An investigation on the effect of offshore islands along the coastal belt of Bangladesh during a storm period

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Abstract - *In this study, a vertically integrated shallow water model is designed to estimate the water level due to nonlinear interaction of tide and surge associated with tropical storms along the coastal belt of Bangladesh. To develop the model, a fine mesh scheme, capable of covering all major islands, has been nested into a coarse mesh scheme covering up to 15°N latitude in the Bay of Bengal. To incorporate accurately thickly populated small and big islands along with highly bending of the coastline along the Meghna estuarine region in the numerical scheme, a very fine mesh scheme for the region is again nested into the fine mesh scheme. Along the northeast corner of the very fine mesh scheme, the fresh water discharge through Meghna river is taken into account. The boundaries of the coast as well as of islands are approximated through proper stair step representation and the model equations are solved by semi-implicit finite difference technique using a staggered grid. The model is then applied to simulate the water levels due to the non-linear interaction of tide and surge associated with the storms SIDR and AILA. The model simulation was quite satisfactory, and then the influence of the offshore islands on the water level along the coast of Bangladesh was tested.*

Key Words: Shallow water model, Non-linear interaction, Storm surge, AILA, SIDR

1. INTRODUCTION

The best technique for the storm surge prediction is the numerical solution of the hydrodynamic equations. The numerical method consists of solving the system of equations governing the sea motion at discrete set of points, called grid points, at a discrete instant of time. For practical forecasting it is superior to the analytical method. With the advent of modern computer and the development of sophisticated numerical techniques, great strides have been made in surge forecasting. The numerical models have the ability to predict surges, its development and spatial variation of the resulting sea surface elevations from the knowledge of the basic parameters describing the generating cyclone and its track. The models can also generate the tidal elevations and their interactions with storm surges. Using numerical methods, various studies on the prediction of storm surges have already been made along the storm affected regions of the world including the Bay of Bengal. Out of them Das *et al.* [1], Johns and Ali [4], Johns *et al.* [5], Talukder *et al*. [13] and Roy [11] considered the interaction of tide and surge phenomena. None of them, except Talukder *et al*. [13] and Roy [11] considered the existence of islands along the head Bay region. But there are many small and big islands in the offshore region of the Bangladesh coast and it is known that there is a significant influence of offshore islands over surge intensity along the coastal belt [7]. Moreover, since the islands are thickly populated it is necessary to estimate the water levels due to tide and surge interaction at these islands. Hence inclusion of islands in a model is essential in order to incorporate the real situation of the head Bay region.

In our present study, we have developed a tide-surge forecasting model to estimate the water levels due to the nonlinear interaction of tide and surge. The model is developed in such a way that it can be able to incorporate the whole coastal belt and offshore islands very accurately in the numerical schemes and the target has been achieved through nested numerical schemes (Rahman *et al*. [8], [9], [10]). For that purpose, a fine mesh scheme (henceforth will be referred to as **FMS**) for the coastal belt of Bangladesh is nested into the coarse mesh scheme (henceforth will be referred to as **CMS**) covering up to 15^0 N latitude in the bay of Bengal. The region between Barisal and Chittagong is full of so many small and big islands and also the bending of the coastline is very high. Considering those facts into account, a very fine mesh scheme (henceforth will be referred to as **VFMS**) for this region is nested into the FMS. Along the northeast corner of the VFMS, the Meghna river water discharge is taken into account. Vertically integrated shallow water equations are solved with the above mentioned doubly nested schemes. The specialty of the nesting is that, the CMS is completely independent, whereas along the open boundaries of the FMS the elevations are prescribed from those obtained from the CMS at each time step of the solution process. Similarly, along the open boundaries of the VFMS the elevations are prescribed from those obtained from the FMS. The model is verified using observed water-level data with severe storms SIDR and AILA that hit the coast of Bangladesh in 2007 and 2009 respectively. For analysis and verification of the results, some locations along the coastal belt of Bangladesh are considered. The locations are Hiron Point, Tiger Point, Kuakata, Patharghata, Char Jabber, Sandwip, Shitakunda and Chittagong.

