De-Noising with Spline Wavelets and SWT

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Abstract - *This paper explores the difference in performance of spline wavelets of the bi-orthogonal type in denoising images corrupted by Additive White Gaussian Noise. The dependence of the peak signal-to-noise ratio and the mean squared error on the filter characteristics of the wavelets, when stationary wavelet transform is used in the de-noising process, is investigated. It is found that the de-noising action augments with use of wavelet of lower effective length for its high pass reconstruction filter. For wavelets with equal effective lengths for their high pass reconstruction filters, a relation similar to the above exists for the high pass decomposition filters. 'Bior1.1'(bi-orthogonal spline wavelet 1.1) is found to be the most suitable wavelet in the family, for de-noising. 'Bior 3.1' is found to be an odd member in the family and is not at all suitable for de-noising, the reason for which is traced to the lack of smoothness of its decomposition scaling function*

Keywords - Spline wavelet, stationary wavelet transform, bi-orthogonal wavelets, DWT, thresholding.

1. INTRODUCTION

1.1 Image Denoising

The digital image processing is concerned with image de-noising. Noise is defined as the transmission medium and error occur during measurement and quantization process of the data for digital storage. This including algorithm and routine goal oriented image Processing. The reduction of degraded images that are Incurred image is being obtained by the image restoration. Degradation comes from blurring as Well as noise due to various sources. Blurring is a form of bandwidth reduction in the image caused by the imperfect image formation process like relative motion between the camera & the object or by an optical system which is out of the focus. For remote sensing purposes when aerial photographs taken atmospheric turbulence introduces blurs, optical system aberration and relative motion between camera and the ground with these blurring effects. The noise can also corrupted by recorded image. Each element in the imaging chain such as film, lenses, digitizer, etc. contribute to the degradation. The field of photography or publishing is used for the purpose of image denoising. In that an image is somehow degraded but it required improving before it can be printed. Such kind of application we need to know about the degradation process in order to design a model for it.

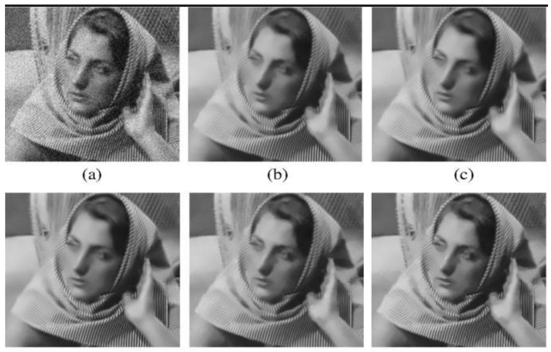

Figure – 1: shows the images with noises

2. LITERATURE SURVEY

The bilateral filtering is applied to the approximation sub-bands. Unlike The difference in performance of spline wavelet of the bi-orthogonal type in denoising images corrupted by Additive White Gaussian noise. The dependence of the peak single to noise ratio and mean squared error of the filter of the wavelet. This stationary wavelet transform is used in the denoising process are investigated. For wavelet with equal effective length for their high pass reconstruction filter similar to the high pass decomposition filter 'Bior1.1' (bi-orthogonal spline wavelet 1.1) is the most suitable wavelet in the family, for de-noising. 'Bior 3.1' is an odd member in the family and is not at all suitable for de-noising, the reason for which is traced to the lack of smoothness of its decomposition scaling function. Image noise reduction or denoising is an active area of research although many of the techniques. The literature mainly target additive white noise. The standard single-level bilateral filtering, this multi resolution bilateral filtering has the potential of eliminating low-frequency noise components. (This will become evident in our experiments with real data.) The approximation sub-bands in addition works in Bilateral filtering it is possible to apply wavelet thresholding to the detail sub-bands, where some noise components can be introduced and removed effectively. The new image denoising framework combines bilateral filtering and wavelet thresholding. Chang and Vetterli [8] proposed an adaptive, data-driven threshold for image denoising using the wavelet soft-thresholding. The application of image processing used in the threshold is derived in a Bayesian framework, and the prior used on the wavelet coefficients is the generalized Gaussian distribution (GGD). The proposed threshold is closed-form and adaptive to each sub-band. This method, so called Bayes Shrink [8], outperforms Donoho and Johnstone‟s Sure Shrink [7] most of the time. Since wavelet coefficients of real images have significant dependencies, Sendur et al. [9] considered the dependencies between the coefficients and their parents in the detail coefficients part. The purpose of the non-Gaussian bivariate distributions is proposed, and corresponding nonlinear threshold functions are derived from the models using Bayesian estimation theory. The new shrinkage functions do not assume the independence of wavelet coefficients. However, the performance of this method is not very well.

