# CONTROL OF TRMS AND MODELLING USING LPV

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**Abstract** – Twin rotor multi input multi output system (TRMS) resembles simplified behaviour of a conventional helicopter with two degrees of freedom (DOF). The system is perceived as a challenging engineering problem owing to its high non-linearity, cross-coupling between its two axes, and inaccessibility of some of its states for measurements.

An attractive solution to represent non-linear systems is Linear Parameter Varying (LPV) models. The main advantage of LPV models is that they allow application of powerful linear
design tools to complex non-linear models. LPV control synthesis fits into the gain scheduling framework, while adding stability and robustness guarantees. The strength of the LPV approach lies in the extension of well-known methods for linear optimal control, including the use of linear matrix inequalities (LMIs), to the design of gain-scheduled LPV controllers .A condition to apply LPV control synthesis is to transform the nonlinear model of the system into an LPV model; hence, LPV modelling becomes a key issue in the design of LPV controllers Many nonlinear systems of practical interest can be represented as quasi-LPV systems, where
quasi is added because the scheduling parameters do not depend only on external signals, but also on system variables. The possibility to embed nonlinear systems into the LPV framework, by hiding nonlinearities within the scheduling parameter, enables the application of linear like control methods to non-linear systems such that, at the same time, stability and desired performance of the closed loop system are guaranteed. An LPV state observer and controller have been designed using LPV pole placement method based on LMI regions. The effectiveness and performance of the LPV control approach have been proved both in simulation and on the real set-up.

***Key Words***: TRMS Control and Representation of non-linear system using LPV models. One of the LPV modelling is Linear matrix inequalities (LMI) is used.

## 1. INTRODUCTION

Recent times have witnessed the development of several approaches for controlling the flight of air vehicle such as Helicopter and Unmanned Air Vehicle (UAV). The modelling and control of the air vehicle dynamics is a highly challenging task owing to the presence of high nonlinear interactions among the various variables and the non-accessibility of certain states. TRMS is an experimental set-up that provides a replication of the flight dynamics. The TRMS has gained wide popularity among the control system community because of the difficulties involved in performing direct experiments with air vehicles. Aerodynamically TRMS consist of two types of rotor, main and tail rotor at both ends of the beam, which is driven by a DC motor and it is counter balanced by an arm with weight at its end connected at pivot. The system can move freely in both horizontal and vertical plane.

### 1.1MOTIVATION

Modelling and controlling of a complex air vehicle such as a helicopter is very challenging task, because of the high nonlinearity, significant cross-coupling between its axes, complex aerodynamics and the inaccessibility of some of its states and outputs for measurements. The motivation for this work stems from the fact that TRMS behaviour, in certain aspects, resembles that of a helicopter. TRMS is a laboratory setup designed for control experiments by Feedback Instruments Ltd. It gained wide popularity among the control system community because of the difficulties involved in performing direct experiments with helicopters.

### 1.2OBJECTIVES

The main objective of designing a controller for Twin Rotor MIMO system is to provide a platform through which flight of Helicopter can be controlled.

- Modelling of TRMS
- Nonlinear modelling of TRMS
- Converting nonlinear model to Quasi-LPV model.
- Design of Controller and Observer for TRMS
- Quasi LPV controller and Observer design.
- Verifying performance of TRMS LPV model with LPV controller and observer (MATLAB/SIMULINk).
- Implementation of LPV control on real time system.

### 1.3 SYSTEM DESCRIPTION

TRMS is a laboratory set-up designed for control experiments by Feedback Instruments Ltd. As the name indicates it consist of two rotors which are perpendicular to each other and joined by a beam pivoted on its base. In

Helicopter, controlling is done by changing the angle of both rotors, while in TRMS it is done by varying the speed of rotors.