2. MATHEMATICAL FORMULATION

2.1 Basic Equations

In the formulation of the model, the sphericity of the earth's surface is ignored. A system of rectangular Cartesian coordinates is used in which the origin, O, is in the undisturbed level of the sea surface as the xy-plane, OX points towards the south, OY points towards the east and OZ is directed vertically upwards. The displaced position of the free surface is considered as $z=\zeta(x,y,t)$ and position of the sea bed as $z=-h(x,y)$ so that the total depth of the water column is $\zeta+h$. Then, following Rahman *et. al.* [10], the vertically integrated shallow water equations can be written as

$$\frac{\partial\zeta}{\partial t}+\frac{\partial}{\partial x}\left[(\zeta+h)u\right]+\frac{\partial}{\partial y}\left[(\zeta+h)v\right]=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-fv=-g\frac{\partial\zeta}{\partial x}+\frac{T_x}{\rho(\zeta+h)}-\frac{C_f\,u\sqrt{u^2+v^2}}{(\zeta+h)} \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+fu=-g\frac{\partial\zeta}{\partial y}+\frac{T_Y}{\rho(\zeta+h)}-\frac{C_f\,v\sqrt{u^2+v^2}}{(\zeta+h)} \tag{3}$$

where

$(u,\ v)$ = velocity components of sea water in x and y directions respectively,
f = Coriolis parameter,
g = gravitational acceleration,
$(T_x,\ T_y)$ = components of wind stress,
h = ocean depth from the mean sea level,
C_f = the friction coefficient,
ρ = water density,

Also, in the above equations u and v are the vertically integrated components given by

$$\left(u,\ v\right)=\frac{1}{\zeta+h}\int_{-h}^{\zeta}\left(\bar{u},\ \bar{v}\right)dz \tag{4}$$

where $\bar{u}$ and $\bar{v}$ are x and y components of the Reynolds averaged velocity.

Using Eq. (1) we may express the Eqs. (2) & (3) in the flux form and thus, Eqs. (1) – (3) may be written as

$$\frac{\partial\zeta}{\partial t}+\frac{\partial\tilde{u}}{\partial x}+\frac{\partial\tilde{v}}{\partial y}=0 \tag{5}$$

$$\frac{\partial\tilde{u}}{\partial t}+\frac{\partial(u\tilde{u})}{\partial x}+\frac{\partial(v\tilde{u})}{\partial y}-f\tilde{v}=-g(\zeta+h)\frac{\partial\zeta}{\partial x}+\frac{T_x}{\rho}-\frac{C_f\,\tilde{u}\sqrt{(u^2+v^2)}}{\zeta+h} \tag{6}$$

$$\frac{\partial\tilde{v}}{\partial t}+\frac{\partial(u\tilde{v})}{\partial x}+\frac{\partial(v\tilde{v})}{\partial y}+f\tilde{u}=-g(\zeta+h)\frac{\partial\zeta}{\partial y}+\frac{T_y}{\rho}-\frac{C_f\,\tilde{v}\sqrt{(u^2+v^2)}}{\zeta+h} \tag{7}$$

where $(\tilde{u},\tilde{v})=(\zeta+h)(u,v)$
Here u and v in the bottom stress terms of Eqs. (2) and (3) have been replaced by $\tilde{u}$ and $\tilde{v}$ in Eqs. (6) and (7) in order to solve the equations numerically in a semi-implicit manner.

2.2 Generation of wind stress

In our study wind field is derived from the empirical formula given by Jelesnianski [3] in the following way :

$$V_a=\begin{cases}V_0\sqrt{(r_a/R)^3}, & \text{for } r_a\le R\\ V_0\sqrt{(R/r_a)}, & \text{for } r_a>R\end{cases}$$

where V_0 is the maximum sustained wind at the radial distance R from the eye of the cyclone and r_a is any radial distance from the eye at which the wind field is desired.

Knowing the wind field, the components of wind stress are derived from :

$$T_x=C_D\rho_a\,u_a\left(u_a{}^2+v_a{}^2\right)^{\frac{1}{2}}\ \text{and}\ \ T_Y=C_D\rho_a\,v_a\left(u_a{}^2+v_a{}^2\right)^{\frac{1}{2}},$$

where C_D and ρ_a are the drag coefficient and air density and u_a and v_a are the x and y components of surface wind, respectively.

