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Construction of control charts based on six sigma initiatives for the number of defectives with varying sample size

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Abstract – *A control chart is a statistical device used for the study and control of repetitive process. W.A. Shewhart (1931) of Bell Telephone Laboratories suggested control charts based on the 3 sigma limits. Now the companies in developed and developing countries started applying Six Sigma initiatives in their manufacturing process, which results in lesser number of defects. The companies practicing Six Sigma initiatives are expected to produce 3.4 or less number of defects per million opportunities, a concept suggested by Motorola (1980). If the companies practicing Six Sigma initiatives use the control limits suggested by Shewhart (1931), then no point fall outside the control limits because of the improvement in the quality of the process. In this paper an attempt is made to construct a control chart based on six sigma initiatives for number of defectives with varying sample size specially designed for the companies applying Six Sigma initiatives in their organization. Suitable Table – 4 is also constructed and presented for the engineers to take quick decisions.*

Keywords: *Control Chart, Process control, Six Sigma, Six Sigma Quality Level.*

1. INTRODUCTION

The concept of Six Sigma was introduced by Motorola (1980) by the engineer M.Harry who analyzed variations in outcomes of the company's internal procedures and realized that by measuring variations it will be possible to improve the working of the system. The procedure was aimed at taking action to improve the overall performance. The companies, which are practicing Six Sigma, are expected to produce 3.4 or less number of defects per million opportunities. Radhakrishnan and Sivakumaran (2008a, 2008b and 2008c) used the concept of Six Sigma in the construction of sampling plans such as single, double and repetitive group sampling plans indexed through Six Sigma Quality Levels (SSQLs) with Poisson distribution as the base line distribution. Radhakrishnan (2009) suggested single sampling plan indexed through Six Sigma quality levels (SSQLs) based on Intervened Random Effect Poisson Distribution and Weighted Poisson Distribution as the base line distributions. Radhakrishnan and Balamurugan (2010a and 2010b) constructed control charts based on six sigma initiatives for defects, mean, average fraction defectives, number of defectives, X bar using standard deviation, Exponentially Weighted Moving Average (EWMA), proportion defectives – number of defectives, Fraction defectives, Standard deviation, Standard deviation with variable sample size, average number of nonconformities per multiple units, number of defects - average number of defects per unit and range. The control charts originated by W.A. Shewhart (1931) was based on 3 sigma control limits. If the same charts are used for the products of the companies which adopt six sigma initiatives in the process, then no point will fall outside the control limits because of the improvement in the quality. So a separate control chart is required to monitor the outcomes of the companies, which adopt six sigma initiatives.

In this paper an attempt is made to construct a control chart based on six sigma initiatives for number of defectives with varying sample size. Suitable Table – 4 is also constructed and presented for the engineers to take quick decisions.

2. CONCEPTS AND TERMINOLOGIES

2.1 Upper specification limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.

2.2 Lower specification limit (LSL)

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

2.3 Tolerance level (TL)

It is the difference between USL and LSL,

TL = USL-LSL

2.4 Process capability (C_p)

This is the ratio of tolerance level to six times standard deviation of the process.

$$c_p = \frac{TL}{6\sigma} = \frac{USL - LSL}{6\sigma}$$

2.5 Subgroup size (k)

It is the number of units in a sample and 'N' is the number of samples.

2.6 Quality Control Constants ($Q_{6\sigma}$)

The constants $Q_{6\sigma}$ introduced in this paper to determine the control limits based on six sigma initiatives for the number of defectives with varying sample size.

3. CONDITIONS FOR APPLICATION

- Number of units in a sample for different lots is further in advance.
- Human involvement should be less in the manufacturing process
- The company adopts Six sigma quality initiatives in its processes

4. CONSTRUCTION OF CONTROL CHART BASED ON SIX SIGMA INITIATIVES FOR THE NUMBER OF DEFECTIVES WITH VARYING SIZE SAMPLE

Fix the tolerance level (TL) and process capability (C_P) to determine the process standard deviation ($\sigma_{6\sigma}$). Apply the value of $\sigma_{6\sigma}$ in the control limits $\bar{c} \pm 4.831\left(\sigma_{6\sigma}/\sqrt{k}\right)$, to get the control limits based on six sigma initiatives for the number of defectives with varying sample size. The value of $Q_{6\sigma}$ is obtained using $p(z \le z_{6\sigma}) = 1 - \alpha_1, \alpha_1 = 3.4 \text{ x } 10^{-6}$ and z is a standard normal variate. For a specified TL and C_P of the process, the value of σ (termed as $\sigma_{6\sigma}$) is calculated from $c_p = \frac{TL}{6\sigma}$ using a C program and presented in Table 4 for various combinations of TL and C_P. The control limits based on six sigma initiatives for the number of defectives with varying size sample are

$$\text{UCL}_{6\sigma} = \bar{c} + \left[Q_{6\sigma} \times \left(\sigma_{6\sigma}/\sqrt{k}\right)\right]$$

$$\text{Central Line CL}_{6\sigma} = \bar{c}$$

$$\text{LCL}_{6\sigma} = \bar{c} - \left[Q_{6\sigma} \times \left(\sigma_{6\sigma}/\sqrt{k}\right)\right]$$

