

# Simulation of integrating systems for direct synthesis approach using PID controller techniques

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**Abstract** -A PID controller is designed using direct synthesis method for various forms of integrating systems. In this method characteristic equation of the integrating system and PID controller with filter with the desired characteristic equation are compared. The desired characteristic equation consists of multiple poles which are placed at the same desired location. To achieve the desired robustness the tuning parameter of the PID controller is adjusted. Tuning rules for process parameter are given for various forms of integrating systems. By specifying  $M$  value tuning parameter can be selected. The proposed controller design method is applied to the nonlinear model equations of jacketed CSTR and various transfer function models to show its effectiveness and applicability.

**Key Words:** Tuning, PID controller, Integrating systems, Time delay systems, Direct synthesis method

## 1. INTRODUCTION

The integrating process can be defined as: The process which consists of at least one pole at the origin. The integrating systems are classified as pure integrating process with time delay (PIPTD), double integrating process with time delay (DIPTD), stable/unstable first order plus time delay integrating process (FOPTDI), stable/unstable first order plus time delay integrating process with a positive/negative zero etc. Bioreactors, boiler steam drum, paper drum dryer cans, level control and composition control loop in distillation column, jacketed CSTR these are some examples of integrating processes.

Many more new methods are proposed in the literature to tune PID controllers for integrating systems. They are Empirical method [1], Internal model control method [2], Equating coefficient method [3], Stability analysis method [4], Two degree of freedom control scheme [5], frequency domain method [6]. It is to be observed that there are some advantages and disadvantages in tuning the PID controller by using these methods. Ajmeri and Ali [7] has considered parallel control structure that decouples servo problem

and regulatory problem for PIPTD, DIPTD and SFOPTDI systems. Shamsuzzoha [8] has reported analytical tuning rules for a PI/PID controller. Jin and Liu [9] have proposed 2DOF IMC with an extra set point filter for PIPTD, DIPTD AND FOPTDI. Viteckova and Vitecek [10] implemented conventional PID controller using multiple dominant pole placement method for FOPTD system with an integrator without time delay and pure integrator plus time delay system. Krishna et al. [11] applied the proposed method to nonlinear model of jacketed CSTR.

## 2. CONTROLLER DESIGN BY DIRECT SYNTHESIS METHOD

Consider a general system given by open loop transfer function:

$$G_p = \frac{k_p (1 + gs) e^{-Ls}}{s(\tau s + p)} \quad (1)$$

Case 1: If  $\tau = 0$ ,  $p = 1$  and  $g = 0$ , the process is a pure integrating system.

Case 2: If  $\tau = 1$ ,  $p = 0$  and  $g = 0$ , the process is a double integrating system.

Case 3: If  $p = 1$  and  $g = 0$ , the process is a integrating stable first order plus time delay system.

Case 4: If  $p = -1$  and  $g = 0$ , the process is a integrating unstable first order plus time delay system.

Case 5: If  $p = 1$  the process is an integrating stable first order plus time delay system with a positive/negative zero. (P negative for positive zero and positive for negative zero).

Case 6: If  $p = -1$  the process is an integrating unstable first order plus time delay system with a positive/negative zero. (P negative for positive zero and positive for negative zero).

The PID controller with a lead lag filter is given by:

$$G_c = k_c \left( 1 + \frac{1}{\tau_I s} + \tau_D s \right) \left( \frac{\alpha s + 1}{\beta s + 1} \right) \quad (2)$$

The characteristic equation is given by:

$$1+GpGc=0 \tag{3}$$

$$1+\frac{k_p(1+gs)e^{-Ls}}{s(\tau s+p)}k_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\left(\frac{\alpha s+1}{\beta s+1}\right)=0$$

(4)

Use first order Pade's approximation in the above equation:

$$1+\frac{k_p k_c(1+gs)(\tau_D \tau_I s^2+\tau_I s+1)(1-0.5Ls)(\alpha s+1)}{\tau_I s^2(\tau s+p)(1+0.5Ls)(\beta s+1)}$$

=0

(5)

