Study of Semi-Circular Beam Used in C-Arm Based Mobile X-Ray Units in Medical Healthcare Sector

Khan Asif, Dr. D.N Raut, Kamat Sumukh

M.Tech Student, Veermata Jijabai Technological Institute, Mumbai, Maharashtra, India
Professor, Veermata Jijabai Technological Institute, Mumbai, Maharashtra, India
Senior Engineer, Siemens Healthcare Private Limited, Goa, India

---***---

Abstract - *The analysis of curved beams (C-Arcs) used in C-Arm based mobile X-Ray Machine units is discussed in this paper. Three dimensional model of C-Arc was created using UGX/NX Software. ANSYS v15.0 is used for the Finite Element Analysis (FEA) and analytical expressions are used to calculate the deflection. MathCAD Prime 3.1 software is used to formulate and solve the equations. Load distance from the profile face is accounted and the deflection expression is derived to study the deflection. The analytical and FEA results are compared. It is observed that the analytical results are close enough to the ANSYS results thereby validating the formula of deflection.*

*Key Words***:** C-Arm, Semi-Circular Curved Beams, Deflection, Castigliano's Theorem, Finite Element Analysis

1. INTRODUCTION

Since the introduction of the first C-arm in 1955 the technology has advanced rapidly. Today, Specialists in fields such as surgery, orthopaedics, traumatology, vascular surgery and cardiology use C-arms for intraoperative imaging. A mobile C-arm is a medical imaging device that is based on X-ray technology and can be used flexibly within a clinic. The name "C-Arm" was derived from the literal shape of the load bearing arm of the device which in fact looks like a large letter "C". Thus the load bearing curved beam is termed as 'C-Arc'. C-Arc is analyzed using Castigliano's theorem and results are compared with Finite Element Analysis. One end of the C-Arc is clamped and force (parallel to the plane of C) acts at the free end.

C-Arc is three dimensional and it involves bending moment, torque and shear. A general solution covering all cases would be complicated. Another complication is introduced by the fact that for certain cross-sections of beam, particularly those of I-Form, there are bending moments induced in parallel plane of the axis of the beam which cannot readily be determined by the use of statics.

The load to C-Arc attached has its center of gravity (cog) away from the face, thus the effect of the cog is also to be studied.

Tore Dahlberg 2004[1] investigated the procedure to calculate deflections of curved elliptical beams using Castigliano's second theorem and compared the beam deflections with handbook formulae.

Yogesh Gangamwar et. al. 2016 [2] compared the deflections of curved beam obtained by using Castigliano's theorem with ANSYS results and found that the overall results lie within the difference of only 7-9%.

B.S. Ramachandra Rao et. al. [3] investigated the stresses in finite curved beams under general equilibrium loading. They described the method to obtain the stresses near a boundary where the load is applied.

Now in this paper the C-Arc is analyzed for the load acting at one of the free ends and parallel to the plane of the arc also the results are compared with finite element analysis.

1.1 Definitions

F = Load (N)

Rc = Radius of curvature (mm)

Rn = Radius of neutral axis of the C-Arc (mm)

x = Distance of cog of load from the free end (mm)

Fa = Axial force component (N)

Fs = Shear force component (N)

M = Moment due to force F (N)

A = Area of cross-section (mm2)

e = Eccentricity (Rc-Rn) (mm)

E = Modulus of elasticity (MPa)

G = Shear modulus (MPa)

C = Shape factor

2. CASTIGLIANO'S THEOREM

Castigliano's theorem states that when force F_i acts on elastic system subject to small displacement δ_i corresponding to the force, in the direction of the force, is equal to the partial derivative of the total strain energy U with respect to that force.

Mathematically,

$$\delta_i = \frac{\delta U}{\delta F_i}$$

2.1 Methodology

The analytical method of determining the deflection by using Castigliano's theorem is described below. The force F acts at a distance of x from the free end. An element bound by angle θ is considered and the force is resolved into axial and shears components.

