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Static and Transient Vibrational Analysis of Functionally Graded Material

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Abstract - *In the present work, Static and Transient vibrational analysis of Functionally Graded Materials under mechanical load are investigated in thermal environment. Functionally Graded Material (FGM) is a class of composite material, usually mixture of Ceramic and Metals. The material properties are assumed to vary continuously through their thickness according to a Power-law distribution. Finite element method is used to study the Static, Free Vibration and Transient vibrational analyses. The numerical results on the transverse deflection in a moderately thick functionally graded plate under uniformly distributed load for various boundary conditions are discussed. The effects of Power-law index for Al/ZrO$_2$ Functionally Graded plates on deflection are commented. Different geometrical shapes of Al/ZrO$_2$ Functionally Graded shells and panels are included to study the free vibration; the effect of volume fractions on their frequency characteristics is discussed. Isotropic plate and Si$_3$N$_4$/SUS304 Functionally Graded square plates are included in the study of transient vibration analysis in thermal environment and the effect of Power-law index is discussed in detail.*

Key Words: Functionally Graded Material, FGM, Transient Vibrational Analysis, FGM Plates, FGM Shells and Panels, Al/ZrO$_2$ FGM, Si$_3$N$_4$/SUS304 FGM

1. INTRODUCTION

Strength of the metal is reduced after it has been in high temperature environment for a period of time; beside metal has a low melting point. Other side, ceramic materials have excellent characteristics in strength and heat-resistance. Due to their low toughness their applications are usually limited.

Composite materials have been widely used in aerospace structural components, automobile industries and many other industrial applications, due to their high specific strength, high specific stiffness, anti-corrosion ability, workability, low density, superior performance reliability, ease in fabrication of complex shapes and several other attributes. But the composite materials have some limitations such as the weakness of interfaces between layers that to de-lamination, extreme thermal loads that may lead to de-bonding between matrix and fibre due to mismatch of mechanical properties, residual stresses that may be present due to difference in coefficients of thermal expansion of the fibre and the matrix, etc.

A new class of materials known as Functionally Graded Materials (FGMs) was proposed in 1980's which overcome the limitations that were found in traditional composite. FGMs are microscopically inhomogeneous composites usually made from a mixture of metals and ceramics. By gradually varying the volume fraction of constituent materials, their material properties exhibit a smooth and continuous change from one surface to another, thus eliminating interface problems and mitigating thermal stress concentrations, which are very common in traditional composites.

For various applications, the FGM approach has been explored to mitigate some of the major problems associated with development of a sharp interface at the join of two dissimilar materials. FGMs are usually more superior to the conventional laminated materials because of no discernable internal interfaces or boundaries, and no internal stress peaks are caused when external loads are applied and thus failure from interfacial debonding or from stress concentration can be avoided.

The continuous change in the microstructure of Functionally Graded Material (FGM) distinguishes them from the fibre-reinforced laminated composite materials. An FGM consist of ceramic on the outside surface exposed to high temperature provides thermal resistance due to its low thermal conductivity while the metallic constituent provides toughness of the plate, that provides thermal corrosion protection and load carrying capability. The FGM are ideal for applications involving severe thermal gradients, ranging from thermal structures in advanced aircraft and aerospace engines to computer circuit boards.

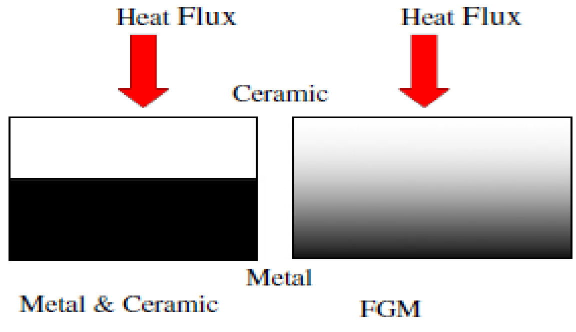

Fig -1: Conventional heat resistant material and FGM

On the macroscopic scale, FGMs are anisotropic, inhomogeneous and possess spatially continuous mechanical properties, which bring the conventional mechanics into a brand-new field. It is extremely complex to manufacture the FGM materials with fully specified profiles of material gradations due to the large number of parameters involved in fabrication such as dispersions in microstructure, volume fraction, porosity etc.