On simplifying and rearranging Eq. (5), we obtain the following equation:

$$\left(\frac{0.5L\beta\tau\tau_I-0.5Lg\alpha\tau_D\tau_I}{k_p k_c}\right)s^5 + \left(\frac{\beta\tau\tau_I+\frac{0.5L\tau\tau_I}{k_p k_c}+\frac{0.5Lp\beta\tau_I}{k_p k_c}-0.5L\alpha\tau_D\tau_I-0.5Lg\tau_D\tau_I+g\alpha\tau_D\tau_I-0.5Lg\alpha\tau_I}{k_p k_c}\right)s^4 + \left(\frac{\tau\tau_I+\frac{p\beta\tau_I}{k_p k_c}+\frac{0.5Lp\beta\tau_I}{k_p k_c}+\alpha\tau_D\tau_I-0.5L\tau_D\tau_I-0.5L\alpha\tau_I+g\tau_D\tau_I-0.5Lg\tau_I+g\alpha\tau_I-0.5Lg\alpha}{k_p k_c}\right)s^3 + \left(\frac{p\tau_I+\tau_D\tau_I+\alpha\tau_I-0.5L\tau_I}{k_p k_c}-0.5L\alpha+g\tau_I-0.5Lg+\alpha g\right)s^2 + (\tau_I+\alpha-0.5L+g)s+1=0 \tag{6}$$

The desired characteristic equation is given by:

$$(\lambda s+1)^5=0 \tag{7}$$

After expanding the Eq. (7) we get the following equation:

$$\lambda^5 s^5 + 5\lambda^4 s^4 + 10\lambda^3 s^3 + 10\lambda^2 s^2 + 5\lambda s + 1 = 0 \tag{8}$$

On comparing the corresponding coefficients of  $s^5, s^4, s^3, s^2$  and  $s$  in Eq. (6) and Eq.(8), we get the following five equations:

$$\lambda^5 - \left(\frac{0.5L\beta\tau\tau_I-0.5Lg\alpha\tau_D\tau_I}{k_p k_c}\right) = 0 \tag{9}$$

$$5\lambda^4 - \left(\frac{\beta\tau\tau_I}{k_p k_c} + \frac{0.5L\tau\tau_I}{k_p k_c} + \frac{0.5Lp\beta\tau_I}{k_p k_c} - 0.5L\alpha\tau_D\tau_I - 0.5Lg\tau_D\tau_I + g\alpha\tau_D\tau_I - 0.5Lg\alpha\tau_I\right) = 0$$

(10)

$$10\lambda^3 - \left(\frac{\tau\tau_I}{k_p k_c} + \frac{p\beta\tau_I}{k_p k_c} + \frac{0.5Lp\beta\tau_I}{k_p k_c} + \alpha\tau_D\tau_I - 0.5L\tau_D\tau_I - 0.5L\alpha\tau_I + g\tau_D\tau_I - 0.5Lg\tau_I + g\alpha\tau_I - 0.5Lg\alpha\right) = 0$$

(11)

$$10\lambda^2 - \left(\frac{p\tau_I}{k_p k_c} + \tau_D\tau_I + \alpha\tau_I - 0.5L\tau_I - 0.5L\alpha + g\tau_I - 0.5Lg + \alpha g\right) = 0$$

(12)

$$5\lambda - (\tau_I + \alpha - 0.5L + g)s + 1 = 0 \tag{13}$$

The above five equations are solved by numerical methods using MATLAB to obtain the parameters  $k_c, \tau_D, \tau_I, \alpha$  and  $\beta$ . It should be note that the parameters obtained by MATLAB solver may require iteration.

The above procedure leads to very bad performance as compared to the literature reported methods for pure integrating system with time delay. Hence, with third order characteristic equation conventional PID controller is tuned by using direct synthesis method.

The characteristic equation is given by:

$$1+\frac{k_p e^{-Ls}}{s}k_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)=0 \tag{14}$$

The desired characteristic equation is given by:

$$(\lambda s+1)^3=0 \tag{15}$$

By comparing the corresponding coefficients of  $s^3, s^2$  and  $s$  of Eqs. (14) and (15), after rearranging the following equations are obtained for PID parameters:

$$\tau_I = 3\lambda + 0.5L \quad (16)$$

$$\tau_D = \left( \frac{3\lambda^2}{2\tau_I} + 0.25L - \frac{\lambda^3}{L\tau_I} \right) \quad (17)$$

$$k_c = \left( k_p \left( \frac{3\lambda^2}{\tau_I} + 0.5L - \tau_D \right) \right)^{-1} \quad (18)$$

Here, the tuning parameter  $\lambda$  is tuned in such a way to obtain the desired robustness of controller.