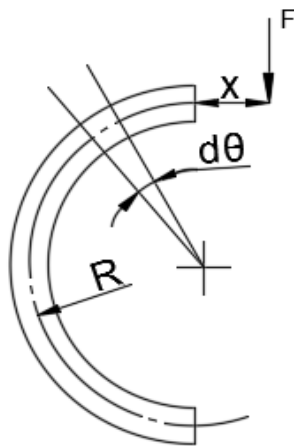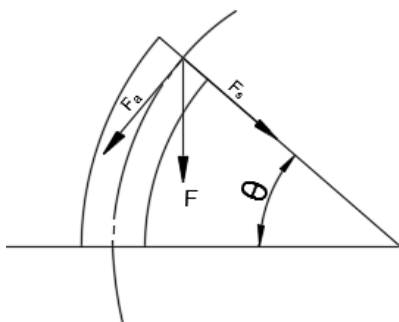**Fig -1**: C-Arc (semi-circular beam of uniform cross section)

Fig -2: C-Arc element bound by angle θ

Moment due to Force (F): M = F (Rsin θ + x)

Axial Force Component: F_a = F sin θ

Shear Force Component: F_s = F cos θ

Analytical calculations follow these three stages:

A) Calculation of strain energy by integration from 0 to π i.e. because of semi-circular shape.

B) Finding deflection in the direction of Force by the partial derivatives of the strain energies with respect to the force.

A)   Strain Energy

1. Due to Bending Moment

$$U_1 = \int \frac{M^2}{2AeE} d\theta$$

$$U_1 = \int_0^\pi \frac{\{F(R\sin\theta + x)\}^2}{2AeE} d\theta$$

2. Due to Axial Force

$$U_2 = \int \frac{Fa^2 R}{2AE} d\theta$$

$$U_2 = \int_0^\pi \frac{(F\sin\theta)^2}{2AE} d\theta$$

3. Due to Moment caused by Axial Force

$$U_3 = -\int \frac{MFa}{AE} d\theta$$

$$U_3 = -\int_0^\pi \frac{\{F(R\sin\theta + x)(F\sin\theta)\}}{AE} d\theta$$

4. Due to Shear Force

$$U_4 = \int \frac{C \, Fs^2 R}{2AG} \, d\theta$$

$$U_4 = \int_0^\pi \frac{(F\cos\theta)^2}{2AG} \, d\theta$$

B) Deflection

By Castigliano's Theorem:

$$\delta = \frac{\delta U_1}{\delta F} + \frac{\delta U_2}{\delta F} + \frac{\delta U_3}{\delta F} + \frac{\delta U_4}{\delta F}$$

Solving the equation gives,

$$\delta = \frac{F}{AeE}\left(\frac{\pi R^2}{2} + \pi x^2 + 4xR\right) - \frac{\pi FR}{2AE} + \frac{\pi CFR}{2AG}$$

2.2 ANALYTICAL CALCULATIONS

Following results are obtained using the deflection formula obtained by Castigliano's theorem.

Data: A=1.25x 10^4 mm^2 , e=10 mm , R=650 mm, E=68.3x 10^3 MPa, G=25.8x10^3 MPa , x=180mm, C=85

Table -1: Load vs. Deflection table (x =180 mm)

SN	Load (kg)	Force (N)	Analytical Deflection (mm)
Analytical Calculations		
1	108	1059	0.437
2	128	1255	0.518
3	148	1452	0.599
4	168	1648	0.679
5	188	1844	0.760

Table -2: Load Distance vs. Deflection table (Load = 108 kg)

SN	Load distance (mm)	Analytical Deflection (mm)
1	75	0.392
2	100	0.402
3	125	0.412
4	150	0.423