Functionally Graded Materials have been used by nature for eons. Bamboo is an excellent example of FGM, where the microstructure consists of a spatially varying concentration of voids and pores in order to maximize bending rigidity, and bending strength, while minimizing mass. Bone has similar Functional Grading, while even human skin is graded to provide certain toughness, tactile and elastic qualities.

2. LITERATURE REVIEW

A brief review of the important literature related to static, dynamic and transient analysis of functionally graded structures is carried out and presented in the following section.

The technique of grading ceramics along with metals initiated by the Japanese material scientist in Sendai has marked the beginning of exploring the possibility of using FGMs for various structural applications [1].

Since the concept of FGM was first proposed, FGMs have been extensively studied by researchers, who have mainly focused on thermo-elastic behavior.

Trung-kien Nguyen et al[2] are proposed the First Order Shear Deformation plate model for modeling structures made of functionally graded material. The material properties of functionally graded material are assumed to be isotropic at each point and vary through the thickness according to a power law distribution.

Kadoli et al[3] studied the static behaviour of FGM beams by using higher order shear deformation theory under ambient temperature. This study reveal that, depending on whether the loading is on the ceramic rich face or metal rich face of the beam, the static deflection and the static stresses in the beam do not remain the same.

Mohammad Talha et al[4] are studied Free vibration and static analysis of functionally graded material (FGM) plates using higher order shear deformation theory. They presented numerical results for different thickness ratios, aspect ratios and volume fraction index with different boundary conditions. For a given thickness ratio non-dimensional deflection increases as the volume fraction index increases. It is concluded that the gradient in the material properties plays a vital role in determining the response of the FGM plates.

Based on the First order Shear deformation Theory (FSDT), Francesco Tornabene et al[5] presented the dynamic behaviour of moderately thick functionally graded shells and annular plates. The solution is obtained by using numerical technique termed as Generalized Differential Qudrature (GDQ) method. Numerical results illustrate the influence of power law exponent of shell and panel structure which are considered.

Xiao-Lin Huang et al[6] presented the non linear Vibration and Dynamic response of functionally graded material plates in thermal environments. They used higher-order shear deformation plate theory and general von Karman-type equation, which includes thermal effects. The results reveal that the temperature field and volume fraction distribution have significant effect on the nonlinear vibration and dynamic response of the functionally graded plate.

3. MATHEMATICAL FORMULATION

The determination of accurate linear behavior of the FGMs are largely depends on the theory used to model the structure. The classical plate theory (CLPT) that is based on the Kirchoff hypothesis may be inaccurate for analysis of FGM plates in which the volume fractions of two or more materials are varied continuously as a function of position in thickness direction. The inaccuracy occurs due to neglecting the effects of transverse shear and normal strains in the plate. To take the effects of gradual change in material properties, the first order shear deformation theory (FSDT) may be considered. Since in the FSDT model the transverse shear strains are assumed to be constant in the thickness direction, therefore the shear correction factor have to be incorporated to adjust the transverse shear stiffness, in order to study the static or dynamic problems of FGM plates. The accuracy of solutions of the FSDT model depends on the shear correction factors. In the present analysis shear correction factor is taken as 5/6. To reduce the computational effort, the FGM plate has been divided into finite number of imaginary layers.