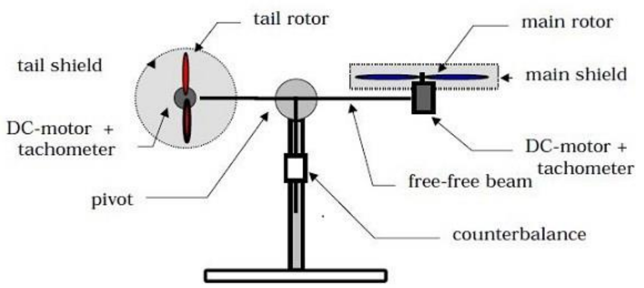


Fig 1.1 Mechanical model of TRMS

It is a multivariable, nonlinear and strongly coupled system, with degrees of freedom on the pitch (vertical) and yaw (horizontal) angle. Both propellers are driven by DC motor and by changing the voltage supplied to beam, rotational speed of propellers can be controlled. For balancing the beam in steady state, counterweight is connected to the system. There is cross-coupling between Main and Tail rotor.

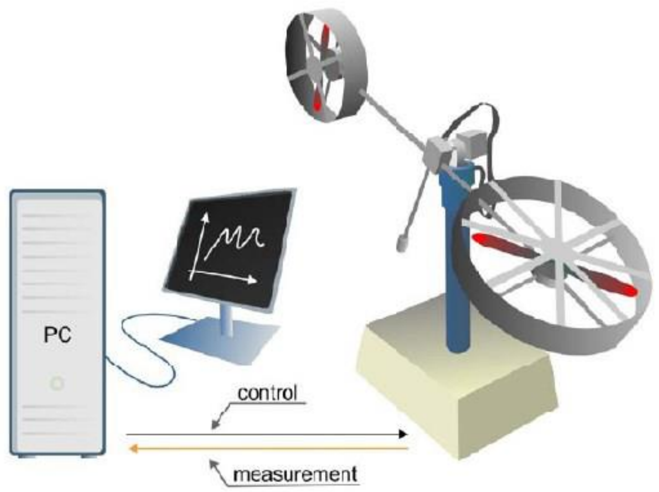


Fig. 1.2 TRMS control system

The complete unit is attached to the tower which ensures safe helicopter control experiments. The electrical unit is placed under the tower which is responsible for communication between TRMS and PC. The electrical unit is responsible for transfer of measured signal by sensors to PC and transfer of control signal via I/O card. The mechanical and electrical unit provide complete control system setup for TRMS as shown in Fig. 1.2.

## 2.1 Nonlinear Modelling of TRMS

The strategy to describe the TRMS would be to split the system into simpler subsystems: the DC-Motors, the propellers and the beam. The first two have independent dynamics, that is, the main motor does not affect the behaviour of the tail motor, and vice-versa. The same is true for the propellers. On the other hand, the dynamics of the beam is strongly non-linear with the presence of interaction phenomena among the horizontal and the vertical dynamics.

### 2.1.1 Modelling of main and tail DC motor

The TRMS possesses two permanent magnet D.C motors, one for the main propeller and the other for the tail. The motors are identical, but with different mechanical loads.

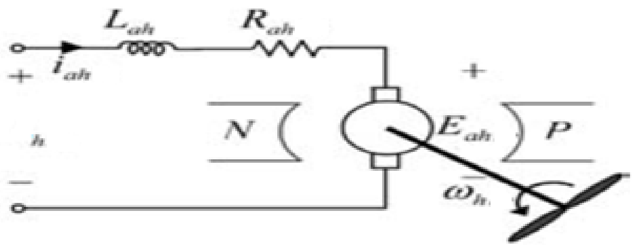


Fig. 2.1 Circuit diagram of the tail DC motor

By neglecting dynamics of current and nonlinear differential equation for tail dc motor is obtained as given in equation.

$$\frac{d\omega_h}{dt} = \frac{K_aK_1}{J_{tr}R_a}u_h - \left(\frac{B_{tr}}{J_{tr}} + \frac{K_a^2}{J_{tr}R_a}\right)\omega_h - \frac{f_{1(\omega_h)}}{J_{tr}}$$

$$\text{Where } f_{1(\omega_h)} = \begin{cases} k_{thp}\omega_h^2 \ if \omega_h \geq 0 \\ -k_{thn}\omega_h^2 \ if \omega_h < 0 \end{cases}$$

