₂, N₂, CO, CO₂ and H₂O, NH₃, CH₄ and air. The selected gases require that the temperature is within a specific interval for not having the molecules dissociation. This interval varies from one gas to another. The gas is considered as perfect. Including the equation of state (P=ρRT) is still valid, except it will be considered calorically imperfect and thermally perfect.

The aim of this work is to develop a new computational program to study the effect of using of propulsion gases at HT on the design and sizing of the 2D supersonic Plug Nozzle (PN) and Expansion Deflexion Nozzle (EDN) giving a uniform and parallel flow at the exit section. Then this study is to allow a suitable choice of gas in accordance with parameters such as the required coefficient C_F, Exit Mach number, choice of construction material and the stress applied on the wall. The selected substances are limited to 9 gases indicated by table 1 [13-16].

Table - 1: Coefficients of C_p(T) function and some constants thermodynamic for selected gases

N	Gas	a' J/(K mol)	b' J/(K ² mol)	c' J K/mol	R J/(kg K)
1	H ₂	27.28	3.26	0.50	4157.250
2	O ₂	29.96	4.18	-1.67	259.828
3	N ₂	28.58	3.76	-0.50	296.946
4	CO	28.41	4.10	-0.46	296.946
5	CO ₂	44.22	8.79	-8.62	188.965
6	H ₂ O	30.54	10.29	0.08	461.916
7	NH ₃	29.75	25.10	-1.55	489.088
8	CH ₄	23.64	47.86	-1.92	519.656
9	Air	Polynomial of 9 th degree [9, 12]			287.102

The following figure 1 shows the flow field in a 2D Plug Nozzle (figure 1a) and the Expansion Deflexion Nozzle (figure 1b). The *PN* has a central body, and the *EDN* has an external wall which must be determined to have a uniform flow and parallel to the exit section. The design method is an accurate method based on the Prandtl-Meyer expansion.

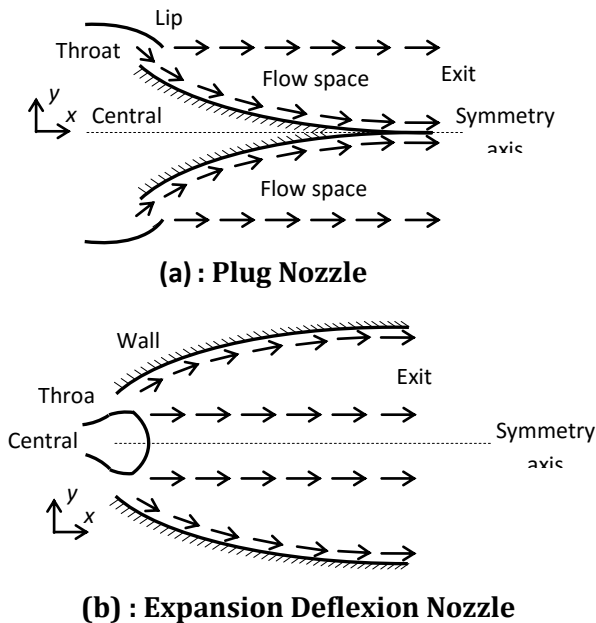


Fig - 1: Flow field in the 2D Plug and Expansion Deflexion Nozzle.

We find a change according to equation (1) of the specific heat at constant pressure. Constants of this function and the thermodynamic constants of gas are shown in the table 1 [13-16].

$$C_p(T) = a' + b'T + \frac{c'}{T^2} \quad (1)$$

Where $C_p(T)$ is the Specific heat temperature at constant pressure (J/kg K).

a', b', c' are the constants of the interpolation of the $C_p(T)$ function.

T is the temperature (K).

For air, the law of variation of $C_p(T)$ is chosen as a polynomial of 9th degree [9, 12].