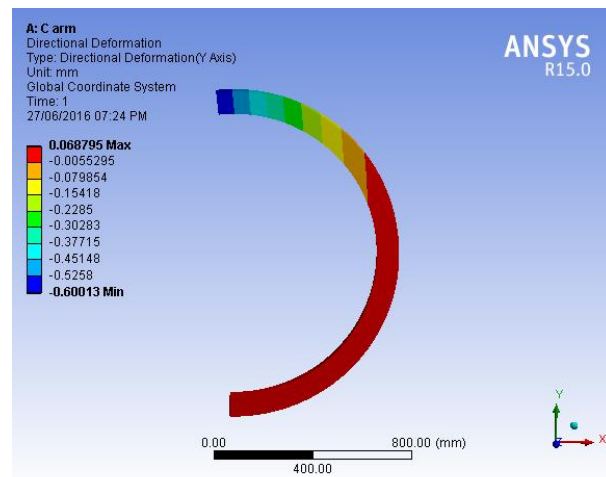| 5	180	0.437
6	210	0.451
7	250	0.471

2. FINITE ELEMENT ANALYSIS

The model is simulated in ANSYS and the directional deformations are obtained.
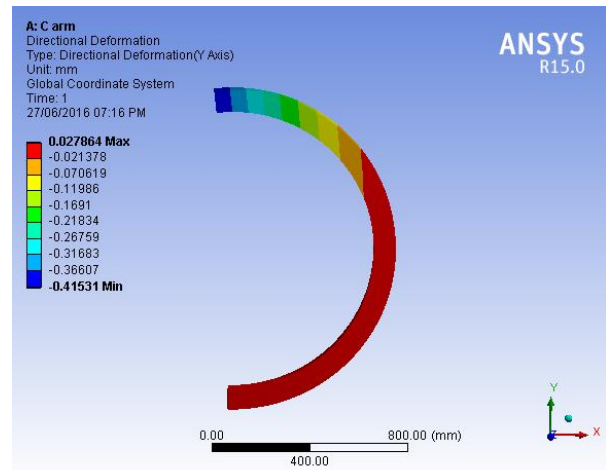
Fig -3: Deflection in Force direction for load = 108 kg

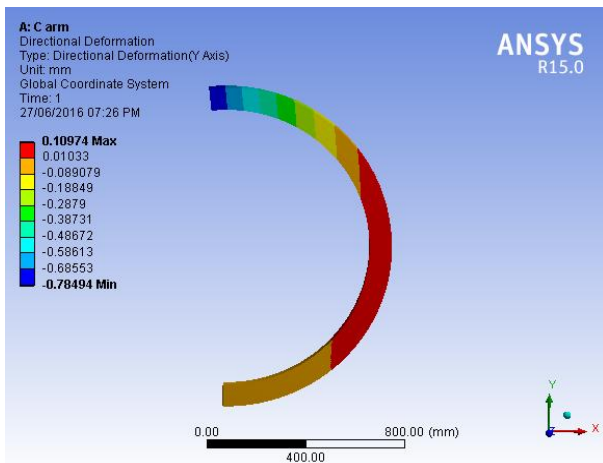**Fig -4**: Deflection in Force direction for load = 128 kg

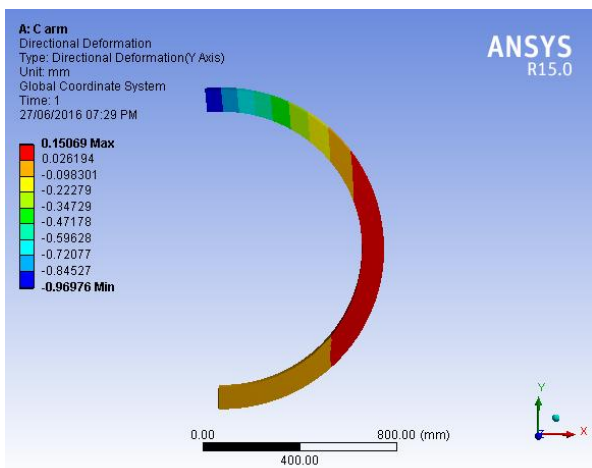**Fig -5**: Deflection in Force direction for load = 148 kg