3.1 First Order Shear Deformation Theory

The first order shear deformation theory (FSDT) is used to describe the kinematics of deformation for the present analysis. The displacement components (u, v, w) at any point (x, y, z) along three perpendicular directions are expressed as follows:

$$u(x,y,z,t) = u_0(x,y,t) + z\phi_x(x,y,t)$$
$$v(x,y,z,t) = v_0(x,y,t) + z\phi_y(x,y,t)$$
$$w(x,y,z,t) = w_0(x,y,t) + z\phi_z(x,y,t)$$

Where *u, v* and *w* are the displacements in the x, y, and z directions, while ϕ_x, ϕ_y and ϕ_z are the rotations.

These equations can be expressed in the matrix form as

$$\{\bar{u}\} = [z'] \{u\}$$

Strain-displacement relation:

$$\varepsilon_{xx} = \varepsilon_{xx}^0 + zk_x$$
$$\varepsilon_{yy} = \varepsilon_{yy}^0 + zk_y$$
$$\varepsilon_{zz} = \varepsilon_{zz}^0$$
$$\gamma_{xy} = \gamma_{xy}^0 + zk_{xy}$$
$$\gamma_{yz} = \gamma_{yz}^0 + zk_{yz}$$
$$\gamma_{xz} = \gamma_{xz}^0 + zk_{xz}$$

Where ε_{xx}, ε_{yy}, ε_{zz}, γ_{xy} etc. are engineering strain components and ε^0_{xx}, ε^0_{yy}, ε^0_{zz}, γ^0_{xy} etc. are generalized strain components and are expressed in terms of displacements (u_0, v_0, w_0, ϕ_x, ϕ_y, ϕ_z)

In case of Functionally Graded Materials, material properties such as modulus of elasticity (E), Poisson ratio (υ), density (ρ) etc. are continuously vary through their thickness. This variation is assumed to according to a power-law distribution of the volume fractions of the plate constituents.

3.2 Power -Law Function

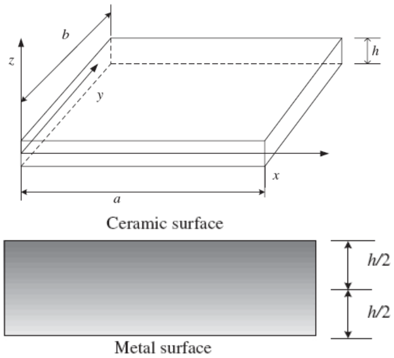

Fig -2: Functionally Graded Plate

Power law function is used in the present analysis. Power law distribution in conjunction with simple law of constituent mixture as follows:

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n,$$
$$V_m = (1 - V_c)$$

Where, V_c and V_m are volume fraction of ceramic and metal respectively, z = distance from mid-surface and n = power law index, a positive real number. The effective material properties P_{eff} are evaluated using the relation,

$$P_{eff}(z,T) = P_m(\mathrm{T})\mathrm{V}_m(\mathrm{z}) + \mathrm{P}_c(\mathrm{T})\mathrm{V}_c(z)$$

Where, P_m and P_c stands for material properties of metals and ceramics respectively. The effective material properties of the plate, including young's modulus E and thermal expansion (α), vary according to above equation. Density (ρ), Poisson ratio ($\boldsymbol{\nu}$) and thermal conductivity $\overline{K}$ are assumed to be independent of temperature, the above equation as follows

$$P_{eff}(\mathrm{z}) = P_m \mathrm{V}_m(\mathrm{z}) + \mathrm{P}_c \mathrm{V}_c(z)$$

3.3 Energy equation

The present analysis involves structural displacements due to external mechanical loading. The total energy of the system can thus be considered as strain energy due to mechanical loading and the kinetic energy due to inertial effects.

3.3.1 Strain energy

The strain energy due to mechanical strain for plate is expressed as

$$U = \frac{1}{2}\int_v \{\varepsilon\}^T [Q]\{\varepsilon\} dv$$

Where *v* is the volume of the plate

3.3.2 Kinetic energy

The kinetic energy is expressed as

$$T_e = \frac{1}{2}\int_v \{\dot{u}\}^T \rho(z)\{\dot{u}\} dv = \frac{1}{2}\int_A \{\dot{u}\}^T [\overline{\rho}]\{\dot{u}\} dA$$

and $\rho(z)$ is the mass density at given depth z and *A* is the area of the plate.