2. DESIGN METHOD

The difference between the *Plug Nozzle* or *Expansion Deflexion Nozzle* presented in our work and the other models like the *MLN* nozzle [17] is that the flow at the throat is inclined by an angle θ^* with respect to the horizontal, which is not the case for the *MLN* nozzle where the flow is

horizontal at the throat [17]. Therefore, the Lip must be inclined at an angle Ψ with respect to the vertical.

In order to obtain the contour shape of the central body geometry, the stream lines determined by the calculation will be replaced by a rigid surface limiting the area of the flow and consequently the shape of the central body will be obtained. Mach waves divergent are issued from the corner of the throat.

The flow at the throat and the exit section is unidirectional. The critical sections ratio remains valid and is taken into consideration for comparing the numerical calculations found by our model and the theory. The calculation of the flow inside the nozzle is rather delicate, since the shape of the nozzle is unknown a priori. The desired shape of the central body accelerates the flow of Mach number $M=1.00$ at the throat to Mach number $M=M_E$ at the exit section. Since the deviation of the flow at the throat is not zero, the flow through the central body only recovers from the angle $\theta=\theta^*$ at the throat at the angle $\theta=0$ at the exit.

The flow calculation and the determination of the contour of the *HT* central body are based on the Prandtl-Meyer expansion by [18-20]:

$$v = \int_T^{T^*} F_v(T) dT \quad (2)$$

with:

$$F_v(T) = \frac{C_p(T)}{2H(T)} \sqrt{\frac{2H(T)}{a^2(T)} - 1} \quad (3)$$

The parameters appearing in relation (2) are presented in [12] and are given by:

$$H(T) = \int_T^{T_0} C_p(T) dT \quad (4)$$

$$M(T) = \frac{\sqrt{2H(T)}}{a(T)} \quad (5)$$

$$a^2(T) = \gamma(T) R T \quad (6)$$

$$\gamma(T) = \frac{C_p(T)}{C_p(T) - R} \quad (7)$$

Where v is the Prandtl Meyer function

M is the Mach number.

P is the Pressure (atm).

H is the Enthalpy (J/kg).

γ is the Specific heats ratio.

R is the thermodynamic constant of gas (J/(kg K)).

a is the Speed of sound (m/s).

T_0 is the Stagnation temperature (K).

T^* is the critical temperature (K).

The angle v is measured with respect to the velocity vector of the throat. In figure 2, the lines AB and AE are

To arrive at designing the nozzle, the discretization of the zone of variation of the temperature in N values including the values of the ends has been chosen and recommended so that the computation is fast.

3. CALCULATION PROCEDURE

For HT perfect gas, the analytical expressions of the calculation of the thermodynamic ratios T^*/T_0 , ρ^*/ρ_0 , and P^*/P_0 and T_E/T_0 , ρ_E/ρ_0 , and P_E/P_0 are presented in [12, 20].

The ratio of the critical sections is given by [12, 20]:

$$\frac{A_E}{A^*} = \text{Exp} \left(\int_{T_E}^{T^*} C_P(T) \left[\frac{1}{\alpha^2(T)} - \frac{1}{2H(T)} \right] dT \right) \quad (17)$$

Where T_E is the exit temperature (K).

A_E is the exit section area (m^2).

A^* is the critical section area (m^2).

The numerical procedure for calculating the integral (17) is illustrated in [12, 20-21]. This ratio will serve as a source of comparison for validation of the numerical calculations.

The value of the Prandtl Meyer function v_E can be calculated using equation (2) by replacing T by T_E . The algorithm for calculating the integral (2) is presented in [12, 20-21].

The deflection Ψ of the Lip with respect to the vertical will be calculated using the equation (9)

Since the calculation method is based on the two successive points, the results must be given at the starting point. The starting point is the point B . We have $M_B=1.00$, $\mu_B=\pi/2$, $v_B=0.0$, $\varphi_B=\pi/2-\Psi-v_B+\mu_B$, $\lambda_B=1.00$.

