The Performance Analysis Of Four Tank System For Conventional Controller And Hybrid Controller

Parvathy Prasad[1]

[1]PG Scholar,Vimal Jyothi Engineering College,chemberi,Kannur

---***---

Abstract - *Design of multivariable control system is very essential in the process industries. The four tank system is a benchmark system. So disturbances are essential to analyze the performance of various controllers. Steady state analysis and linearization done using mathematical operations. Mathematical modeling of system is done using first principle. Here the performance of the system in non minimum phase is analyzed for conventional PI controllers and hybrid fuzzy PI controller. Servo and regulatory response of both of the controllers analyzed .It is found that hybrid fuzzy PI controller controls the system effectively than conventional controller.*

Key Words: **four tank system, MIMO system, Conventional controller, hybrid fuzzy PI controller, Fuzzy logic controller**

1.INTRODUCTION

Industrial control problems are usually non-linear in nature and have multiple control variables. The systems involved in such industrial process show significant uncertainties, non-minimum phase behaviour and a lot of interactions. To understand the non-idealities of such industrial processes there is a need for laboratory equipment to carry out experiments. The quadruple tank system is a benchmark system used to analyse the nonlinear effects in a multivariable process[1]. This helps in realizing the multi loop systems in industries. It is a level control problem based on four interconnected tanks and two pumps. The inputs to the process are the voltage to the pumps and outputs are the water levels in the lower two tanks. Many times the liquid will be processed by chemical or mixing treatment in the tank, but always the level of fluid in the tank must be controlled and the flow between tanks must be regulated[2].

The Quadruple tank was developed in the 1996 at Lund institute of Technology, Sweden in order to illustrate the importance of multivariable zero location for control design[3]. The quadruple tank process is thus used to demonstrate coupling effects and performance limitations in multivariable control systems. The multivariable dynamic property in a quadruple tank system is the way in which each pump affects both the outputs of the system. The quadruple tank system is widely used in visualizing the dynamic interactions and non- linearities exhibited in the operation of power plants, chemicalindustries and biotechnological fields [4].The control of such interacting multivariable processes is of great interest in process Industries. Multivariable system involves a number of control loops which interact with each other where one input not only affects its own output but also influences other outputs of the system[5].

The performance of the four tank system for conventional PI and I controller was very effective[6].The system performs well for PID controller also[7],[8].It has been discussed in various papers. The system is suitable for the analysis of some advanced controllers also. It is possible to control the quadruple tank system using Fuzzy logic PI controllers[8].The four tank system provides better result for the interval type -2 fuzzy logic controller(IT2FLC)[9].There were some studies to design a decentralized Fuzzy pre compensated PI controller for multivariable laboratory quadruple tank system[10].Some papers describes the implementation of model based methods for the control of interacting four tank systems[11].FMMRAC is proposed and applied to the four tank system to test its effectiveness[12].Simple Fuzzy logic controllers can provide better control of the Quadruple tank system[13],[14]. Here mathematical modelling of the system done by applying first principles ,Taylor series and Jacobian matrix transformations[15].Quadruple tank system used for analysis of conventional PI controller and hybrid PI Fuzzy controller.

In this paper section 2 and 3 describes the quadruple tank system and modelling respectively. Section 4 contains the design of controllers and 5 is simulation analysis. Section 6 is the conclusion

2. FOUR TANK SYSTEM

The quadruple tank system is a MIMO system used to analyze different control strategies. It is considered as a two double-tank process. The setup consists of four interacting tanks, two pumps and two valves. The two process inputs are the voltages v_1 and v_2 pumps. Tank 1 and tank 2 are placed below tank 3 and tank 4 to receive water flow by the action of gravity. To accumulate the outgoing water from tank 1 and tank 2 a reservoir is present in the bottom .

The action of pumps 1 and 2 is to suck water from the reservoir and deliver it to tanks based on the valve opening. Pump 2 delivers water to tank 2 and tank 3. Similarly the pump 1 delivers water to tank 1 and tank 4. The system aims at controlling the liquid levels in the lower tanks. The controlled outputs are the liquid levels in the lower tanks (h_1, h_2). The valve positions are γ_1 and γ_2. These valve positions give the ratio in which the output from the pump is divided between the upper and lower tanks. The valve position is fixed during the experiment and only the speed of pump is varied by changing the input voltages to the pumps. Flow of the liquid can be seen using a rotameter.