3. MATERIALS AND METHODS

3.1. Bi-orthogonal wavelets

The meaning of 'bi-orthogonal' is two functions or 'bases' which are mutually orthogonal to each other , but each of these two functions need not form an orthogonal set. Two different scaling functions and two different wavelet functions are used for bi-orthogonal wavelets. The decomposition step scaling and wavelet functions (Φ and Ψ) are used and in the reconstruction step the other set (Ψ and Ψˆ) is used. This provides interesting features are not possible by using one and same filters for decomposition and reconstruction in the orthogonal case. Also, filter banks comprising bi- orthogonal filters are more flexible and can be designed easily. The linear phase which is good for reconstruction of images has Bi-orthogonal wavelets. The bi-orthogonal spline wavelets listed as: 'bior 1.1', 'bior 1.3', 'bior 1.5', 'bior 2.2', 'bior 2.4', 'bior 2.6', 'bior 2.8', 'bior 3.1', 'bior 3.3', 'bior 3.5', 'bior 3.7', 'bior 3.9', 'bior 4.4', 'bior 5.5' and 'bior 6.8'

3.2. Stationary Wavelet Transform

The Stationary wavelet transform (SWT) is similar to the DWT. Signal is never sub-sampled and instead the filters are up sampled at each level of decomposition. The stationary wavelet transform (SWT) is a wavelet transform algorithm designed to overcome the lack of translation-invariance of the discrete wavelet transform (DWT). Translation-invariance is achieved by removing the down samplers and up samplers in the DWT and The up sampling the filter coefficients by a factor of in the level of algorithm. The SWT is an inherently redundant scheme as the output of each level of SWT contains the same number of samples as the input – so for a decomposition of N levels there is a redundancy of N in the wavelet coefficients.

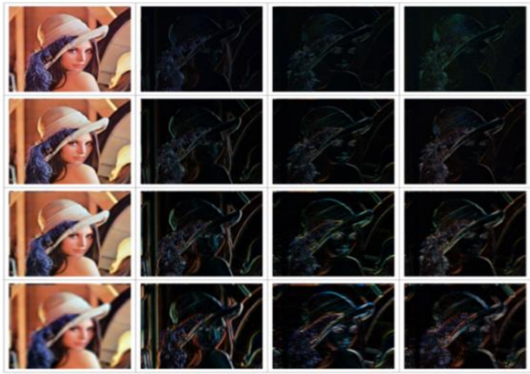

Figure- 2: Haar Stationary Wavelet Transform of Lena

3.3 Thresholding :

Thresholding is a simple non-linear technique, these operates on one wavelet coefficient at a time. In its most basic form of each coefficient which is smaller than threshold set to zero. The small co-efficient are dominated by noise, while coefficient with large absolute value carry more signal information than noise. Replacing noise co-efficient (small coefficients below a certain threshold value) by zero and an inverse wavelet transform. This thresholding idea is based on the following:

1) The de-correlating property of wavelet transform creates a sparse signal. Most untouched coefficient is zero or close to zero.
2) Noise is spread out equally along all co-efficient.
3) The noise level is not too high so that one can dis-tinguish the signal wavelet coefficients from binary ones.
This method is an effective and thresholding is simple and efficient method for noise reduction

4. RESULTS AND DISCUSSION

The noisy image is shown in Fig.1, Fig. 2 and Fig.3 The de-noised images with the obtained maximum and minimum values of PSNR, respectively. The MSE and PSNR corresponding to de-noising with the different bi-orthogonal wavelets are shown in Table1.The decomposition process using SWT involves convolution of the image matrix with a low pass filter and a High pass filter. These filters are LoD and HiD and the values of their effective lengths are in Table1. Similarly the reconstruction process involves convolution of the image matrix with another set of filters containing a low pass filter and a high pass filter indicated as LoR and HiR. The values of the effective lengths of these filters are given in Table 1. Usually the high frequency components in an image comprise the noise in the image.