The position of the first point (point B) of the wall is given by:

$$\frac{x_B}{\lambda_B} = \cos(\varphi_B) \quad (18)$$

$$\frac{y_B}{\lambda_B} = \sin(\varphi_B) \quad (19)$$

Where x_B, y_B is the coordinate of the point B of the throat (m). φ_B is the polar angle at point B of the throat.

The angle of deflection of the flow θ^* at the throat is given by:

$$\theta^* = \theta_B = \varphi_B - \mu_B \quad (20)$$

The second step consists in assigning the results obtained at point B as the first numerical calculation point for $i=1$. Here the temperature at point B is equal to T^* .

For each Mach line, it is necessary to know the temperature at the expansion center A which also represents the temperature on the wall.

Since the number of points chosen is equal to N , then N -panels are obtained, hence the temperature at point i is given by:

$$T_i = T^* - \frac{i-1}{N-1} (T^* - T_E) \quad i=1, 2, \dots, N \quad (21)$$

Where N is the number of the Mach wave issued from the point B .

T_i is the temperature at the point i of the central body.

By incrementing the counter from $i=2$ to N , one can determine the thermodynamic and the physical properties along all the Mach lines selected at the outset and hence the shape of the central body will be obtained.

From the results of the last point when $i=N$, one can fix the following results:

The position of the point E of the exit section is given in non-dimensional form by:

$$\frac{x_E}{\lambda_B} = \frac{x_N}{\lambda_B} \quad (22)$$

$$\frac{y_E}{\lambda_B} = \frac{y_N}{\lambda_B} \quad (23)$$

Where x_E, y_E is the coordinate of the point E of the exit (m).

The computed total length of the central body and the critical sections ratio are given, in non-dimensional form, by:

$$\frac{L}{\lambda_B} = \frac{x_E}{\lambda_B} - \frac{x_B}{\lambda_B} \quad (24)$$

$$\frac{A_E}{A^*} (\text{Computed}) = \frac{y_E}{\lambda_B} - \frac{y_N}{\lambda_B} \quad (25)$$

Where L is the length of the central body of the PN and EDN Nozzle.

4. MASS OF THE PLUG AND EXPANSION DEFLEXION NOZZLE

The calculation of the mass of the structure is linked with the calculation of the curvilinear length of the wall of the central body. Then, by unit of depth and in non-dimensional form, one obtains:

$$C_{Mass} = \sum_{i=1}^{i=N-1} \sqrt{\left(\frac{x_{i+1} - x_i}{\lambda_B} \right)^2 + \left(\frac{y_{i+1} - y_i}{\lambda_B} \right)^2} \quad (26)$$

Where C_{Mass} is the adimensional mass of the central body.

5. THRUST OF THE PLUG AND EXPANSION DEFLEXION NOZZLE

The total axial pressure force exerted on the central body per unit of depth is calculated as the sum of all the axial pressure forces exerted on all the sections. The central body consists of two parts by reason of symmetry. In non-dimensional form, we obtain:

$$C_F = \sum_{i=1}^{i=N-1} \left(\frac{P}{P_0} \right)_{(i)} \left[\frac{y_{i+1}}{\lambda_B} - \frac{y_i}{\lambda_B} \right] \quad (27)$$

Where C_F is the thrust coefficient.

By varying the thermodynamic properties of the different gases used by considering the specific heat $C_p(T)$ according to the table 1, the central body shape of the Plug Nozzle or the external wall of the Expansion Deflexion Nozzle and the corresponding design parameters can be determined.

6. RESULTS AND COMMENTS

Figures 4, 5, 6 and 7 illustrate the effect of the propellant gas on the shape of the 2D Plug and Expansion Deflexion nozzle, giving M_E at the exit section, respectively for $M_E=2.00$, 3.00, 4.00 and 5.00 when $T_0=2000$ K. The numerical design parameters are represented respectively in tables 2, 3, 4 and 5.