Similarly differential equation of main dc motor is obtained as

$$\frac{d\omega_v}{dt} = \frac{K_aK_1}{J_{mr}R_a}u_v - \left(\frac{B_{mr}}{J_{mr}} + \frac{K_a^2}{J_{mr}R_a}\right)\omega_v - \frac{f_{4(\omega_v)}}{J_{mr}}$$

$$\text{Where } f_{4(\omega_h)} = \begin{cases} k_{tvp}\omega_h^2 \ if \omega_h \geq 0 \\ -k_{tvn}\omega_h^2 \ if \omega_h < 0 \end{cases}$$

### 2.1.2 Modelling of vertical beam dynamics

The mathematical model of vertical beam is derived by applying Newton's second law of motion in vertical plane of system shown in Fig. 2.2.

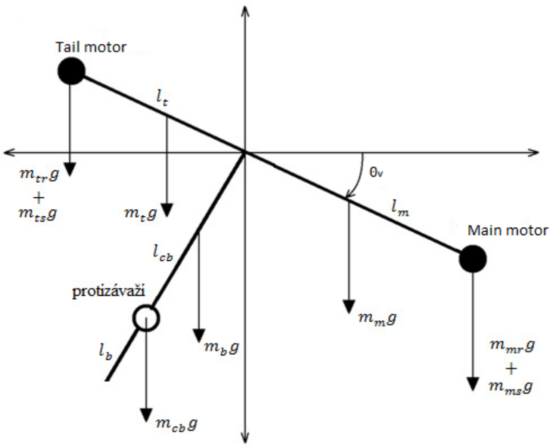


Fig. 2.2. Schematic front view of TRMS with gravitation forces

The mathematical model of the vertical dynamics of the system

$$\frac{dS_v}{dt} = \frac{M_v}{J_v}$$

$$= \frac{l_m f_5(\omega_v) + g\left((K_A - K_B)\cos\theta_v - K_C \sin\theta_v\right) - k_{ov}\Omega_v + k_g \Omega_h f_5(\omega_v)\cos\theta_v - \Omega_h{}^2 K_H \sin\theta_v \cos\theta_v}{J_v}$$

### 2.1.3 Modelling of horizontal beam dynamics

The mathematical model of horizontal beam is derived by applying Newton's second law of motion in horizontal plane of system Fig. 2.3.

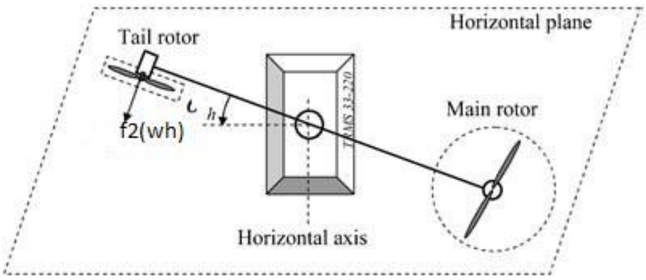


Fig. 2.3 Schematic top view of TRMS

The mathematical model of the horizontal dynamics of the system

$$\frac{dS_h}{dt} = \frac{M_h}{J_h}$$

$$= \frac{l_t f_2(\omega_h)\cos(\theta_v) - k_{oh}\Omega_h - f3(\theta_h) + f6(\theta_v)}{K_D \cos^2(\theta_v) + K_E \sin^2(\theta_v) + K_F}$$

### 2.2 PROBLEM DEFINITION

From the nonlinear model of the system inputs, states, input and output vectors are obtained as below

- System input vector: $u = [uh\ uv]$
- System state vector: $x = [\omega h\ \Omega h\ \theta h\ \omega v\ \Omega v\ \theta v]$
- System output vector :[ $\theta h\ \theta v$ ]

The step responses of the TRMS for yaw and pitch angles without any controller are shown in Fig. 2.4 and 2.5.