3.3.3 Potential energy

The potential energy is expressed as

$$-W_e = \int \{u\}^T \{p\} dA$$

Where, {*p*} is the surface force vector

3.3.4 Variational principle for dynamic problem

The fundamental equations of the present analysis are derived from the Hamilton's variational principle. The Hamilton's principle states that the variation of the Lagrangian during any time interval t_1 to t_2 must be equal to zero

$$\delta\int_{t_1}^{t_2} (T_e - U - W_e) dt = 0$$

Using above equations all together, the global equilibrium equations are obtained after assembling Equations with respect to the global axes.

$$[M]\{\ddot{d}\}+[K]\{d\}=\{F\}$$

Where [M] and [K] are global mass and stiffness matrices and $\{d\}$, $\{\ddot{d}\}$ and $\{F\}$ are global displacement, acceleration and force vectors, respectively.

It is already mention that the FGM structures mostly exposed to thermal environments. Then the governing equation written including thermal load vector $\{F_t\}$ as,

$$[M]\{\ddot{d}\}+[K]\{d\}=\{F\}+\{F_t\}$$

4. STATIC ANALYSIS OF FUNCTIONALLY GRADED PLATE

FGM are meant for use where temperature fluctuations are severe. However, their performance for loads under ambient conditions is also necessary to be investigated since, there can exist situations where temperature fluctuations may be present for a short duration. Hence it is essential to examine the static behaviour of functionally graded plates.

The governing equilibrium equation for the static analysis without thermal load can be written as

$$[K]\{d\}=\{F\}$$

Where, $[K]$ is the stiffness matrix

4.1 Transverse deflection of Thick FGM Plate

In this section, Transverse deflection characteristics for various plates such as ceramic ZrO_2 (Zirconia), metal Al (Aluminum) and functionally graded material (Al/ZrO_2) were examined. The material properties of ceramic ZrO_2 and metal Al are given in table 1. The variation of young's modulus of FGM (Al/ZrO_2) with Power Law index n=1, n=2 and n=5 are shown in Fig -3.

Table -1: Material properties of Al/ ZrO_2 FGM Component

Material	Young's modulus (E) Gpa	Density (ρ) kg/m³	Poison's Ratio (υ)
Aluminum (Al)	70	2707	0.3
Zirconia (ZrO_2)	151	5700	0.3

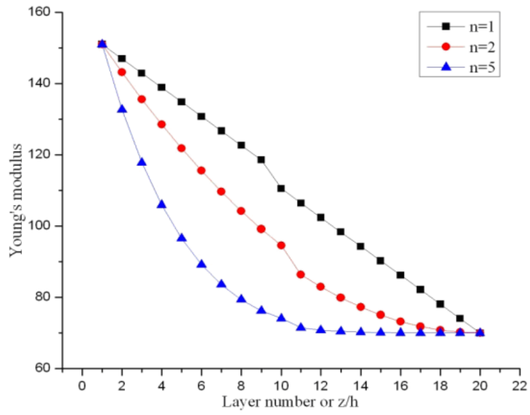

Fig -3: Variation of Al/ZrO_2 FGM's young's modulus with layer number

A Functionally Graded plate (Al/ZrO_2) with SCSC (S-Simply supported, C-Clamped) boundary condition is subjected to unit distributed load and it's aspect ratio and thickness ratio are taken as a/b=0.5 and a/h=20 respectively.