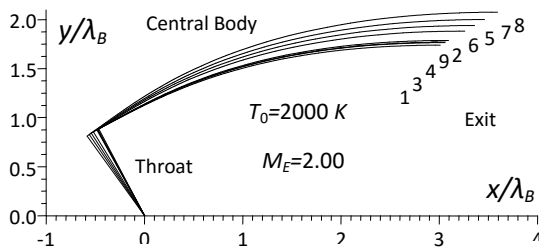


Fig - 4: Effect of gas at HT on the 2D PN central body and external EDN wall shape giving $M_E=2.00$.

Table - 2: Numerical design values for figure 4.

N	Gas	θ^* (deg)	L/λ_B	C_{Mass}	C_F	y_E/λ_B	x_{Plug}/λ_B
1	H ₂	27.800	3.477	3.626	0.313	1.737	3.011
2	O ₂	29.052	3.579	3.745	0.338	1.785	3.094
3	N ₂	28.472	3.531	3.689	0.326	1.763	3.055
4	CO	28.600	3.542	3.701	0.329	1.768	3.063
5	CO ₂	32.766	3.898	4.121	0.419	1.937	3.357
6	H ₂ O	31.437	3.780	3.981	0.388	1.880	3.259
7	NH ₃	34.158	4.023	4.269	0.450	1.997	3.462
8	CH ₄	35.806	4.177	4.454	0.490	2.073	3.592
9	Air	28.646	3.547	3.708	0.330	1.770	3.068

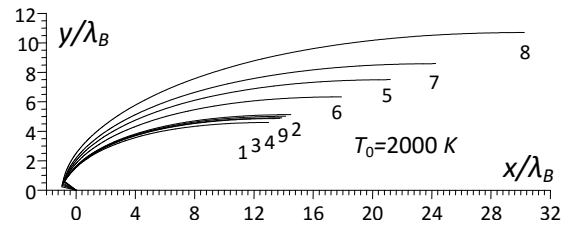


Fig - 5: Effect of gas at HT on the 2D PN central body and external EDN wall shape giving $M_E=3.00$.

Table - 2: Numerical design values for figure 5.

N	Gas	θ^* (deg)	L/λ_B	C_{Mass}	C_F	y_E/λ_B	x_{Plug}/λ_B
1	H ₂	52.457	13.775	14.813	0.815	4.582	12.982
2	O ₂	55.770	15.313	16.554	0.905	5.113	14.487
3	N ₂	54.203	14.561	15.703	0.862	4.853	13.750
4	CO	54.478	14.681	15.839	0.869	4.895	13.867
5	CO ₂	66.883	22.113	24.279	1.233	7.476	21.194
6	H ₂ O	62.155	18.793	20.506	1.087	6.319	17.908
7	NH ₃	70.880	25.207	27.808	1.356	8.556	24.262
8	CH ₄	77.006	31.216	34.655	1.555	10.661	30.241
9	Air	54.989	14.972	16.167	0.884	4.995	14.153

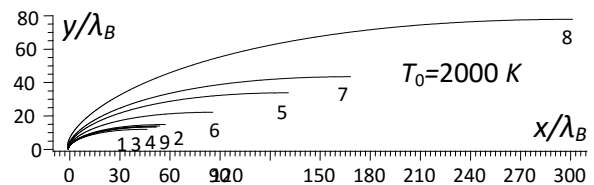


Fig - 6: Effect of gas at HT on the 2D PN central body and external EDN wall shape giving $M_E=4.00$.

Table - 3: Numerical design values for figure 6.