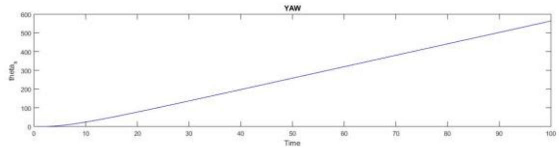


Fig. 2.4 Step response – Yaw

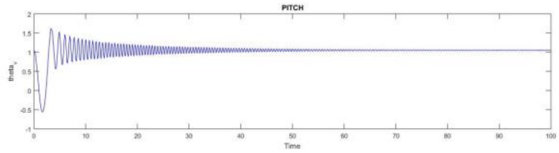


Fig. 2.5 Step response – Pitch

Responses show that the system is unstable without any controllers because the yaw response increases without any bound. Instability could be contributed by nonlinearity, cross coupling etc. Thus for TRMS a controller needs to be developed

### 3. QUASI-LPV REPRESENTATION OF TRMS

The nonlinear model of TRMS expressed is converted to quasi-LPV form following parameter non-linear embedding approach proposed by Kwiatkowski

$$\begin{bmatrix} \dot{\omega}_h(t) \\ \dot{\Omega}_h(t) \\ \dot{\theta}_h(t) \\ \dot{\omega}_v(t) \\ \dot{\Omega}_v(t) \\ \dot{\theta}_v(t) \end{bmatrix} = A(\psi(t)) \begin{bmatrix} \omega_h(t) \\ \Omega_h(t) \\ \theta_h(t) \\ \omega_v(t) \\ \Omega_v(t) \\ \theta_v(t) \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22}(p(t)) \\ 0 & 0 \\ 0 & b_{42} \\ b_{51} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_h(t) \\ u_v(t) \end{bmatrix}$$

where

$$\psi(t) = [a_{11}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}, a_{44}, a_{52}, a_{54}, a_{56}]^T$$

is the vector of varying parameters scheduled by

$$p(t) \models [\theta_h(t), \theta_v(t), \omega_h(t), \omega_v(t), \Omega_h(t)]^T$$

### 3.1 Open Loop Simulation Results On LPV TRMS Model

The implementation of the simulation model for the TRMS system is done using the Matlab/Simulink environment. The Matlab and Simulink environment are integrated into one entry, enabling to analyse, calculate, simulate and revise the models in either environment at any point.. Open loop simulation results obtained from LPV TRMS model are shown in Fig. 3.1 and Fig 3.2 for yaw and pitch responses respectively.

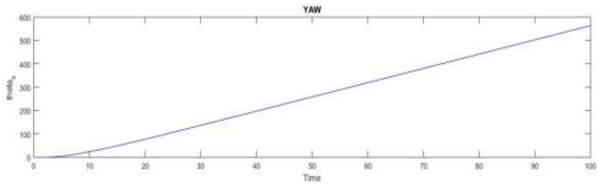


Fig 3.1 LPV Model Step Response: Yaw

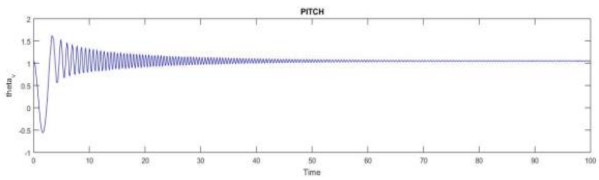


Fig 3.2 LPV Model Step Response: Pitch

Comparing step response obtained from both nonlinear model and LPV model it is evident that both are similar. So it is inferred that by converting nonlinear model to LPV model, the nonlinear dynamics of system is not changed. So controller designed based on this LPV model will be more effective than that of linearized models.

## 4. DESIGN OF LPV CONTROLLER AND OBSERVER

The Quasi-LPV system represented by

$$x(k+1) = Ad\ (\psi)x(k) + Bdu(k)$$
$$y(k) = Cx(k)$$

can be controlled by a state feedback controller with tracking reference input. The control law can be expressed as:

$$u(k) = ur\ (k) + kd(\psi(k))(\hat{x}(k) - xr\ (k))$$

The state reference $xr\ (k)$ and feed-forward control action $ur(k)$ corresponds to an equilibrium point for reference r(k). Matrix $kd\ (\psi)$ is gain of LPV controller. LPV controller matrix $kd(\psi)$ can be designed using LMI pole placement approach. Therefore an LPV state observer is used to provide state estimation. The observer system can be represented as:

$$\hat{x}(k+1) = Ad\ (\psi)\hat{x}(k) + Bdu(k) + Ld(\psi)(y(k) - \hat{y}(k))$$

$$\hat{y}(k) = C\hat{x}(k)$$

$\hat{x}(k)$ and $\hat{y}(k)$ are estimated state and output. The matrix $Ld(\psi)$ is the gain of LPV state observer which can be obtained using LMI pole placement approach.