### 3. SIMULATION RESULTS

To show the effectiveness of controller, proposed method is tested on various transfer function models (linear and non-linear). The performance is compared in terms of input usage and (TV) and IAE for servo problem.

#### 3.1 Example 1

An example of pure integrator plus time delay is considered.

$$G_p = \frac{0.05e^{-5s}}{s} \quad (19)$$

In this case study  $\lambda$  is tuned to 5.9962, so as to match the Mvalue of 2 as given by reported methods. The PID tuning parameters of proposed method are  $k_c = 3.727, \tau_I = 18.94, \tau_D = 1.89$ . The tuning parameter of Viteckova and Vitecek [1] method are  $k_c = 3.14, \tau_I = 18.66, \tau_D = 1.32$  and Ajmeri and Ali [7] are  $k_c = 3.3209, \tau_I = 20.5750, \tau_D = 1.7705$ . The servo response of PIPTD is shown in Figure 1. From servo response Integral Absolute Error (IAE), Input usage (TV) and overshoot (OS) is calculated and reported in Table 1.

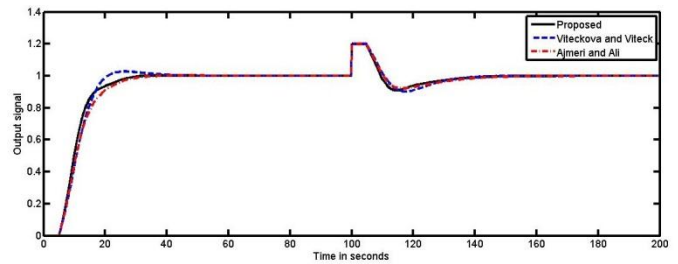


Fig-1: Servo response of the Example 1

Table-1: Performance comparison in terms of IAE,TV and OS

Method	IAE	TV	OS
Proposed	<b>13.8714</b>	<b>3.19</b>	<b>0.002</b>
Viteckova and Vitecek	14.60	1.81	0.027
Ajmeri and Ali	14.92	2.61	0.002

The Table 1 and Figure 1 shows that the proposed method produces better result compared with reported methods in terms of less IAE, TV and OS for servo problem.

#### 3.2 Example 2

An example of double integrator plus time delay is considered.

$$G_p = \frac{e^{-s}}{s^2} \quad (20)$$

In this case study  $\lambda$  is tuned to 2.1351 so as to match the Mvalue of 2 as given by reported methods. The PID tuning parameters of proposed method are  $k_c = 0.1378, \tau_I = 9.7264, \tau_D = 3.8211, \beta = 1.0392$ . The tuning parameter of Jin and Liu [9] method are  $k_c = 0.016, \tau_I = 21.281, \tau_D = 7.255$  and Ajmeri and Ali are  $k_c = 0.0414, \tau_I = 22.48, \tau_D = 7.8415$  [7]. The servo response of DIPTD is shown in Figure 2. From servo response Integral Absolute Error (IAE), Input usage (TV) and overshoot (OS) is calculated and reported in Table 2.