Figure-3: Noisy image

Figure-4: Image de-noised with 'bior 1.1'

Figure -5: Image de-noised with 'bior 3.1'

Therefore it is reasonable for us to examine the features of HiD and HiR to relate the same to the variations in the denoising performance of the different wavelets used for the study. The output of low pass filter contains approximation of the image. It is observed that the estimated values of the PSNR (and MSE) vary with the different wavelets used in the SWT for the de-noising

PSNR, MSE and effective filter lengths of the wavelets						
Wavelet	MSE	PSNR dB	Effective length of filters			
			LoD	HiD	LoR	HiR
bior 1.1	12.4334	37.1849	2	2	2	2
bior 1.3	13.0208	36.9844	6	2	6	2
bior 1.5	13.2655	36.9036	10	2	2	10
bior 2.2	13.0943	36.9600	5	2	2	5
bior 2.4	13.2460	36.9100	9	3	3	9
bior 2.6	13.3893	36.8632	17	3	3	17
bior 2.8	13.5105	36.8241	4	4	4	4
bior 3.1	22.9163	34.5294	8	4	4	8
bior 3.3	13.6426	36.7818	12	4	4	12
bior 3.5	13.4364	36.8480	16	4	4	16
bior 3.7	13.5213	36.8206	20	4	4	20
bior 4.4	13.4691	36.8374	9	7	7	9
bior 5.5	13.6570	36.7773	9	11	11	9
bior 6.8	13.8360	36.7207	17	11	11	17

Table-1: Variation on the PSNR value

Process. From Table 1 it can be seen that the variations in the PSNR values have some amount of relationship with the effective lengths of HiR and HiD. A detailed inspection of the corresponding values leads to the following observations:

1. The PSNR decreases with increase in the effective Length of HiR. This fact is observed to be true in all the denoising cases under consideration, except in the cases Of denoising with 'bior 3.1', 'bior 3.3' and 'bior 3.5'. 'Bior 3.1' gives the lowest PSNR (34.5294) even though this wavelet has a low value 4 for effective length of HiR. In fact, 'bior 3.1' has the second lowest effective length of HiR when we consider the corresponding values of all the other members in the bi-orthogonal spline wavelet family. Thus 'bior 3.1' is found to have an odd behavior, the reasons for which shall be explored later.

Hence the following discussion skips 'bior 3.1', for the time being. As we move from 'bior 2.8' to 'bior 3.3', the PSNR decreases even though the effective length of HiR has decreased. The reason for this is an increase in the actual values of HiR represented by the increased value of HiRmax (maximum value of HiR) given in Table 2. Due to this increase in values of HiR, high amplitude coefficients containing noise are retained. An effect just opposite to this is observed in the case of 'bior 3.5'. 'Bior 1.1' gives the maximum value of PSNR which is 37.1849. Also, the effective length of HiR has the lowest value for 'bior 1.1'. This fact agrees with our above observation regarding relation between PSNR and effective length of HiR. Large effective length of HiR means large number of nonzero filter points in the filter. Since this high pass Filter with the large number of non-zero coefficients is convolved with the coefficients resulting from decomposition of the noisy digital image which have subsequently been thresholded such a convolution gives rise to high frequency components spread over a large extent and carries the noise components that have not been removed in the thresholding process. This explains the reduction in PSNR with increase in effective length of HiR.