### 4.1 LMI POLE PLACEMENT

Main motivation for seeking LMI pole placement approach is that stability is guaranteed and a satisfactory transient response can be ensured. Definition for LMI Regions: A subset $\mathcal{D}$ of complex plane is called LMI region if there exist a symmetric matrix $\alpha = [\alpha kl\ ]\epsilon\mathbb{R}m\times m$ and matrix $\beta = [\beta kl\ ]\epsilon\mathbb{R}m\times m$ Є R such that

$$\mathcal{D} = \{z\epsilon\mathbb{C}\colon f\mathcal{D}(z) < 0\}$$

with

$$f\mathcal{D}(z) \coloneqq \alpha + z\beta + \bar{z}\beta T$$

Using Gutman's theorem for LMI region, pole location in a given LMI region can be characterized in terms of m x m block matrix given by:

$$Z\mathcal{D}(Y,X) \coloneqq \alpha\otimes X + \beta\otimes(YX) + \beta T\otimes(YX)T$$

Gutman's theorem for stability: A matrix Y is D-stable if and only if there exists a symmetric matrix X>0 such that

$zD(Y,X)$<0
$zD(Y,\ \ X)$ and $f\mathcal{D}(z)$ are related by substitution$(X,YX,XYT\ ) \longleftrightarrow (1,z,\bar{z})$.

$$f_{\mathcal{D}}(z) = \begin{pmatrix} S_{min} - (z+\bar{z})/2 & 0 \\ 0 & (z+\bar{z})/2 - S_{max} \end{pmatrix}$$

Such that

$$z_{\mathcal{D}}(Y,X) = \begin{pmatrix} S_{min}X - (YX+XY^T)/2 & 0 \\ 0 & (YX+XY^T)/2 - S_{max} \end{pmatrix} < 0$$

### 4.2 DESIGN OF CONTROLLER FOR TRMS

Consider a discrete time LPV system described by

$$x(k+1) = Ad\ (\psi d)x(k) + Bdu(k)\ ($$
$$y(k) = Cx(k)$$

under state feedback control law $u(k) = kd\ (\psi d)x(k)$.The problem to be solved consist in finding a state feedback gain $kd$scheduled by $\psi d$ that places the closed loop poles in some LMI region $\mathcal{D}$ with characteristic function

$$f\mathcal{D}(z) \coloneqq \alpha + z\beta + \bar{z}\beta T$$

Thus pole-placement in the intersection between the disk of radius $rk$ and center $(-qk$ , 0 and vertical strip with extreme values $Smin$ and $Smax$ is obtained by solving the following system of LMIs:

$$Q = diag(Q_1, Q_2, Q_3) < 0$$

$$X>0$$

Where

$$Q_1 = \begin{pmatrix} -r_k X & q_k X + A_d X + B_d \Gamma \\ q_k X + X A_d{}^T + \Gamma^T B_d^T & -r_k X \end{pmatrix}$$

$$Q_2 = S_{min} X - \frac{1}{2}(A_d X + X A_d^T + B_d \Gamma + \Gamma^T B_d^T)$$

$$Q_3 = \frac{1}{2}(A_d X + X A_d^T + B_d \Gamma + \Gamma^T B_d^T) - S_{max} X$$

Once this system of LMIs are solved the controller gains $kd = \Gamma X{-}1$

## 4.3 DESIGN OF OBSERVER FOR TRMS

Consider a discrete-time LPV state observer described by

$$\hat{x}(k+1) = Ad\,(\psi)\hat{x}(k) + Bdu(k) + Ld(\psi)(y(k) - \hat{y}(k))$$
$$\hat{y}\,(k) = C\hat{x}(k)$$