3. NUMERICAL METHOD

3.1 Set-Up of the Nested Schemes

In our study, following Rahman *et. al.* [10], we have taken the study area considerably big so that, a storm can move over the area at least for 3 days before crossing the coast. This is because the surge response along the coast becomes significant well before a storm reaching the coast. On the other hand, in order to include the major islands in the estuary the mesh size (the distance between two consecutive grid points) should be considerably smaller whereas, this is unnecessary away from the estuary. Consideration of very fine mesh over the whole analysis area involves, unnecessarily, more memory and more CPU time in the solution process and invites problem of numerical instability. Considering the above facts, a high-resolution numerical scheme (FMS) is nested into a coarse mesh scheme (CMS). In the fine mesh scheme, all the major islands are incorporated through proper stair step representation. For the existence of so many small and big islands and also for high bending of the coastline along the Meghna estuary, a very fine mesh scheme

(VFMS) for the region between Barisal and Chittagong is again nested into the fine mesh scheme. Information on the grids is given in table 1.

Table 1. Model grid specifications

Schemes	Scheme area	Grid size	Grid spacing	
			North-South (Δx)	East-West (Δy)
CMS	15^0N to 23^0N (latitude) 85^0E to 95^0E (longitude)	60×61	15.08 km	17.52 Km
FMS	21^0-15′ N to 23^0N (latitude) 89^0E to 92^0E (longitude)	92×95	2.15 Km	3.29 Km
VFMS	21.77^0N to 23^0N (latitude) 90.40^0E to 92^0E (longitude)	190×145	720.73 m	1142.39 m

3.2 Boundary conditions

In the outer model (CMS), there are three open sea boundaries along 15^0N latitude, 85^0E longitude and 95^0E longitude. According to Johns *et al.* [5], the following radiation type boundary conditions are taken for open sea boundaries:

$$\text{At the west boundary}: \quad v+\left(\frac{g}{h}\right)^{\frac{1}{2}}\zeta=0. \tag{9}$$

$$\text{At the east boundary}: \quad v-\left(\frac{g}{h}\right)^{\frac{1}{2}}\zeta=0. \tag{10}$$

At the south boundary:

$$u-\left(\frac{g}{h}\right)^{1/2}\zeta=-2\left(\frac{g}{h}\right)^{1/2}a\sin\left(\frac{2\pi t}{T}+\phi\right), \tag{11}$$

where a and ϕ denote, respectively, the amplitude and phase related to the tidal constituent and T is its period. The above type of boundary conditions are taken in order to prevent the artificial reflection at the open boundaries of the disturbances generated within the model area.

In the inner model (VFMS), the open boundary condition along the inlet between 90.46^0E and 90.61^0E longitudes is given by

$$u_b=u+\frac{Q}{(\zeta+h)B} \tag{12}$$

where Q is the discharge of the fresh water through the Meghna river in $\mathrm{m^3s^{-1}}$, and B is the breadth of the river in meter.

3.3 Model data set-up (with grid generation)

The governing equations given by Eqs. (5)-(7) as well as the boundary conditions given by Eqs. (9)-(11) are descritized by finite difference (forward in time and central in space) by considering the discrete points in the $x-y$ plane define by

$$x_i=(i-1)\Delta x,\quad i=1,2,3,\ \dots,\ m(\text{even}),$$
$$y_j=(j-1)\Delta y,\quad j=1,2,3,\ \dots,\ n(\text{odd}),$$

where Δx and Δy are grid increments. A sequence of time instants is defined by

$$t_k=k\Delta t,\quad k=1,2,3,\ \dots,$$

where Δt represent the increment in time. The details of the grid generation and model data set-up are similar to that discussed in Rahman *et. al* [10]. It can be noted here that the unstaggered grid cannot handle properly the high frequency waves, particularly two grid waves [2]. So we have used a suitable arrangement of grid points (staggered grid system) in $x-y$-plane.