2. When the effective lengths of HiR for two different Wavelets are equal, the PSNR is found to decrease with Increase in the effective length of HiD. This is evident by Observing the PSNR values of the set of wavelets comprising 'biro 2.4', 'biro 4.4' and 'biro 5.5', each of which has an effective length 9 for HiR. The PSNR values obtained on denoising with these wavelets decrease regularly as the effective lengths of HiD increase. This is shown separately in Table 3 for easy reference. An identical effect is noticed on observing the de-noising performance of 'bior 2.8' and 'bior 6.8'. Both of these wavelets have effective length 17 for HiR. The PSNR is found to have decreased as the effective length of HiD has increased

3. The influence of effective length of HiD on de-noising Performance is considerably less than that of HiR. This can be established in the following way. We have already established above that (i) the PSNR decreases with increase in the effective length of HiR and that (ii) when the effective lengths of HiR of 2 bi-orthogonal spline wavelets are equal, the PSNR decreases with increase in the effective length of HiD. As we move from 'bior 1.5' to 'bior 2.2' the effective length of HiD increases from 2 to 3, effective length of HiR

Table- 2: Maximum values of HiR for 'bior 2.8','bior 3.3' and 'bior3.5'

Wavelet	HiRmax
bior 2.8	0.4626
bior 3.3	0.9944
bior 3.5	0.9667

Table-3: PSNR and effective lengths of HiD for wavelet

PSNR and effective lengths of HiD for wavelets with HiR of effective length 9.		
Wavelet	Effective length of HiD	PSNR dB
bior 2.4	3	36.9100
bior 4.4	7	36.8374
bior 5.5	11	36.7773

Decreases from 10 to 5 and PSNR increases. Also when we move from 'bior 3.9' to 'bior 4.4', the effective length of HiD undergoes an increase from 4 to 7; at the same time, effective length of HiR decreases from 20 to 9 and the PSNR value increases. Here the effective length of HiD has increased by 1 point in the former case and by 3 points in the latter case. On the other hand, the effective lengths of HiR in these cases have had decrease and that by considerably larger numbers of points. In both the instances the PSNRs have only increased; the increase in PSNR accompanies the decrease in effective length of HiR. In this context it may be noted that the aforesaid increases in effective lengths of HiD have had nonoticeable effect on the PSNR. This establishes that effective length of HiD has considerably lesser influence on de-noising performance, compared to effective length of HiR. The apparent dominance of the dependence of effective length of HiR on PSNR, compared to that of HiD, is consequent of the larger value of effective length of HiR compared to that of HiD, or in other words, due to the larger numbers of non-zero filter points of HiR when compared to those of HiD; it can be seen that in most cases, the effective length of HiR is 2 to 4 times that of HiD. Now, we may investigate the reason for the odd behavior of 'bior 3.1'. The decomposition scaling function of 'bior 3.1' is shown in Figure 4. As what can be seen from Figure 4, this function is not at all a smooth one. It is scaling function bases that generate the wavelet basis functions [2]. Hence the decomposition wavelet function of 'bior 3.1' is also not smooth. The basic two-scale relation in MRA is:

$$\Phi\ (t) = \Sigma_k\ p\ (k)\ \Phi\ (2t - k),\ k\varepsilon\ Z,$$

Where Φ (t) is the scaling function and p (k) is the discrete

Sequence of coefficients resulting from the decomposition.

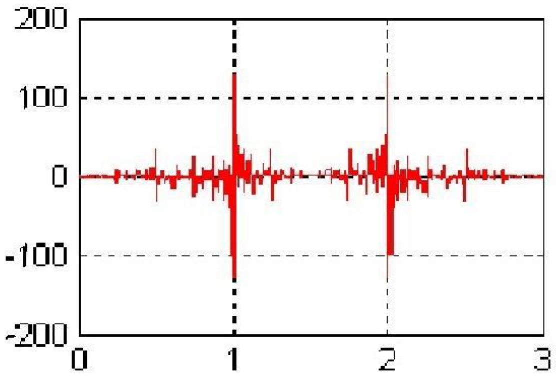

Figure - 6: Decomposition scaling function of 'bior 3.1'

Table -4: Variance of the effective lengths of LoD

Variances of the effective lengths of LoD	
Wavelet	Var (LoD)
bior1.1	0
Bior1.3	0.1396
Bior1.5	0.0956
Bior2.2	0.2208
Bior2.4	0.1350
Bior2.6	0.0983
Bior2.8	0.0777
Bior3.1	0.6667
Bior3.3	0.2662
Bior3.5	0.1696
Bior3.7	0.1255
Bior3.9	0.1001
Bior4.4	0.0934
Bior5.5	0.0567
Bior6.8	0.0565