N	Gas	θ^* (deg)	L/λ_B	C_{Mass}	C_F	y_E/λ_B	x_{Plug}/λ_B
1	H ₂	68.947	47.212	50.271	1.218	11.907	46.278
2	O ₂	74.019	58.170	62.101	1.368	14.714	57.209
3	N ₂	71.586	52.580	56.067	1.296	13.282	51.631
4	CO	71.934	53.237	56.779	1.306	13.450	52.286
5	CO ₂	92.772	131.862	141.548	1.956	33.596	130.863
6	H ₂ O	83.702	86.661	92.865	1.665	22.016	85.667
7	NH ₃	98.765	169.092	181.696	2.147	43.133	168.104
8	CH ₄	111.051	302.085	324.718	2.543	77.147	301.152
9	Air	72.624	54.840	58.513	1.327	13.861	53.886

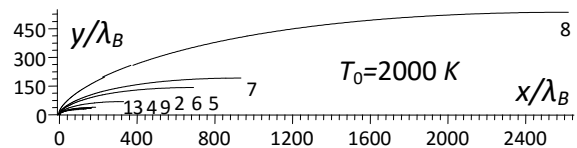


Fig - 7: Effect of gas at HT on the 2D PN central body and the and external EDN wall shape giving $M_E=5.00$.

Table - 4: Numerical design values for figure 7.

N	Gas	θ^* (deg)	L/λ_B	C_{Mass}	C_F	y_E/λ_B	x_{Plug}/λ_B
1	H ₂	80.120	138.216	144.841	1.505	27.842	137.231
2	O ₂	86.419	187.275	196.399	1.694	37.766	186.277
3	N ₂	83.371	161.394	169.205	1.602	32.531	160.401
4	CO	83.739	163.688	171.622	1.613	32.996	162.694
5	CO ₂	111.487	691.605	725.237	2.462	139.516	690.675
6	H ₂ O	98.321	331.298	347.666	2.058	66.878	330.309
7	NH ₃	118.165	934.287	979.783	2.662	188.424	933.405
8	CH ₄	136.310	2620.78	2744.66	3.174	526.941	2620.09
9	Air	84.262	166.491	174.598	1.630	33.565	165.496

It is clear that the propellant gas effect on the nozzle shape, and therefore on all design parameters. For example CH₄ gas provides a large length and mass as well as the very high C_F . While the H₂ gas, N₂, O₂ and CO give small parameters. For supersonic nozzles construction applied to missiles and aircraft is recommended for use for example the H₂ gas, N₂ or CO instead of CH₄, NH₃ even the air. The influence of M_E and T_0 is noted on the form and the parameters. H₂, N₂, O₂ and CO gases gives better performance compared to the air for applications in aerospace manufacturing. The shapes of the nozzles in the case of air (curve 9) can be found in reference [9].

Figures 8, 9, 10, 11, 12 and 13 show the effect of gas on the variation of various design parameters as a function of the exit Mach number M_E of the 2D PN, for $T_0=2000$ K. At low M_E until about 2.00, it is not influenced by a choice remark such gas on the design parameters. While most M_E becomes big and starts to exceed 2.00 about the choice of such a gas is essential for propulsion.

In figures 9, 10, 12 and 13, the representation is preferred in the form of the logarithmic scale, given the large values of these parameters accompanied by small values for certain gases.

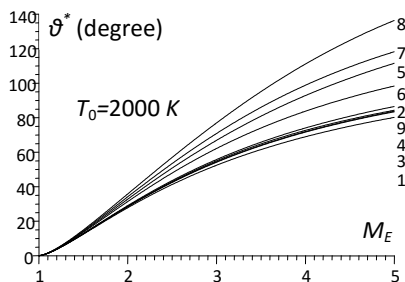


Fig - 8: Effect of gas at HT on θ^* of the throat for 2D PN central body and the external EDN wall.

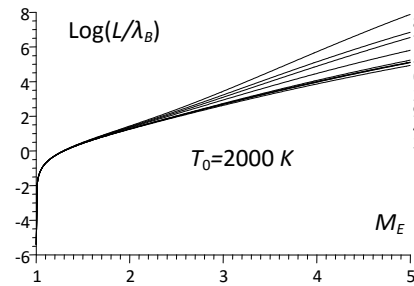


Fig - 9: Effect of gas at HT on the length L/λ_B for 2D PN central body and the external EDN wall.