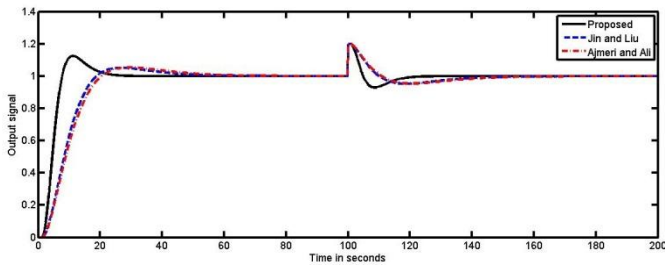


Fig-2: Servo response of the Example 2

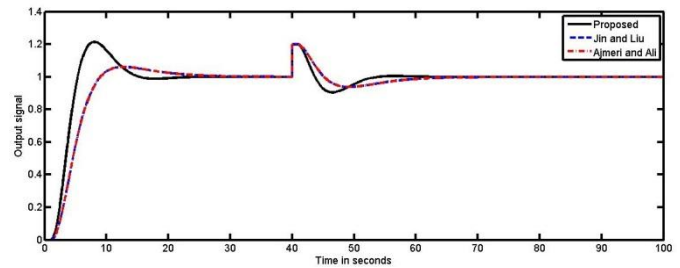


Fig-3: Servo response of the Example 3

Table-2: Performance comparison in terms of IAE,TV and OS

Method	IAE	TV	OS
Proposed	<b>6.8560</b>	<b>0.2141</b>	<b>0.12</b>
Jin and Liu	12	0.1344	0.05
Ajmeri and Ali	12.92	0.1306	0.055

Table-3: Performance comparison in terms of IAE, TV and OS

Method	IAE	TV	OS
Proposed	<b>5.65</b>	<b>5.8973</b>	<b>0.215</b>
Jin and Liu	6.60	3.7072	0.061
Ajmeri and Ali	6.61	3.6967	0.060

The Table 2 and Figure 2 shows that the proposed method produces better result compared with reported methods in terms of less IAE, TV and OS for servo problem.

The Table 3 and Figure 3 shows that the proposed method produces better result compared with reported methods in terms of less IAE, TV and OS for servo problem.

### 3.3 Example 3

An example of stable first order plus time delay with an integrator is considered.

$$G_p = \frac{0.2 e^{-s}}{s(4s + 1)} \quad (21)$$

In this case study  $\lambda$  is tuned to 1.4191 so as to match the Mvalue of 2 as given by reported methods. The PID tuning parameters of proposed method are  $k_c = 5.7422, \tau_I = 5.9046, \tau_D = 1.9519, \beta = 0.4915$ .

The tuning parameter of Jin and Liu [9] method are  $k_c = 3.686, \tau_I = 10.392, \tau_D = 2.43$  and Ajmeri and Ali are  $k_c = 3.67, \tau_I = 10.4221, \tau_D = 2.4769$  [7]. The servo response of FOPTDI is shown in Figure 3. From servo response Integral Absolute Error (IAE), Input usage (TV) and overshoot (OS) is calculated and reported in Table 3.

### 3.4 Example 4

An example of stable first order plus time delay with an integrator with a negative zero is considered.

$$G_p = \frac{(1 + 10s) e^{-s}}{s(2s + 1)} \quad (22)$$

In this case study  $\lambda$  is tuned to 1.1992 so as to match the Mvalue of 2.35 as given by reported methods. The PID tuning parameters of proposed method are  $k_c = 1.1601, \tau_I = 5.1536, \tau_D = 1.2458$ . The tuning parameter of Shamsuzzoha[8] method are  $k_c = 0.217, \tau_I = 6.255, \tau_D = 0.359$ . The servo response of FOPTDI and with a negative zero is shown in Figure 4. From servo response Integral Absolute Error (IAE), Input usage (TV) and overshoot (OS) is calculated and reported in Table 4.

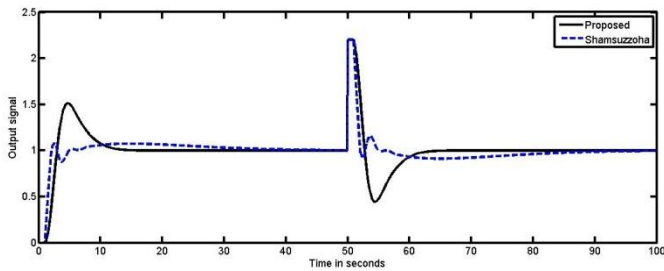


Fig-4: Servo response of the Example 4

Table-4: Performance comparison in terms of IAE, TV and OS

Method	IAE	TV	OS
Proposed	<b>9.03</b>	<b>0.0086</b>	<b>0.51</b>
Shamsuzzoha	7.14	0.22	0.076

The Table 4 and Figure 4 shows that the proposed method produces better result compared with reported methods in terms of less IAE, TV and OS for servo problem.