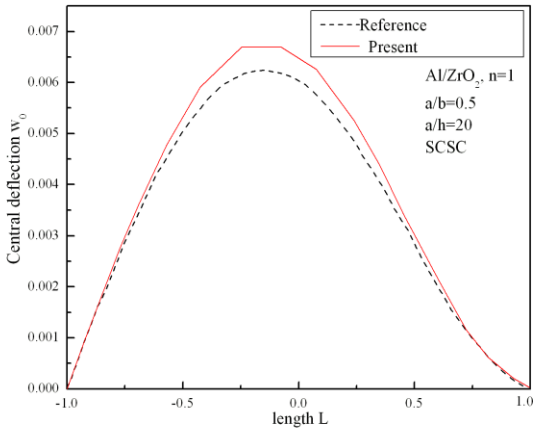

Fig -4: Comparison central deflection along length with reference [4]

Central deflection of Al/ ZrO_2 FGM plate with CCFF (C-Clamped, F- Free) boundary condition under unit UDL presented below.

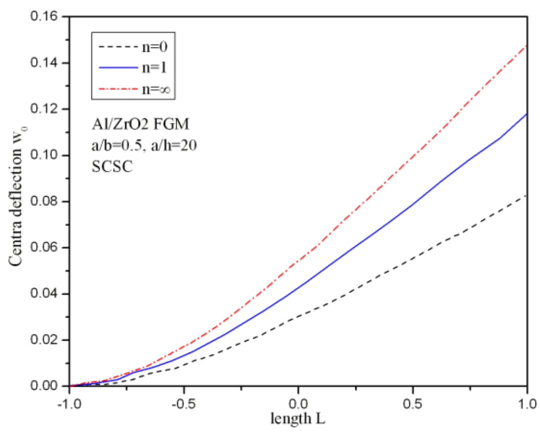

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Fig -5: Central deflection of Al /ZrO_2 FGM plate along length under unit UDL

Table -2: Comparison of Present study with Ferriera[8]

Aspect Ratio(a/b)	Al/Zro2 FGM plate with n=1 Deflection (W).	
	ANSYS	Ferriera et al[8]
0.5	0.112846	0.1626
1	0.677966	0.7352
2	1.505	1.9062

From Fig -4, it can be observed that, the difference between present and reference [4] results because reference [4] used higher order shear deformation theory to solve the problem. FGM is assumed as finite number of layers where each layer treated as homogeneous and isotropic. These assumptions may cause some non linearity in material gradation.

From Fig -5, the central deflection profile increases with increasing Power Law index (n) because metal content increases with increasing Power Law index. Table -2 shows that the comparison of maximum central deflection of Al/ZrO_2 FGM plate with CFCF boundary condition under unit uniformly distributed load with reference [8]

5. FREE VIBRATION ANALYSIS OF FUNCTIONALLY GRADED CONICAL, CYLINDRICAL SHELLS AND ANNULAR PLATE

The study of free vibration behavior is very important to analyze the transient response of graded materials which will help in designing sensors and actuators to control vibrations. Aluminum/Zirconia Functionally Graded Material is used to analyze present study, the material properties are given in Table 1.

The governing equation for free vibration analysis is given below,

$$[M]\{\ddot{d}\}+[K]\{d\}=0$$

Where [M] and [K] are global mass and stiffness matrices and $\{d\}$, $\{\ddot{d}\}$ are global displacement and acceleration respectively.

5.1 Free Vibration of Functionally Graded Annular Plate

In this section, free vibration of Al/ZrO_2 Functionally Graded annular plate with C-F (C-Clamped, F- Free) boundary condition is analyzed. The first 6 natural frequencies and mode shapes are verified with available literature.

Annular plate(C-F): R_b=0.5m, h=0.1m

R_e-R_b=1.5m, where R_e is external radius.

R_b is internal radius.