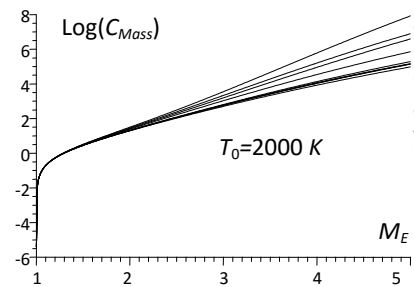


Fig - 10: Effect of gas at HT on C_{Mass} of the 2D PN central body and the external EDN wall.

Figures 14, 15, 16, 17 and 18 represent the variation of the thermodynamic parameters and the Mach number through the wall of the central body of the Plug Nozzle and the external wall of the Expansion Deflexion Nozzle of figure 5 for $T_0=2000$ K and $M_E=3.00$. It is noted an expansion of gas from $M=1.00$ at the throat to the exit for $M=M_E$. In parallel there is a decrease of the deviation of the flow from $\theta=\theta^*$ at the throat to $\theta=0.0$ at the exit according to the figure 15. It also shows that the flow is horizontal to the exit section.

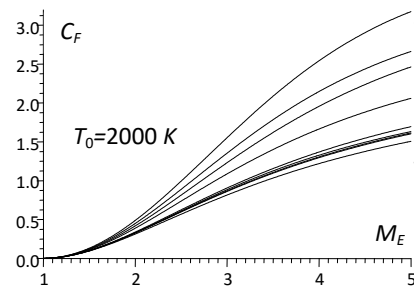


Fig - 11: Effect of gas at HT on the C_F of 2D PN central body and the external EDN wall.

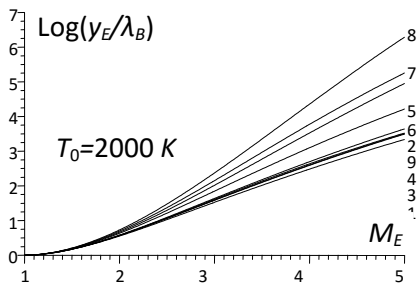


Fig - 12: Effect of gas at HT on y_e/λ_B of the 2D PN central body and the external EDN wall.

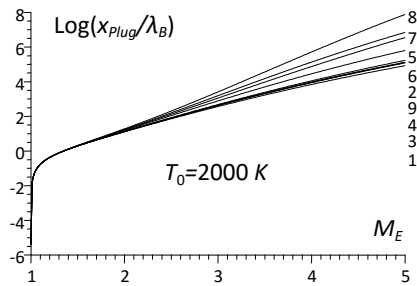


Fig - 13: Effect of gas at HT on x_{Plug}/λ_B of the 2D PN central body and the external EDN wall.

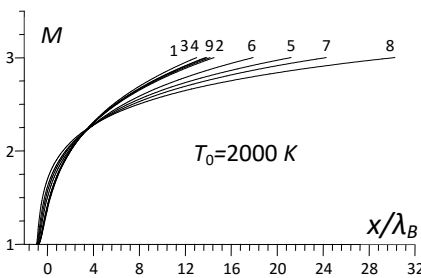


Fig - 14: Effect of gas at HT on the variation of the Mach number through the 2D PN central body and the external EDN wall.

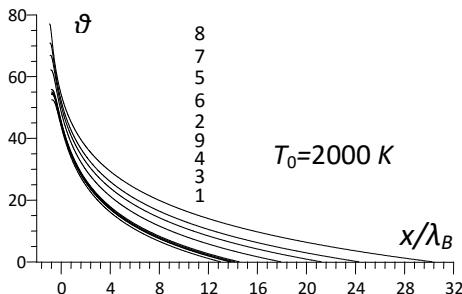


Fig - 15: Effect of gas at HT on the variation of θ through the 2D PN central body and the external EDN wall.

In figure 16 we see that the temperature through the wall is high enough for the CH_4 gas, NH_3 , CO_2 and H_2O relative to

the air. While for the H_2 gas, N_2 , and CO is cold enough with respect to air. So we must chose a suitable building material resistant to the distribution shown in figure 17 according to the selected gas.