The necessary meteorological inputs used in our study are supplied from Bangladesh Meteorological Department (BMD). In shallow water, surge height is very sensitive to the depth of ocean. Therefore, bed topography should specify accurately. The bed topography for the region that covers FMS is specified for CMS, FMS and VFMS from the soundings carried out by the survey group of Land Reclamation Project (LRP) under Bangladesh Water Development Board (BWDB). There are some grid points where sounding is not available. We used weighted interpolation to interpolate the bottom for other necessary grid points. Outside the FMS region, the water depth data for CMS for some representative points of the model domain are compiled from the figure quoted in Johns et al. [5]. The bottom for other necessary grid points, again we used weighted interpolation. In the solution process, the values of the friction coefficient c_f and the drag coefficient C_D are taken uniform throughout the physical domain, which are 0.0026 and 0.0028 respectively [7]. A stable tidal regime over the model domain was generated first by applying the most energetic tidal constituent M_2, the semidiurnal principal lunar tide. The period of the tidal oscillation is taken as 12.4 h. It is to be noted here that the period of tidal oscillation in the region of interest is not exactly periodic but the mean period is found to be nearly 12.4 h [11]. The initial values of amplitude, a and phase, φ are prescribed through Eq. (11) along the southern open boundary of the CMS following McCammon and Wunsch [6]. Then using the constants involved with constituent M_2 from initial state of rest in the absence of atmospheric pressure gradient force, tide generating forces, and wind stress, a stable tidal regime was achieved after 4 tidal cycles. But the generation of a pure tidal condition depends on the exact values of a and φ, and we followed the technique of Roy [11] for the precise specification of the values of the constants. The initial values of ζ, u and v are taken as zero to represent a cold start and from this start a stable tidal regime over the analysis area is generated. To take into account the nonlinear interaction between the tide and surge, this tidal regime provides the initial condition of the sea at the model time $t=0$. The time step is taken as 60 seconds that ensures Courant-Friedrichs-Levy (CFL) criterion of stability of the numerical schemes.

4. RESULTS

For the purpose of model verification, the model had been applied for some major storms that hit the coast of Bangladesh in the last ten years. Records show that SIDR 2007 and AILA 2009 storms are the most severe cyclonic storms having maximum wind speed 240 km/h and 120 km/h respectively. So we give more stress on the data of SIDR and AILA for computation of results. The tracks (paths) of the chosen storms are shown in Fig. 1.

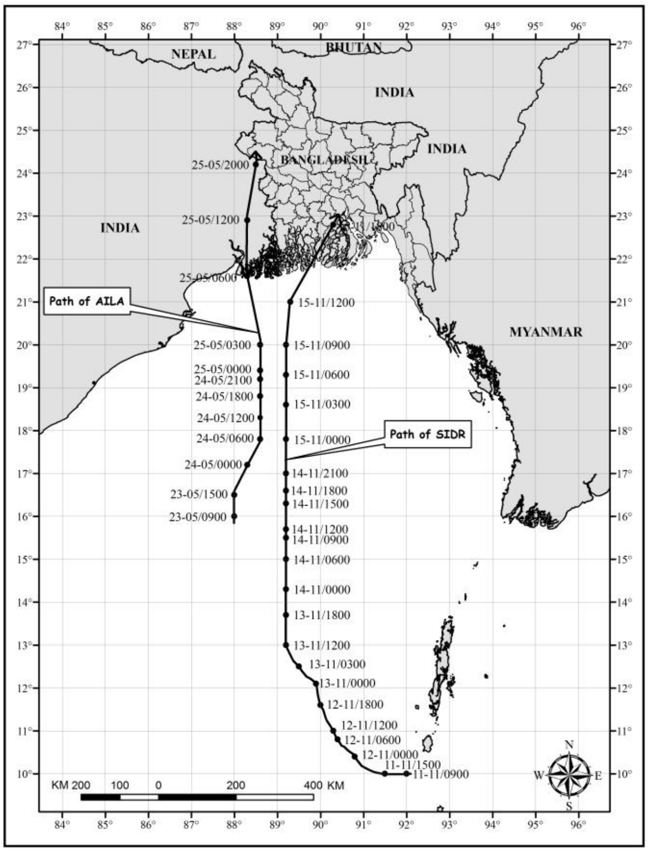


Fig - 1: Observed track (path) of cyclonic storms SIDR and AILA.