The problem to be solve consists of finding gain such that it places closed loop poles of observer in an LMI region with characteristic function . $D$-detectability of the pair $(Ad\,(\psi), C)$ is equivalent to $D$-stabilizability of of $(AdT\,(\psi), CT\,)$. Thus problem reduces in computing a state feedback gain $Ld$ and a Lyapunov matrix X>0 such that

$$zD(AdT + CTLdT\,, X) < 0$$

Thus pole-placement in the intersection between the disk of radius $rL$and center $(-qL$ , 0 and vertical strip with extreme values $Smin\ L$ and $Smax\ L$is obtained by solving the following system of LMIs:

$$R = diag(R_1, R_2, R_3) < 0$$

$$X>0$$

Where

$$R_1 = \begin{pmatrix} -r_L X & q_L X + A_d{}^T X + C^T \Gamma \\ q_L X + X A_d + \Gamma^T C & -r_L X \end{pmatrix}$$

$$R_2 = S_{\min L} X - \frac{1}{2}(A_d{}^T X + X A_d + C^T \Gamma + \Gamma^T C)$$

$$R_3 = \frac{1}{2}(A_d{}^T X + X A_d + C^T \Gamma + \Gamma^T C) - S_{\max L} X$$

## 4.4 PERFORMANCE ANALYSIS OF LPV CONTROLLER

A controller needs to be evaluated for its robustness. Here parameter variations are already taken care in the controller design.

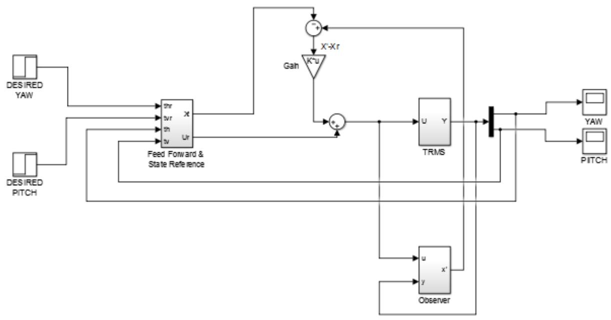


Fig 4.1 Simulation Framework in Simulink

### 4.4.1 Setpoint Variation

Steps of 0.2 and 0.25 rad respectively are taken as desired yaw and pitch path and response to step input and corresponding control signals

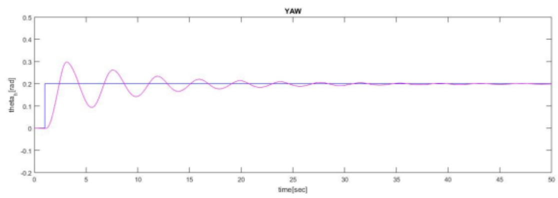


Fig. 4.2 Yaw response to step input

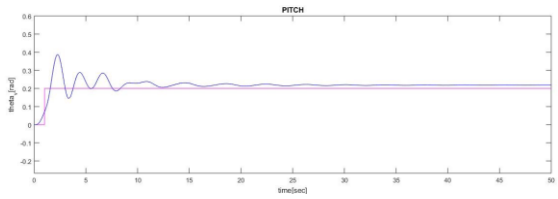


Fig 4.3 Pitch response to step input.

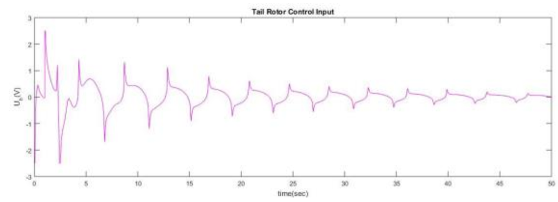


Fig 4.4 Tail motor control input

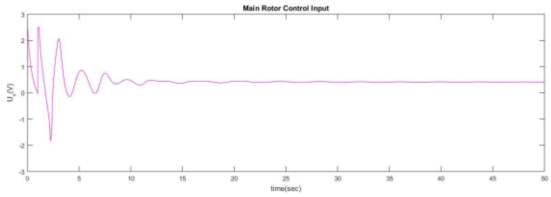

Fig 4.5 Main motor control input

Results shows that controller provides satisfactory results for tracking of step path