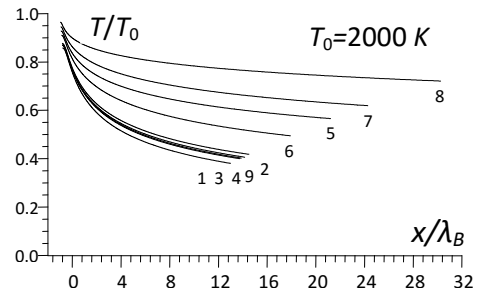


Fig - 16: Effect of gas at HT on the variation of T/T_0 through the 2D PN central body and the external EDN wall.

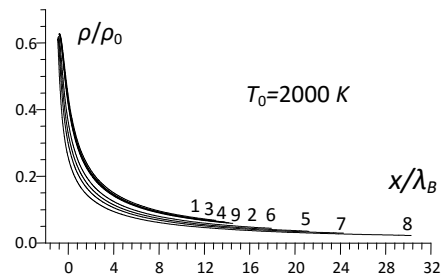


Fig - 17: Effect of gas at HT on the variation of ρ/ρ_0 through the 2D PN central body and the external EDN wall.

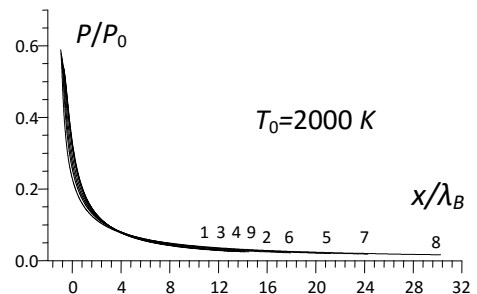


Fig - 18: Effect of gas at HT on the variation of P/P_0 through the 2D PN central body and the external EDN wall.

7. CONCLUSIONS

This work allows us to study the effect of using gas of propulsion on the design parameters of the supersonic two-dimensional plug and Expansion Deflexion Nozzle at HT. We can draw the following conclusions

The computer program can do any gas found in the nature. It should be added in this case, the $C_p(T)$ and the gas constant R and calculate the corresponding $H(T)$.

The $C_p(T)$ function, R and $\gamma(T)$ characterize the calorific value of the gas and mainly affects all design parameters.

For applications of missiles and supersonic aircraft, it is recommended to use propellant gas having the smallest ratio γ to have a small mass and large C_f . Among the selected gas, CH_4 is a bad choice and the H_2 is a good choice.

The stagnation temperature and the exit Mach number also affect the shape of the nozzle and all design parameters.

If we keep the given form of the nozzle for the air, the use of another gas instead of air will lose the condition of uniform and parallel flow at the exit section. Among the selected gas, the use of H_2 , N_2 , CO by increasing the exit Mach number and the use of gas CH_4 , NH_3 , CO_2 and H_2O will degrade the exit Mach number.

The convergence of results is controlled by the convergence of the ratio of critical sections, calculated numerically by the relationship (23), to that given by equation (17). All others design parameters also converge.

The stagnation temperature and the exit Mach number also affect the shape of the nozzle and all design parameters.

Endless shaped of Plug and Expansion Deflexion Nozzles can be found by varying three parameters which are M_E , T_0 and the gas itself ($C_p(T)$, R).

The propellant gas affects all the thermodynamic and the design parameters of the flow.

The truncation of the nozzle is sometimes useful to gain a large portion of the mass of the nozzle and in parallel we will see a small loss in C_f and M_E .

The design method and the results for design parameters is the same for the Plug and Expansion Deflexion Nozzles.

As prospects, we can study the effect of gas of propulsion at HT on the design and sizing of the axisymmetric PN and EDN at HT by the method of characteristics.

ACKNOWLEDGEMENT

The authors acknowledges Khaoula, AbdelGhani Amine, Ritadj and Assil Zebbiche and Mouza Ouahiba for granting time to prepare this manuscript.

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