Figure 2 depicts the computed water levels due to tide-surge interaction associated with SIDR 2007 storm at Hiron Point, Tiger Point, Patharghata, Kuakata and Chittagong. It may be observed that, the maximum water level is increasing with time as the storm approaches towards the coast and finally there is recession. At Hiron Point a strong recession is occurred after 15 hrs of 15th November, earlier than in any other location and about 3 hrs before landfall of the storm. The recession takes place due to backwash of water from the shore towards the sea. In fact, Hiron Point is situated far left (west) of the storm path and so the direction of the anti-clock wise circulatory wind becomes northerly (i.e. towards the sea) at Hiron Point long before the storm reaches the coast and thus driving the water towards the sea. It may be noticed that the beginning of recession delays as we proceed towards east as is expected. At every location, the peak surge is attaining before the land falls time of the storm. This is expected; as the circulatory wind intensity is highest along the coast when the storm reaches near the coast. The maximum elevation varies between 3.9 m (at Hiron Point) to 6 m (at Chittagong). According to the Wikipedia website, the entire cities of Patuakhali (kuakata), Barguna (Patharghata) and Jhalokati district were hit hard by the storm surge of over 5 meters (16 ft). Thus, the results that come out through our model at these locations compare well with the results stated in Wikipedia website.

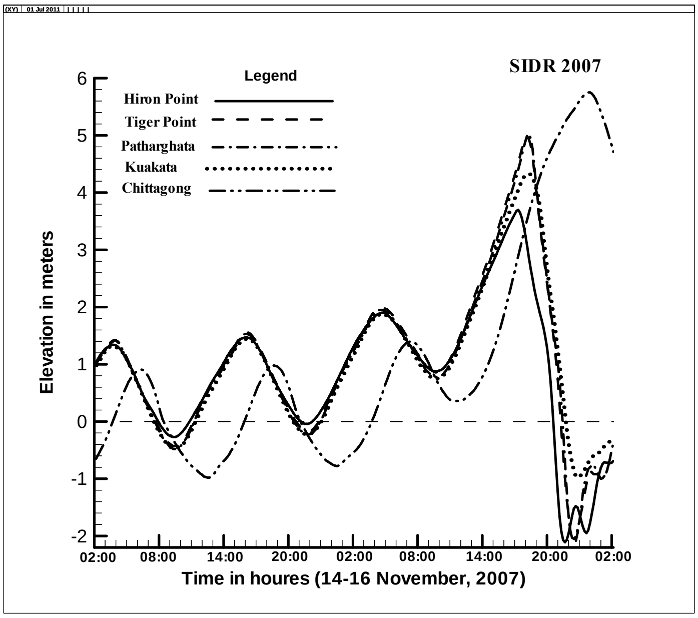


Fig -2: Computed water levels due to the interaction of tide and surge at different locations associated with storm SIDR.

Figure 3 depicts the computed water levels due to tide-surge interaction associated with AILA 2009 storm at Patharghata, Char Jabber, Sandwip, Shitakunda and Chittagong. The maximum overall water level due to the interaction of tide and surge at these locations are between

2.6 m and 3.4 m (Fig. 3). According to the Wikipedia website [http://en.wikipedia.org/wiki/Cyclone_Aila], there was 3 m (10 ft) surge height at the western regions of Bangladesh. According to the report of NASA's Goddard Space Flight, there was storm surge between 10-13 feet (3.048 m- 3.9624 m) high along the western Bangladesh coastline while land fall occurred. Thus the computed water levels for western coastal locations are found to be in good agreement with the observed ones.