### 3.5 Example 5

An example of higher order system is considered.

$$G_p = \frac{0.5(1 - 0.5s)e^{-0.7s}}{s(0.4s + 1)(0.1s + 1)(0.5s + 1)} \quad (23)$$

In this case study  $\lambda$  is tuned to 1.1992 so as to match the Mvalue of 2.81 as given by reported methods. The PID tuning parameters of proposed method are  $k_c = 1.003$ ,  $\tau_I = 6.3792$ ,  $\tau_D = 0.6573$ . The tuning parameter of Jin and Liu [9] method are  $k_c = 0.97$ ,  $\tau_I = 8.698$ ,  $\tau_D = 1.12$ . The servo response of higher order system is shown in Figure 5. From servo response Integral Absolute Error (IAE), Input usage (TV) and overshoot (OS) is calculated and reported in Table 5.

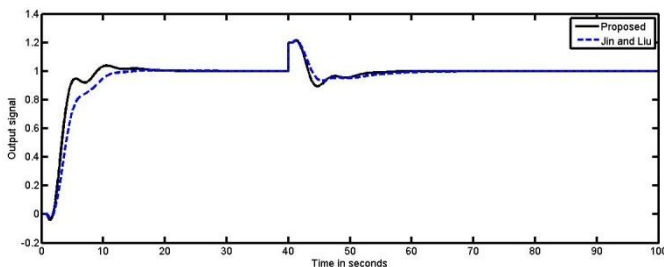


Fig-5: Servo response of the Example 5

Table-5: Performance comparison in terms of IAE, TV and OS

Method	IAE	TV	OS
Proposed	<b>4.8548</b>	<b>0.5942</b>	<b>0.004</b>
Shamsuzzoha	5.9051	0.453	0.002

The Table 5 and Figure 5 shows that the proposed method produces better result compared with reported methods in terms of less IAE, TV and OS for servo problem.

### 3.6 Example 6

An example of non-linear jacketed CSTR, unstable FOPTDI with a zero is considered.

$$G_p = \frac{6.83 \times 10^5 (1 + 766.0752s)e^{-s}}{s(1112.099s - 1)} \quad (24)$$

In this case study  $\lambda$  is tuned to 1.1992 so as to match the Mvalue of 3.52 as given by reported methods. The PID tuning parameters of proposed method are  $k_c = 6.954 \times 10^5$ ,  $\tau_I = 5.1155$ ,  $\tau_D = 2.0962$ . The tuning parameter of Krishna et. al [11] method are  $k_c = 3.7102 \times 10^5$ ,  $\tau_I = 2221.4$ ,  $\tau_D = 3.7967$ . The servo response of higher order system is shown in Figure 6. From servo response Integral Absolute Error (IAE), Input usage (TV) and overshoot (OS) is calculated and reported in Table 6.

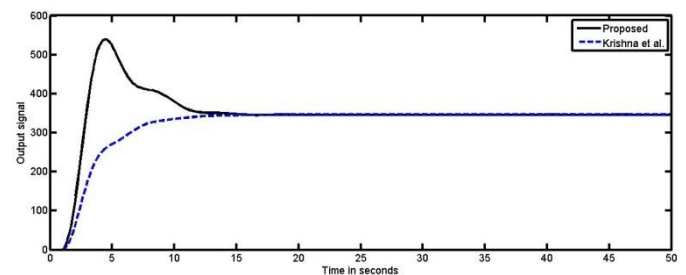


Fig-6: Servo response of the Example 6

Table-6: Performance comparison in terms of IAE, TV and OS

Method	IAE	TV	OS
Proposed	<b>161.17</b>	<b>14868</b>	<b>1.078</b>
Shamsuzzoha	152.56	52074	0.006

The Table 6 and Figure 6 shows that the proposed method produces better result compared with reported methods in terms of less IAE, TV and OS for servo problem.

#### 4. CONCLUSIONS

From all of the simulation results, the performance of the integrating systems with PID controller are obtained and discussed. The controller designed by the proposed method is robust even the parameters of the model changes. A single tuning parameter is selected to obtain desired robustness by specifying  $M$  value. Furthermore performances of proposed method and recently reported method are compared in terms of IAE, input usage (TV) and overshoot (OS) and reported. The proposed method is proved to be effective both in terms of performance and input usage.

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