Table -3: Comparison of Present study with Tornabene[5]

Mode shape	Al/ZrO_2, n=1		ZrO_2, n=0		Al, n=∞	
	A	B	A	B	A	B
1	66.543	68.71	70.203	70.13	65.758	65.59
2	121.11	124.95	127.67	127.5	119.59	119.43
3	121.11	124.95	127.67	127.5	119.59	119.43
4	202.17	208.35	213.14	212.58	199.65	199.12
5	202.17	208.35	213.14	212.58	199.65	199.12
6	283.29	291.13	298.66	296.99	279.75	278.19

A- Present study

B- Reference[5], Percentage error= 2.6- 3.15%

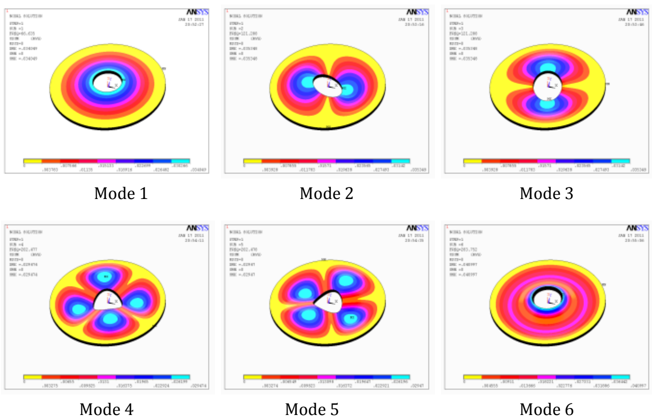


Fig -6: First 6 mode shapes of Functionally Graded annular plate

5.2 Free Vibration of Functionally Graded Cylinder and Conical dome

In this section, free vibration of Al/ZrO_2 Functionally Graded cylinder and conical shell with C-F boundary condition is analyzed. The first 5 natural frequencies and mode shapes are given below.

Cylinder(C-F): R_b=1m, h=0.1m and L_0=2m

Where L_0 is length of cylinder

R_b is radius of cylinder

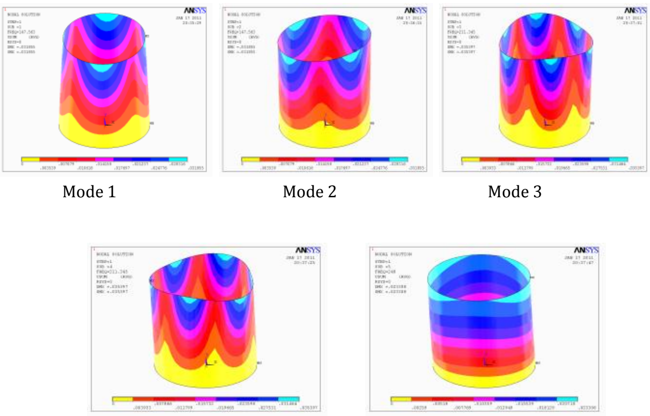
Conical dome(C-F): R_b=0.5m, h=0.1m and L_0=2m

Where L_0 is length of conical dome

R_b is radius of bottom portion

Table -4: First 5 natural frequencies of Functionally Graded Cylinder with C-F boundary condition

Mode shape	ZrO_2, n=0	Al/ ZrO_2, n=1	Al, n=∞
1	151.72	147.56	142.11
2	151.72	147.56	142.11
3	219.27	211.34	205.39
4	219.27	211.34	205.39
5	253.90	248.00	237.82

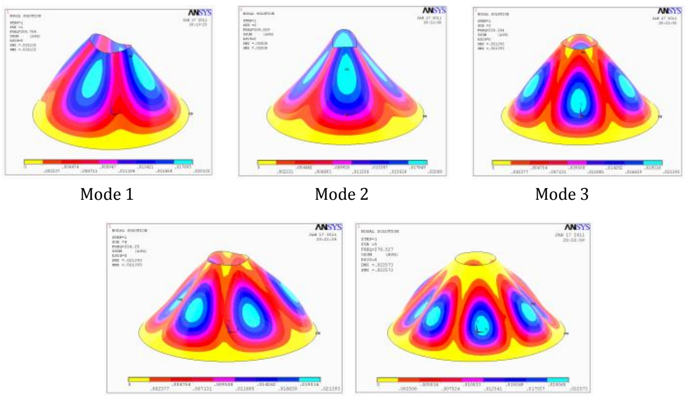


Mode 1 Mode 2 Mode 3
Mode 4 Mode 6

Fig -7: First 5 mode shapes of Functionally Graded Cylinder with C-F boundary condition