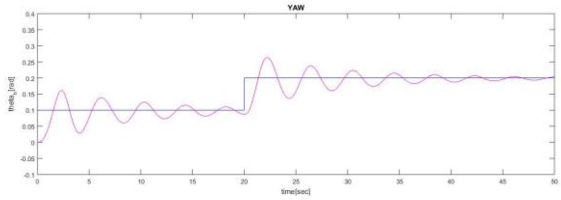

Fig 4.6 Yaw response for combination step wave input

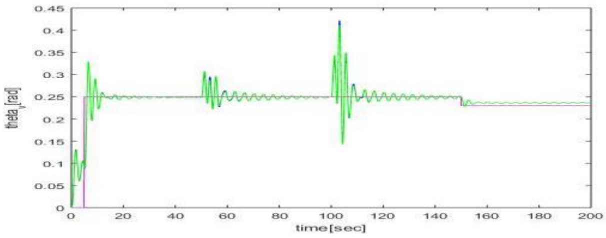

Fig 4.7 Pitch response for combination step wave input

### 4.4.2 Disturbance Rejection

In this section performance of the controller is analysed with various disturbances at pitch, yaw and the corresponding control inputs.

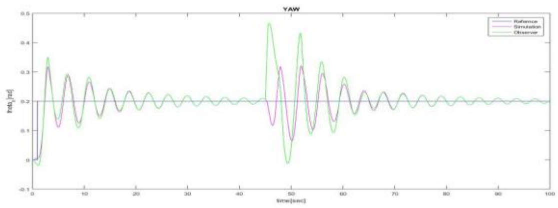

Fig 4.8 Yaw response for step input with a pulse disturbance

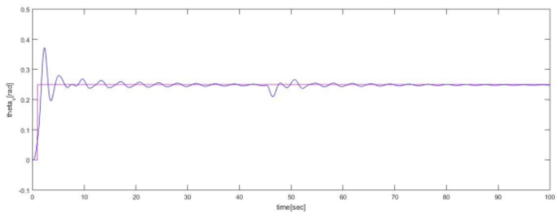
Fig 4.9 Pitch response for step input with a pulse disturbance

### 4.5 REAL TIME IMPLEMENTATION OF LPV CONTROLLER ON TRMS

The LPV controller designed has been implemented on the nonlinear real time TRMS setup in the laboratory and analysed performance of the controller

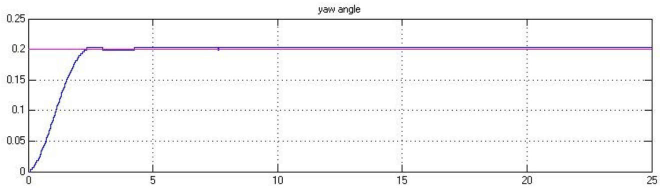

Fig 4.10 Yaw response to step input of amplitude 0.2 rad.

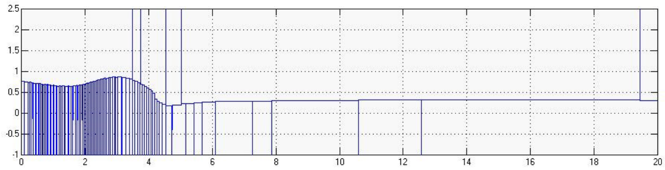
Fig 4.11 Tail rotor control input

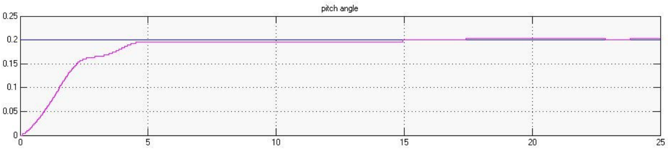

Fig 4.12 Pitch response to step input of amplitude 0.2 rad.