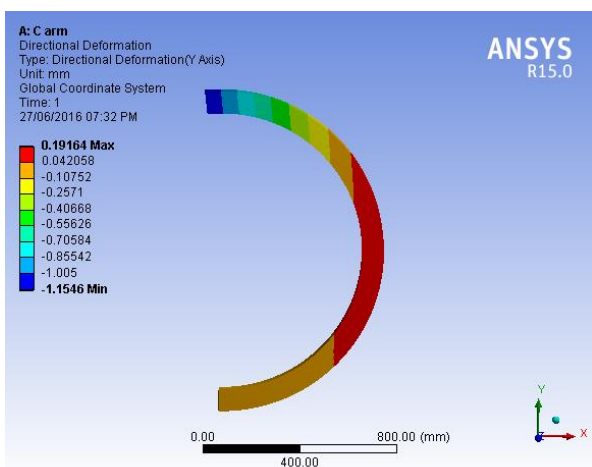**Fig -6**: Deflection in Force direction for load = 168 kg

Fig -7: Deflection in Force direction for load = 188 kg

3. RESULTS

The results obtained by analytical calculation and ANSYS are compared.

Table -3: Deflection results: Analytical vs. ANSYS

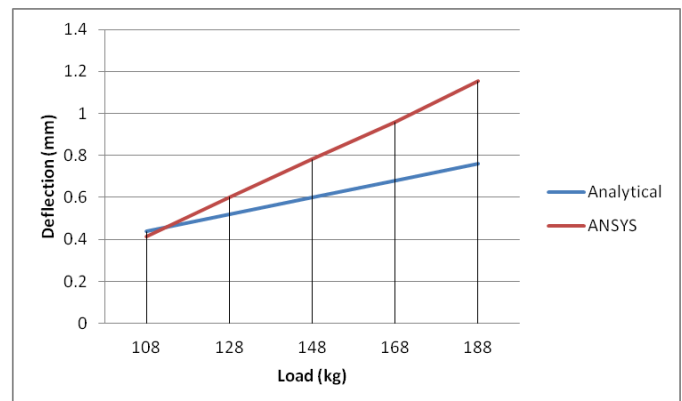| SN	Load (kg)	Deflection (mm)	
		Analytical	ANSYS
1	108	0.437	0.415
2	128	0.518	0.6
3	148	0.599	0.78
4	168	0.679	0.96
5	188	0.760	1.154

Chart -1: Result comparison graph

4. CONCLUSIONS

Castigliano's Theorem has been used to determine the deflection in the C-Arc also the Finite Element Analysis is done to validate the results. The load distance from the face of the C-Profile is also accounted in the calculations and Table-3 shows that it has not much of influence on the deflection of the C-Arc. It is observed that the analytical results and ANSYS results are close enough. ANSYS results show the maximum deformation zone and these zones are to be externally supported for stiffening.

REFERENCES

[1]   Tore DahlbergD, "Procedure to calculate deflections of curved beams," Int J. Engng Ed., vol. 20, pp. 503-513, 2004.

[2]   Yogesh Gangamwar, Sumit Chate, Makarand Bhandare, Vinit Deo and H.N Deshpande, "Analytical, Experimental determination of deflection of curved beams and its validation" International Journal of Innovative Research

in Science, Engineering and Technology, vol. 05, Jun. 2016.

[3] P.V.B.A.S. Sarma, B.S Ramachandra Rao and Gopalacharyulu. S., "Eigen function solutions for plane stress problems of curved beams" Int. J. Engng. Sci., pp. 149-159, 2016.

[4] Ben T. Yeh, Donjoo Kim and John L. Wilson, "Evaluation of Displacements and Stresses in Horizontally Curved Beams" ATLSS Reports, pp. 74, 2006.

[5] D. Bharadwaj and A. Purushotham, "Numerical Study on Stress Analysis of Curved Beams" International Journal of Mechanical Engineering and Technology, vol. 06, pp. 21-27, Jul. 2016.

[6] Tadashi Horibe and Kotaro Mori, "In-Plane and Out-of-Plane Deflection of J-Shaped Beam" Journal of Mechanical Engineering and Automation, vol. 5, pp. 14-19, 2015.

BIOGRAPHIES

Khan Asif Amin
M.Tech Production Engineering
Veermata Jijabai Technological Institute
Mumbai

Dr. D. N. Raut
Professor
Department of Production Engineering
Veermata Jijabai Technological Institute
Mumbai

Kamat Sumukh
Senior Engineer
Department of Research & Development
Siemens Healthcare Private Limited
Goa