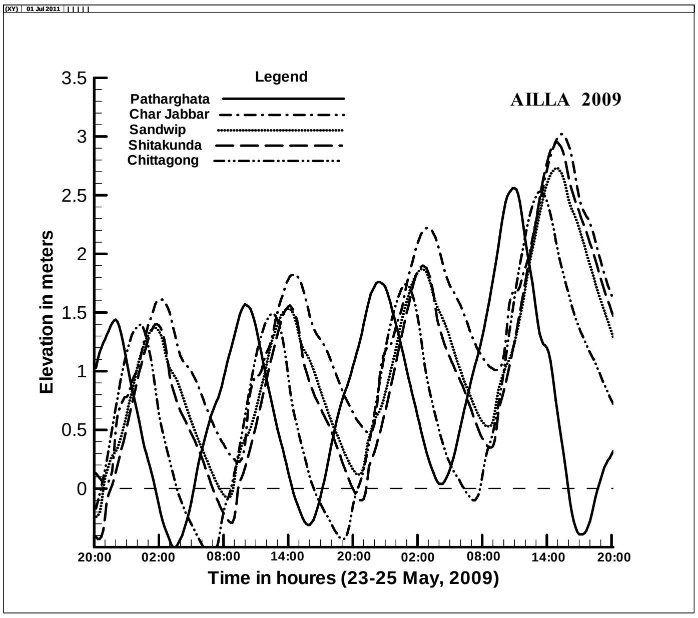
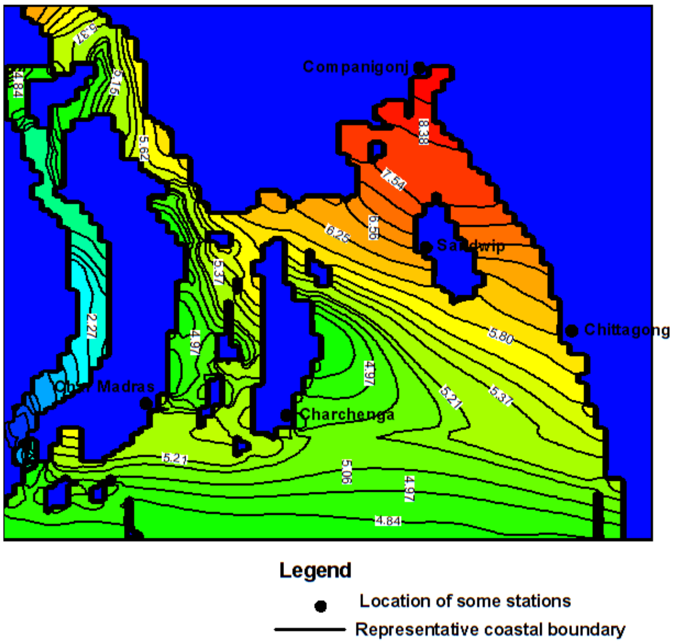
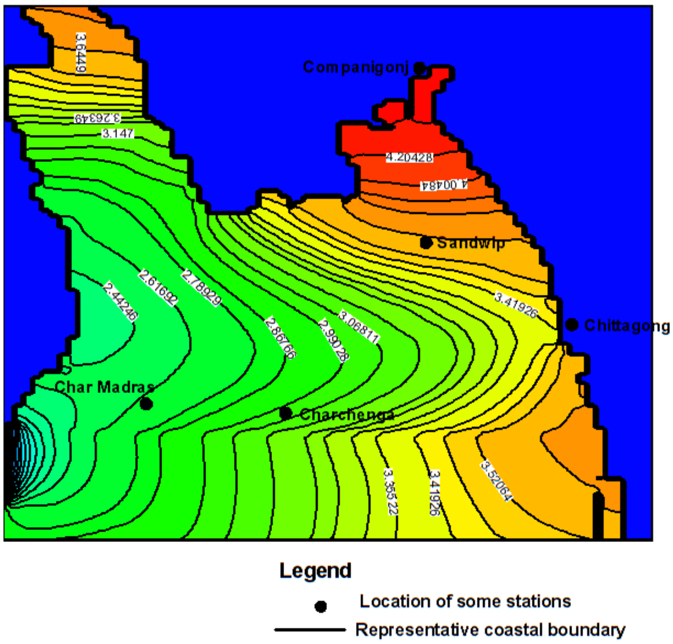


Fig -3: Computed water levels due to the interaction of tide and surge at different locations associated with storm AILA.