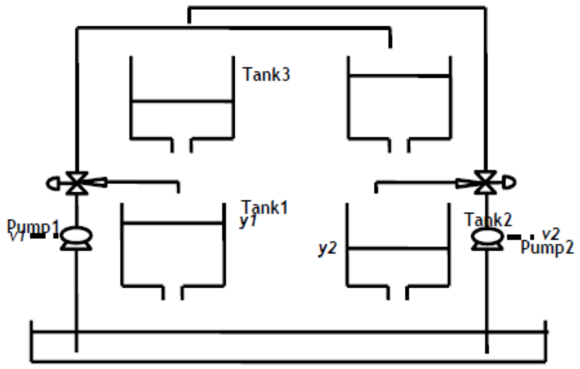

Fig- 1: *Quadruple Tank System*

The operation of quadruple tank system can be in two phases minimum phase and non- minimum phase. It is seen from the Fig.1 that the output of tank 1 is influenced by tank 3 and the position of valve 1. The output of tank 2 is influenced by tank 4 and also by the position of valve 2. When the fraction of liquid entering the lower tanks is less than that of upper tanks then the system starts operating in non minimum phase. When the fraction of liquid entering the upper tanks is less than that of lower tanks then the system starts operating in minimum phase.

3. MODELLING

Here we are representing the system in terms of a set of mathematical equations, whose solutions gives the dynamic behavior of the system.

Initially we write the mass balance equations using Bernoulli's law.

$$A\frac{dh_i}{dt} = q_{in\ i} - q_{out\ i} \qquad (1)$$

Each pump i=1, 2 gives a flow proportional to the control signal as follows

$$q_{in\ i} = k_i v_i \gamma \quad \text{for} \quad i=1,2 \qquad (2)$$

$$q_{in\ i} = k_i v_i (1-\gamma) \quad \text{for } i=3,4 \qquad (3)$$

Where k_i is the pump constant. Considering the flow in and out of all tanks simultaneously, the non-linear dynamics of the quadruple tank process is given Where A_i denotes the cross sectional area of the tank, hi is the water level, $q_{in\ i}$ is the in-flow of the tank and $q_{out\ i}$ is the out-flow of the tank.

$$q_{out\ i} = a_i\sqrt{2gh_i} \qquad (4)$$

Where a_i denotes the cross-sectional area of the outlet hole, g is the acceleration due to gravity

Final equations are

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \qquad (5)$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \qquad (6)$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \qquad (7)$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1 \qquad (8)$$

Non linear relationship in the equation is due to square root term present in it. The equation is solved using Taylor series followed by Jacobian matrix transformation to obtain a state space form of quadruple tank process[15].

$$A=\begin{bmatrix} \frac{-1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & \frac{-1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & \frac{-1}{T_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{T_4} \end{bmatrix} \qquad (9)$$

$$B=\begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} \qquad (10)$$

$$C=\begin{bmatrix} kc & 0 & 0 & 0 \\ 0 & kc & 0 & 0 \end{bmatrix} \qquad (11)$$

$$D=0 \qquad (12)$$

State space model can be converted to transfer function model by using a simple conversion technique.

$$G(s) = C(sI - A)^{-1} + B \quad (13)$$

$$(SI-A)^{-1} = \begin{bmatrix} \frac{T_1}{1+sT_1} & 0 & \frac{A_3 T_1}{A1(1+sT_1)(1+sT_3)} & 0 \\ 0 & \frac{T_2}{1+sT_2} & 0 & \frac{A_4 T_2}{A_2(1+sT_2)(1+sT_4)} \\ 0 & 0 & \frac{T_3}{1+sT_3} & 0 \\ 0 & 0 & 0 & \frac{T_4}{1+sT_4} \end{bmatrix} \quad (14)$$

$$C(SI-A)^{-1} + B = \begin{bmatrix} \frac{T_1 k_1 k_c \gamma_1}{A_1(1+sT_1)} & \frac{T_1 k_1 k_c k_2\,(1-\gamma_2)}{k_1 A_1\,(1+sT_1)(1+sT_3)} \\ \frac{T_2 k_1 k_c k_2\,(1-\gamma_1)}{k_2 A_2\,(1+sT_2)(1+sT_4)} & \frac{T_2 k_2 k_c \gamma_2}{A_2(1+sT_2)} \end{bmatrix} \quad (15)$$

$$G(s) = \begin{bmatrix} \frac{C_1 \gamma_1}{(1+sT_1)} & \frac{C_1 k_2\,(1-\gamma_2)}{(1+sT_1)(1+sT_3)k_1} \\ \frac{C_1 k_1\,(1-\gamma_1)}{(1+sT_2)(1+sT_4)k_2} & \frac{C_2 \gamma_2}{(1+sT_2)} \end{bmatrix} \quad (16)$$