Table -5: First 5 natural frequencies of Functionally Graded Conical dome with C-F boundary condition

Mode shape	ZrO_2, n=0	Al/ ZrO_2, n=1	Al, n=∞
1	209.76	204.78	196.47
2	209.80	204.78	196.52
3	231.9	224.24	217.21
4	231.9	224.25	217.22
5	287.79	276.53	269.57



Mode 1 Mode 2 Mode 3
Mode 4 Mode 5

Fig -8: First 5 mode shapes of Functionally Graded conical dome with C-F boundary condition

Table -3 shows the comparison of first 6 natural frequencies of annular plate with Tornabene[5]. The percentage difference between present study and Tornabene[5] because the reference solution is obtained by Generalized Differential Quadrature(GDQ) numerical technique.

From Table -4 and 5, the natural frequencies are given for cylinder and conical dome. The natural frequencies from ceramic to metal (i.e. n=0 to n= ∞) decreases with increasing Power law index for various FGM geometries.

6. TRANSIENT VIBRATION ANALYSIS OF FUNCTIONALLY GRADED MATERIAL

Transient response analysis is the most general method for computing forced dynamic response. The study of dynamic behavior of FGM plate is very much important for the design of advanced structure.

The governing equation for transient analysis with thermal effect can be written as,

$$[M]\{\ddot{d}\}+[K^*]\{d\}=\{F\}$$

Where, [K*] Includes thermal preload as geometric stiffness

The above problem becomes transient when the mechanical force, {*F*} varies with time. It is solved by using Newmark's method.

6.1 Transient Response of Isotropic Square Plate

Transient response of a simply supported square plate subjected to a suddenly applied load is compared with available literature [15]. Transient response is carried out with time step Δt = 0.005 and capturing response over a period 0.25 sec.

Isotropic square plate properties:

Length a =283.8 cm

Thickness h= 0.635 cm

Poison's ratio (υ) = 0.25

Young's modulus (E) = 7.031×10^{5} kg/cm^{2}

Density (ρ) = 2.547×10^{-6} kg sce^{2}/cm^{4}

Load (q) = 4.882×10^{-4} kg/cm^{2}

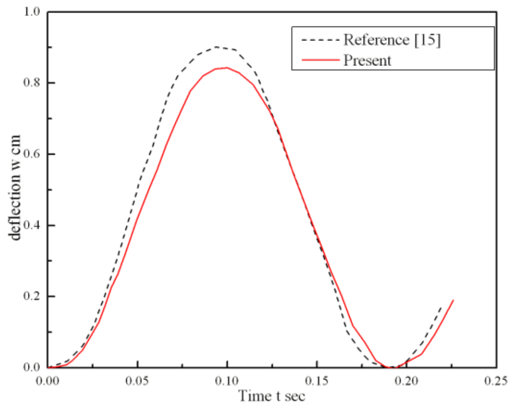


Fig -9: Comparison of transient response of isotropic plate with Reference [15]

6.2 Transient Response of Si_3N_4/SUS304 FGM Plate

In this section, Si_3N_4/SUS304 functionally graded plate's transient response is analyzed with time step Δt=2μs. Si_3N_4/SUS304 functionally graded plate is subjected 400K and 300K temperature to ceramic (Si_3N_4, Top) and Metal (SUS304, Bottom) respectively. A sudden pressure of 50 Mpa is applied in ceramic surface and capturing response is shown below. Material properties and time step are taken from reference [14]

Table -6: Material properties of Si_3N_4/SUS304 FGM

Material	Young's modulus (E) Gpa	Density (ρ) kg/m³	k	Poison's Ratio (υ)
Si_3N_4	322 Gpa	2370	9.19 W/mK	0.3
SUS304	193Gpa	8166	12.04W/mK	0.3