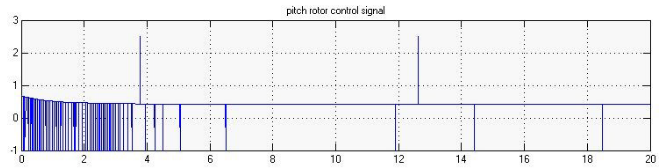

Fig 4.13 Main rotor control input

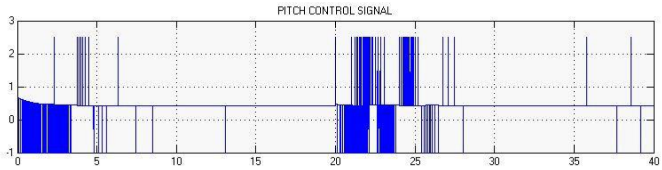

Fig 4.14 Main rotor control input.

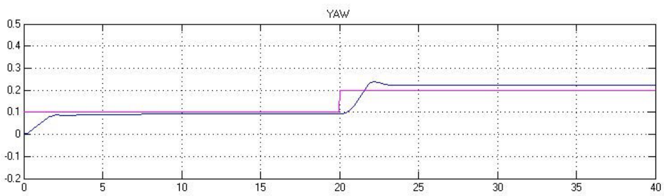

Fig 4.15 Yaw response to combination of step wave input.

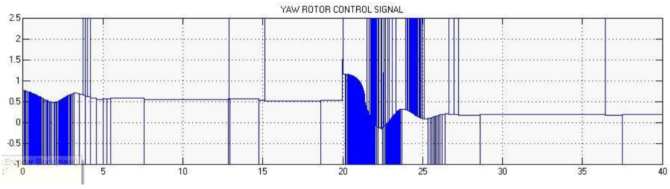


Fig 4.16 Tail rotor control input

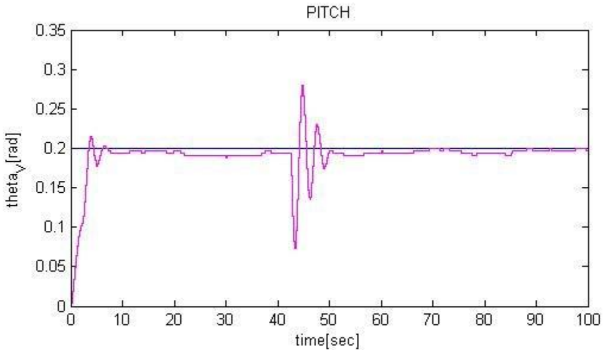


Fig 4.17 Pitch response with an external disturbance

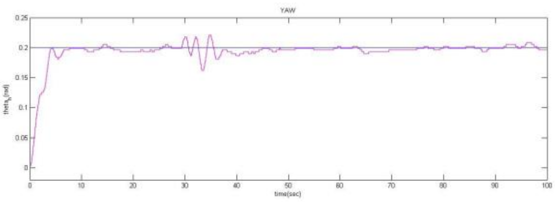


Fig 4.18 Yaw response with an external disturbance

## 5.1 CONCLUSION

In this journal, Linear Parameter Varying (LPV) modelling and controller design for highly nonlinear and cross coupled Twin Rotor Multi Input Multi Output System (TRMS) is investigated. Quasi-LPV model is obtained from the nonlinear dynamical model. This model is validated against simulation results obtained from the nonlinear model. Then using LPV pole placement approach based on LMI regions, LPV state feedback controller is designed for the LPV system model. Effectiveness of controller is validated in simulation environment. In the next phase LPV state feedback observer is designed using LPV pole placement approach based on LMI regions and validated the performance of observer with nonlinear model of the system. Performance of the LPV controller is evaluated for its robustness by checking output response of nonlinear model in presence of variation in set point and with presence of disturbances. At the final stage of work the designed LPV controller is tested on Real time lab setup of TRMS available in laboratory and effectiveness of the approach is validated. In real time environment a better performance is obtained with low overshoot and settling time than simulation. The system stability and desired performance are guaranteed with the proposed LPV controller even in the presence of disturbances and set point variations.

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