$$C_1 = \frac{T_1 k_1 k_c}{A_1} \quad (17)$$

$$C_2 = \frac{T_2 k_2 k_c}{T_2} \quad (18)$$

Table -1: System Parameter Values

Parameter(Unit)	Value
$A_1, A_3 [cm^2]$	28
$A_2, A_4 [cm^2]$	32
$a_1, a_3 [cm^2]$	.071
$a_2, a_4 [cm^2]$	.057
k_c	1
g	981
h_1	10.43
h_2	15.98
h_3	6.60
h_4	9.57
v_1	3.15
v_2	3.15
k_1	3.14
k_2	3.29
γ_1, γ_2	.35

$$G(s) = \begin{bmatrix} \frac{2.26}{(57.74s+1)} & \frac{2.91}{(57.74s+1)(38.14s+1)} \\ \frac{3.61}{(88.34s+1)(56.47s+1)} & \frac{3.17}{(88.34s+1)} \end{bmatrix} \quad (19)$$

4. . DESIGN OF CONTROLLERS

4.1 PI CONTROLLER

PI controller is a conventional controller widely used in industries. It will eliminate forced oscillations and steady state error resulting in operation of on–off controller and proportional controller respectively.

It is generally used in the areas where speed of the system is not an issue. Here we are using two PI controllers for controlling two tanks. Tuned values for both Kp, Ki are given in the table.

Table 2-Tuned values of PI controller

PI controller	Kp	Ki
PI_1	-0.042871	-0.001319
PI_2	0.610421	0.02343

4.2 FUZZY PRE COMPENSATED PI CONTROLLER

Here fuzzy methods are used to enhance the performance of system using PI controller. Basic configuration of FPPIC is shown in the figure.FLC has just a supplementary role to enhance the existing control system when control conditions changes. When disturbance acting on the system ,the performance of PI controller become poor. The replacement or tuning of existing controller may provide better results. But this is not possible all the time. So FLC used to provide a modified control action to existing PI controller.

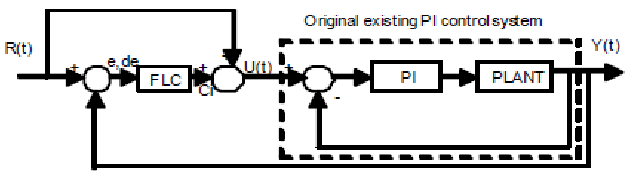

Fig-2: Basic configuration of FPPIC

Basic configuration of FLC comprises of three components: fuzzification interface, decision making logic and a defuzzification interface.Fuzzy logic uses linguistic variables instead of numerical variables. The process of converting a numerical variable in to a linguistic variable is called fuzzification. In this work the error and change in error of level outputs (h_1, h_2) are taken as inputs and the pump voltages(v_1,v_2) are the controller outputs. The error and change in error is converted into seven linguistic values namely NB, NM,NS,ZR,PS,PM and PB. Similarly controller output is converted into seven linguistic values namely NB,NM,NS,ZR,PS,PM and PB. Triangular membership function is selected and the elements of each of the term sets are mapped on to the domain of corresponding linguistic variables. The membership functions for error change in error and controller output is shown in the figure.

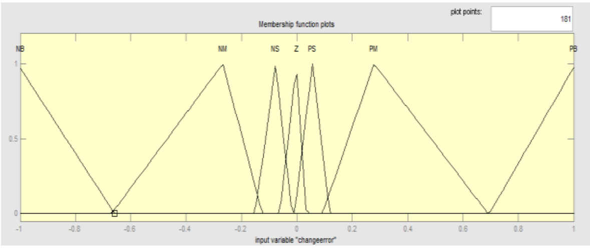

Fig -3: Membership function of error and change in error

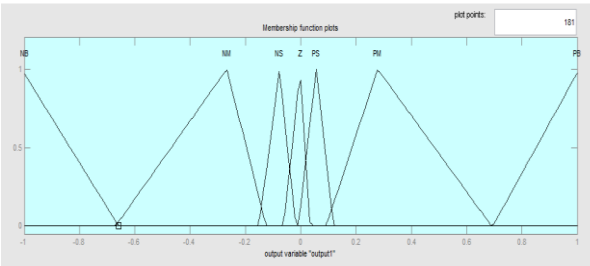

Fig-4: Membership function of controller output

Table-3: Rule base for the fuzzy logic compensator

de e ci	NB	NM	NS	ZR	PS	PM	PB
NB	NB	NB	NB	NM	NS	NS	ZR
NM	NB	NB	NM	NS	NS	ZR	PM
NS	NB	NM	NS	NS	ZR	PS	PM
ZR	NM	NM	NS	ZR	PS	PM	PB
PS	NM	NS	ZR	PS	PS	PM	PB
PM	NS	ZR	PS	PS	PM	PB	PB
PB	ZR	PS	PS	PM	PB	PB	PB