Finally, a sensitivity test is done to examine the effect of offshore islands on the surge intensity along the coast. Figures 4 and 5 show our model simulated water levels along the Meghna estuarine region with and without inclusion of offshore islands for the storms SIDR and AILA respectively. It can be shown from these figures that when offshore islands are taken into account, water levels are found to be reduced. Thus it can be concluded here that offshore islands reduce surge intensity. Roy [12] in his study also obtained the similar result. But he pointed that more investigation is needed before making any final conclusion. It is to be noted here that the study was conducted only including two major islands Sandwip and Bhola and the very fine resolution was not taken into account, where our present study is conducted including all the small and big offshore islands accurately in our model.

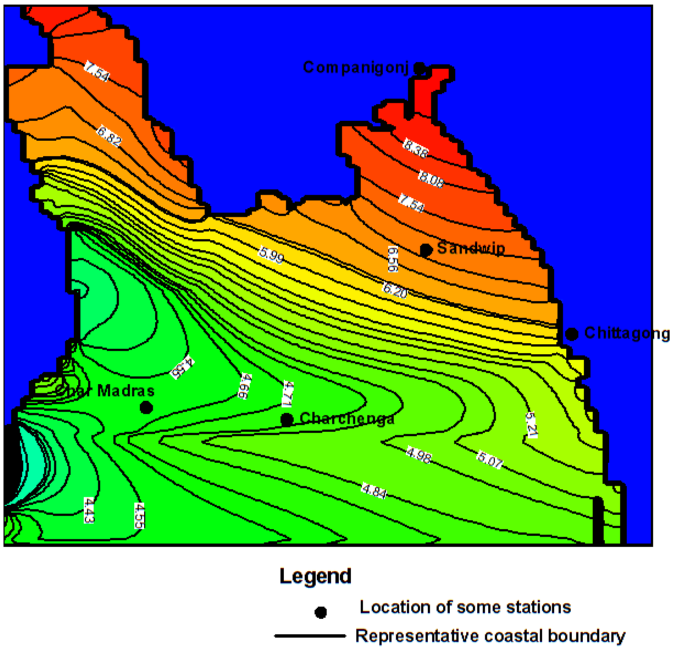


(a)

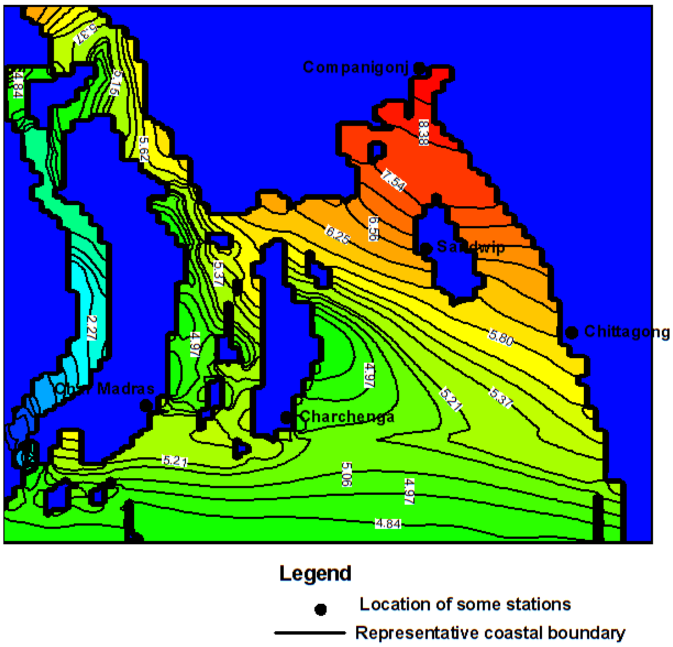


(b)

Fig -4: Contours of our computed water levels (in the **VFMS** region of the computed domain) associated storm with SIDR 2007 (a) with islands and (b) without islands



(a)



(b)

Fig -5: Contours of our computed water levels (in the **VFMS** region of the computed domain) associated with storm AILA 2009 (a) without islands and (b) with islands

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