This equation indicates that the scaling function at a particular resolution can be decomposed in to a linear combination of scaling functions at the next higher resolution [2]. The discrete sequence p (k) of the coefficients resulting from the decomposition constitutes the low pass filter in the wavelet decomposition. Since the decomposition scaling function is not smooth, its regularity is poor and the decomposition low pass filter has a high variance. This is also evident from Table 4 Which shows the variances of LoD (Var (LoD)) of the different wavelets. It can be seen that 'bior 3.1' has the highest value for "variance" or "dispersion" of LoD. This explains the reason for the odd behavior and the poor de-noising performance of 'bior 3.1'. Also the visual quality of the denoised images is found to have changes following the changes in the PSNR values

5. CONCLUSIONS

This paper explores de-noising performance of the different bi-orthogonal spline wavelets, when SWT is used as the transform for the de-noising operation. The denoising action is found to improve with the use of bi-orthogonal wavelet of lower effective length for its high pass reconstruction filter. When the effective lengths of high pass reconstruction filter for any two bi-orthogonal spline wavelets are equal, the PSNR decreases with increase in the effective length of high pass decomposition filter. The influence of effective length of high pass decomposition filter on denoising performance is considerably less than that of high pass reconstruction filter; this is due to the fact that the latter has larger number of non-zero filter points than the former. The maximum value of PSNR is obtained by de-noising with the bi-orthogonal spline wavelet with the minimum effective reconstruction filter length which is 'bior 1.1'. 'Bior 1.1' is hence the most suitable bi-orthogonal spline wavelet for de-noising images corrupted by AWGN. 'Bior 3.1' is found to be an odd member in the bi-orthogonal spline wavelet family. This wavelet gives the lowest PSNR. Therefore 'bior 3.1' is not at all suitable for de-noising. The odd behavior and the worst de-noising performance of 'bior3.1' are traced to be consequent of the lack of smoothness of its decomposition scaling function. It is also found that the visual quality of the images resulting from de-noising using the different bi-orthogonal spline wavelets follow the changes in the PSNR values obtained.

REFERENCES

[1] I.W. Selesnick, R.G. Baraniuk, N.G. Kingsbury, "The dual-tree complex wavelet transform". IEEE Sigl. Proc. Mag. 2005; 22: 123-151.

[2] Y. Sheng, The Transforms and Applications Handbook, 2 nd ed., Boca Raton, FL, USA, CRC Press, 2000.

[3] S.G.Mallat, "A theory for multi-resolution signal decomposition: the wavelet representation", IEEE Trans. Patt. An. & Mac. Int., 1989; 11: 674 — 693.

[4] I. Daubechies, Ten Lectures on Wavelets. Philadelphia, PA 19104 — 2688 USA; SIAM, 1992.

[5] I. Daubechies, "Orthonormal bases of compactly supported wavelets", J Com. Pure App. Math., 1988, XLI: 909 — 996.

[6] G. Strang, "Wavelets", Am. Sci. J, 1994, 82: 250 — 255.

[7] M. Vetterli, C. Herley, "Wavelets and filter banks: theory and design", IEEE Trans. SigL. Proc. 1992, 40: 2207－2232.

[8] Krishnakumar, Basantkumar, R.Shah, "Analysis of efficient waveletbased volumetric image compression", Int. J Im. Proc., 2012, 6: 113－122.

[9] M. Chui, Y Feng, W.Wang, Z. Li, X. Xu, "Image de-noising method with adaptive bayes threshold in no subsampled contourlet domain",2012 AASRI Conf.on Comp. Intelligence and Bioinformatics, 1－2 July 2012; Changsha, China: ELSEVIER. pp. 512－518..

[10] S.D. Ruikar, D.D. Doye, "Wavelet based image de-noising technique",Int. J Adv. Comp., Sc. & App., 2011, 2: 49－53.