Basically, the decision logic stage is similar to a rule base consisting of fuzzy control rules to decide how FLC works. The rules are generated heuristically from the response of the conventional controller , 49 rules are derived for each fuzzy controller from careful analysis of trend obtained from the simulation of conventional controller and known process knowledge. The rules are given in Table 3.The decision stage processes the input data and computes the controller outputs. The output of the rule base is converted into a crisp value, this task is done by defuzzification module.

5. SIMULATION ANALYSIS

The performance of the system for PI controller and Fuzzy pre -compensated PI controller can be analyzed easily from the simulation results

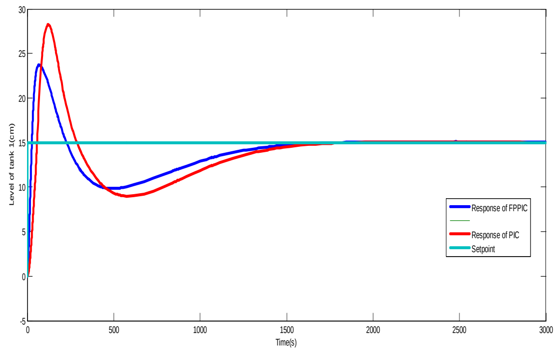

Chart -1: Regulatory response of the system for tank 1

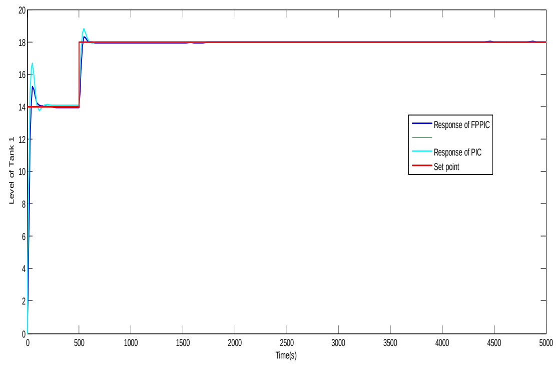

Chart-2: Servo response of the system for tank 1

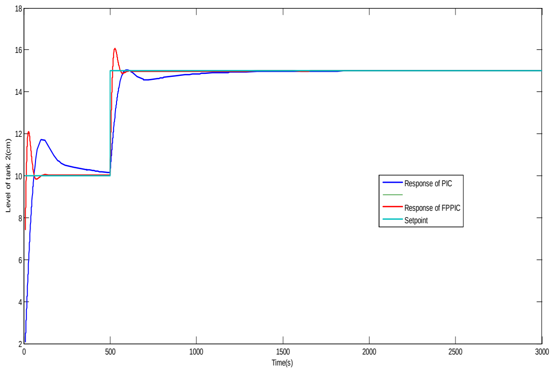

Chart-3: Servo response of the system for tank 2

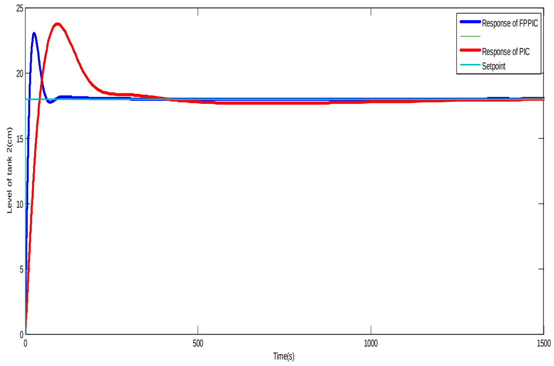

Chart-4: Regulatory response of the system for tank 2

It is clear from the results that the hybrid controller provides less steady state error and fast settling compared to conventional controller.