5. EXAMPLE

The example provided by Acheson J. Duncan (1958) is considered here. The following table gives the number of missing rivets noted at aircraft final inspection:

$$\text{Weightage average}(\bar{c}) = \frac{\sum c}{\sum k} = \frac{21.98}{8.4} = 2.62$$

Inspection Lot No.	Square Yards Oilcloth Inspected	No. of defects per 100 square Yards	Number of units in a sample (k)	Number of defects in Total Lot (c)
1	200	2.5	2.0	5
2	250	2.8	2.5	7
3	100	3.0	1.0	3
4	90	2.2	0.9	1.98
5	120	3.3	1.2	3.96
6	80	1.3	0.8	1.04
Total			**8.4**	**21.98**

Table 1: Missing rivets noted at aircraft

5.1 Three Sigma Control limits for the number of defectives with varying sample size

The 3σ control limits suggested by Shewhart [22] are $\bar{c} \pm 3\sqrt{\bar{c}/k}$. The values are presented in Table 2.

Table 2: UCL and LCL values for 3 sigma control limits

Inspection Lot number	k	$3\sqrt{\bar{c}/k}$	$UCL = \bar{c} + 3\sqrt{\bar{c}/k}$	$LCL = \bar{c} - 3\sqrt{\bar{c}/k}$
1	2.0	3.43	6.05	-
2	2.5	3.07	5.69	-
3	1.0	4.86	7.48	-
4	0.9	5.12	7.74	-
5	1.2	4.43	7.05	-
6	0.8	5.43	8.05	-

From the resulting Figure 1, it is clear that the process is in control, since the entire subgroup numbers lie inside the control limits.

5.2 Control limits based on six sigma initiatives for the number of defectives with varying sample size

For a given TL = 2 (USL-LSL =3.3-1.3) & C_P = 2.5, it is found from the Table-4 that the value of $\sigma_{6\sigma}$ is 0.13. The control limits for the number of defectives with varying sample size based on six sigma initiatives for a specified TL and $Q_{6\sigma}$ is $\bar{c} \pm 4.831\left(\sigma_{6\sigma}/\sqrt{k}\right)$. The UCL and LCL values are presented in Table 3.

Table 3: UCL and LCL values for control chart based on six sigma Initiatives

Inspection Lot number	k	$4.831 \times \frac{0.13}{\sqrt{k}}$	$UCL = \bar{c} + \left(4.831 \times \frac{0.13}{\sqrt{k}}\right)$	$LCL = \bar{c} - \left(4.831 \times \frac{0.13}{\sqrt{k}}\right)$
1	2.0	0.44	3.06	2.18
2	2.5	0.40	3.02	2.22
3	1.0	0.63	3.25	1.99
4	0.9	0.66	3.28	1.96
5	1.2	0.57	3.19	2.05
6	0.8	0.70	3.32	1.92

From the resulting Figure 1, the lot number 5 goes above the upper control limit and the lot number 6 goes below the lower control limit. Therefore the process does not exhibit statistical control.

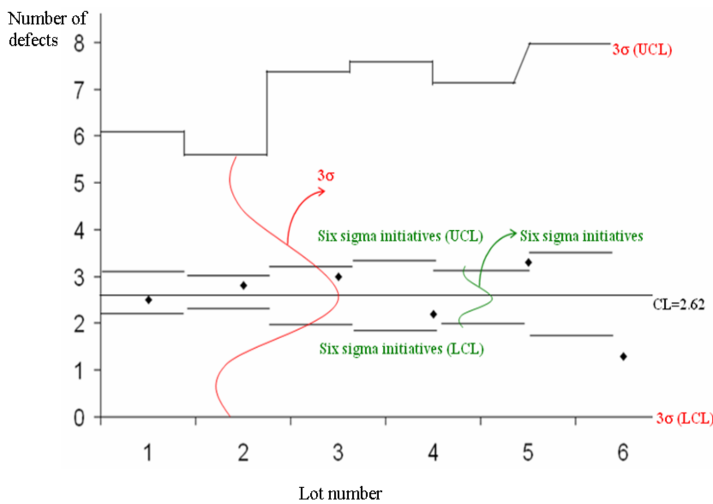

Figure 1: Comparison of the process: 3σ limits and control limits using six sigma initiatives

6. CONCLUSION

In this paper, a procedure is given to construct a control chart based on six sigma initiatives for the number of defectives with varying sample size with an example. It is found that the process was in control even when Six Sigma initiatives are adopted but it is very clear from the comparison that when the process is centered with reduced variation than the 3 sigma control limits, which indicate that the process is not in the level it was expected. So a correction in the process is very much required to reduce the variations. The charts suggested in this paper will be very useful for the companies practicing Six Sigma initiatives in their process. These charts will replace the existing Shewhart (1931) control charts in future when all the companies started implementing Six Sigma Initiatives in their organization.

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