Si_3N_4/SUS304 functionally graded plate (SSB):

Length (a) =0.2 m

Thickness (h) = 0.025m

Applied loads: Pressure (q_0) = -50 Mpa

T_t=400K

T_b=300K

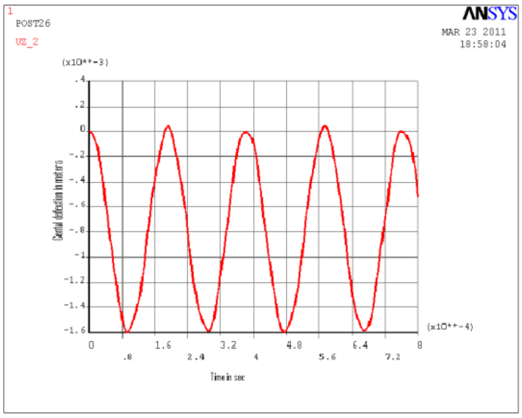


Fig -10: Transient response of Si3N4/SUS304 Functionally Graded plate (n=1) under thermal environment

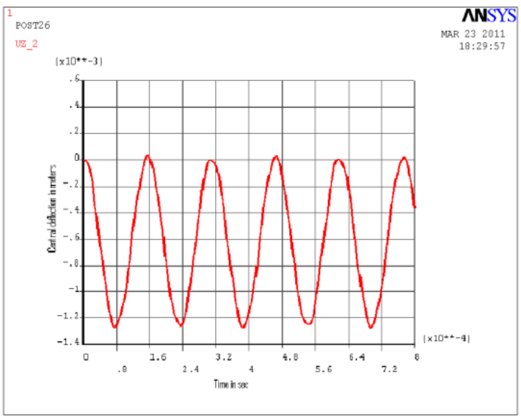


Fig -11: Transient response of the Si_3N_4 (n=0) under thermal environment

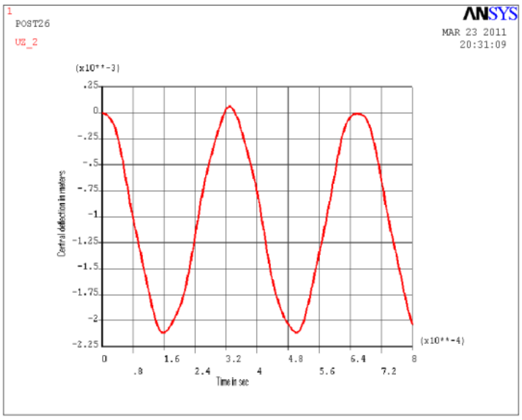


Fig -12: Transient response of the SUS304 (n= ∞) under thermal environment

Numerical analysis carried out with same load and boundary conditions with different Power Law index i.e. n=0, 1 and ∞. Above Fig -10, 11 and 12 results state that the amplitude of vibration is maximum for the metallic plate and a minimum for the ceramic plate. The amplitude of vibration increases smoothly as the amount of metal in the plate increases. Also, it is clear that the frequency of vibration of the ceramic plates is much higher than that of the metallic plates. The natural frequencies are reduced by increasing Power law index n and temperature rise.

7. CONCLUSION

The static, free vibration and transient vibration analysis is carried out to investigate the performance of Functionally Graded Material due to mechanical load.

From the static analysis it was observed that for Functionally Graded plates of Al/ZrO_2, the deflections for given boundary conditions increases with increasing Power law index.

From the free vibration analysis, it is found that as the Power Law index (n) increases, the frequencies of all modes decreases. This effect is independent of geometrical shape.

The transient analysis states that the amplitude of vibration is maximum for the metallic plate and a minimum for the ceramic plate. The natural frequencies are reduced by increasing Power law index n and temperature rise. Also, it is clear that the frequency of vibration of the ceramic plates is much higher than that of the metallic plates.

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