Table-3:Analysis of Performance indices

Performance indices	Tank 1				Tank 2			
	PI servo	FPPI servo	PI regulatory	FPPI regulatory	PI Servo	FPPI servo	PI regulatory	FPPI regulatory
ITAE	*2675*	*1327*	*1875*	*1012*	*2533*	*352*	*1013*	*122*
ISE	*277*	*106*	*2762*	*118*	*604*	*199*	*2762*	*1668*

The performance of the two controllers can be evaluated using performance indices namely Integral Square error (ISE) and Integral Absolute Error (IAE). A control system is considered optimal when it minimizes the above integrals. Table 4 summarizes the integral error values for the two control schemes. Fuzzy pre compensated PI controller has the least ISE, and IAE values.

3. CONCLUSIONS

This work clearly shows the advantages of using fuzzy pre compensated PI controller for a Four Tank System. The comparison of the two controllers reveals that fuzzy pre compensated PI controller performs well for the system.

REFERENCES

[1] Prof. D. Angeline Vijula, Anu K,Honey Mol P, PoornaPriya "Mathematical Modelling of Quadruple Tank System", International Journal of Emerging Technology and Advanced Engineering ,Volume 3, Issue 12, December 2013

[2] Jaya prakash J, Davidson D, Subha Hency Jose P, "Comparison of Controller Performance for MIMO Process", International Journal of Emerging Technology and Advanced Engineering, Volume 3, Issue 8, August 2013

[3] Johansson KH, Horch A, Hansson A. "Teaching multivariable control using the quadruple tank process". In: Proceeding of the 38th conference on decision & control; December 1999.

[4] Qamar Saeed, Vali Uddin and Reza Ketabi " Multivariable predictive PID control for quadruple tank", in Proceedings of World Academy of Science, Engineering and Technology, Beijing, China, Vol.67, No.4, pp.861-866.

[5] Jayaprakash J, SenthilRajan T, Harish Babu T,"Analysis of Modelling Methods of Quadruple Tank System", International Journal of Advanced Research in Electrical Electronics and Instrumentation Engineering, Vol. 3, Issue 8, August 2014

[6] Yousof Gholipour, Ahmad Zare, "Comparing the effect of PI and I controllers on a four tank process", International Journal of Recent Research in Electrical and Electronics Engineering (IJRREEE) Vol. 1, Issue 1, pp: (1-6), Month: April - June 2014.

[7] W. Belhaj , O. Boubaker, "On MIMO PID Control of the quadruple-tank process via ILMIs Approaches : Minimum and Non-Minimum Case studies", 10th IFAC International Symposium on Dynamics and Control of Process Systems, The International Federation of Automatic Control ,December 18-20, 2013. Mumbai, India

[8] S.K.Lakshmanaprabu, N. Sivaramakrishnan, U. Sabura Banu, "Closed Loop Control of Quadruple Tank Process using Fuzzy Logic PI Controller", National Conference on Computational Intelligence for the Engineering Quality Software (CiQS 2014)

[9] Karthick S, Lakshmi P, Deepa T, "Comparison of Fuzzy Logic Controllers For a Multivariable Process", International Journal of Electrical and Electronics Engineering (IJEEE) ISSN):pp 2231 – 5284, Volume-3, Issue-1, 2013

[10] R.Suja Mani Malar , T.Thyagarajan, "Design of Decentralized Fuzzy Pre compensated PI Controllers for Quadruple Tank system".International Journal of Recent Trends in Engineering, Vol 2, No. 5, November 2009

[11] Edward Gatzke, Edward S. Meadows, Chung Wang, Franc is J. Doyle III P, "Model Based Control Of a Four Tank System", Computers and Chemical Engineering pp 1503-1509.

[12] ` Sankata B. Prusty, Umesh C. Pati and Kamala K. Mahapatra, "A Novel Fuzzy based Adaptive Control of the Four Tank System"

[13] Shebeera C. A, Shijoh V, M. V. Vaidyan, " Non-linear Modeling and Fuzzy Control for a Quadruple Tank Process", Proceedings of National Conference EEM 2014

[14] "Non-linear Modeling and Fuzzy Control for a Quadruple Tank Process", Proceedings of National Conference EEM 2014.

[15] R.SujaMani Malar,T.Thyagarajan,"Modeling of Quadruple Tank System Using Soft Computing Techniques",European Journal of Scientific Research ISSN 1450-216X Vol.29 No.2(2009),pp.249-264

[16] Jayaprakash J, SenthilRajan T, Harish Babu, "Analysis of Modelling Methods of Quadruple Tank System" , Engineering International Journal of Advanced Research in Electrical Electronics and Instrumentation, Vol. 3, Issue 8, August 2014