

Time-Frequency Analysis methods: A Comparative study

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Abstract - The natural real world signals are mostly non-linear and non-stationary. For processing such signals, one needs tools that are adaptive to work with such signals. Here, a novel tool, viz., Hilbert-Huang Transform is discussed and compared with the traditional and conventional tools available for dealing with real world signals. This paper synthesizes a non-stationary signal and analyzes with Fourier transform, Short Time Fourier transform and wavelet analysis. Similarly, a non-linear and non-stationary signal is synthesized and analyzed with the above mentioned tools in MATLAB. The results show that wavelet analysis performs better in analyzing non-stationary signals compared to traditional Fourier analysis methods, but degrades when non-linearity is introduced into the synthesized non-stationary signal. Hilbert-Huang transform (HHT) is then applied to the synthesized non-linear and non-stationary signal and the time-frequency graphs show that HHT is clearly able to interpret the dynamics of the signal under consideration. Thus HHT proves to be a promising tool to analyze many real world natural signals.

Key Words: Wavelet Transform; Spectrogram; Empirical Mode Decomposition; Time-Frequency; Non-linear; Hilbert-Huang transform; Non-Stationary

1. INTRODUCTION

A signal is essentially a bearer of data. Depending on the area of engineering, it could be electric current, voltage, voice, Electrocardiogram (ECG), Electroencephalogram (EEG), and so on, which provide valuable information about the system generating these signals. All the real world signals are non-Gaussian, non-stationary, and non-linear unless otherwise the respective systems generating these signals are in controlled stable state. To better understand the systems one need to process these non-linear and non-stationary signals using tools that are capable of analysing them.

Most of the techniques currently used to analyze these signals generalize and assume that the process or the signals being investigated are linear and stationary which may not be valid in all situations. Traditionally Fourier analysis like Fast Fourier Transform (FFT), Short Time Fourier Transform (STFT) have been very widely used in the area of signal

processing. In Fourier analysis it is assumed that a signal is linear and stationary, but this may not be valid for all signals. Anugom, Julian U, et.al have developed time-frequency analysis system using Fourier transform methods and wavelet transform [1]. STFT gives the spectrogram of the signal. The spectrogram operation is similar Fourier analysis wherein the analysis is carried within certain window limit that is moved slowly over the signal to obtain the time-frequency plot [2]. The underlying assumption in this method is that the signal is piecewise stationary. But there are certain difficulties in locating the changes in frequency due to interplay between time and frequency resolution depending on the window size, which is usually called as Uncertainty principle. Wigner-Ville technique also has a drawback that the inherent cross-term interferences usually conceals the actual frequency and time of the signal being analyzed [3]. Wavelet transform is well received among researchers in the recent years as it is effective in analyzing the non-stationary signals, but it is also limited by the uncertainty principle. It also fails poor when non-linearity is introduced to non-stationary class of signals.

In this review paper, a powerful technique, viz., Hilbert-Huang transform (HHT), is used to analyze non-stationary and non-linear signals. Simulations are performed in MATLAB to check and compare the capabilities of the techniques like FFT, STFT, and Wavelet Transform in comparison with HHT.

2. HILBERT-HUANG TRANSFORM

An innovative technique to study and evaluate signals which are both non-stationary and non-linear was developed by Dr.N.E.Huang et. al [2]. It is called as Hilbert Huang Transform (HHT). The most important breakthrough in this technique is the development of empirical mode decomposition (EMD) which breaks down a complex data into decomposed, set of signals, called Intrinsic Mode Functions (IMFs). Furthermore, HHT uses Hilbert Transform to apply it to the IMFs and produce a spectra with time and frequency which is also called as HHT. In essence, the HHT is carried out in two steps, by applying EMD to the given data first and later applying Hilbert transform to the resultant IMFs.

2.1 Empirical Mode Decomposition

The extraction of IMFs from the given signal, is carried out by Empirical Mode Decomposition (EMD). As the name suggests, this method is an empirical method with certain number of steps to be followed for completion of the EMD procedure. The extracted IMF should meet certain conditions viz.

1. The amount of extrema's encountered should be equal or may vary by one with the amount of zero crossings.
2. The mean value calculated from the envelopes of upper and lower curves must be zero [2].

The steps involved in extraction of IMFs via the EMD technique are listed below:

1. Find the local maxima and local minima present in the given signal.
2. An upper and lower envelope is constructed by interpolating the identified maxima's and minima's respectively, using cubic spline technique. The mean of both the envelopes is designated as $m(t)$.
3. An intermediate signal designated as $h_1(t)$ is calculated by subtracting the original signal with the $m(t)$. Depending the nature of the signal, sometimes, $h_1(t)$ may seem to satisfy the IMF conditions but there may be inherent disturbances present in the signal. Therefore algorithm is repeated k number of times, treating this intermediate signal as the input signal.
4. During the iteration process, any particular intermediate signal, $h_{1k}(t)$, may satisfy the IMF conditions and this signal is designated as first IMF $C_1(t) = h_{1k}(t)$. Also subtracting the initial input signal with first IMF gives residue, $r(t) = X(t) - C_1(t)$.
5. This iterative loop is again continued with residue as input signal, $X(t) = r(t)$ for getting the next batch of IMFs.
6. This empirical procedure is continued till they meet some stopping criteria like, standard deviation of residue is less than predefined standard deviation or no further IMF may be possible to extract from the residue.

The extracted IMFs and residue can be reconstructed to obtain the same original input signal. Therefore, mathematically it can be written in the form of equation as shown below.

$$X(t) = \sum_{i=1}^n c_i(t) + r_n(t) \tag{1}$$

where, $C_1(t), C_2(t) \dots C_n(t)$ designates the IMFs and $r_n(t)$ indicates the residue

2.2 Hilbert Spectral Analysis

This is the second stage, of the Hilbert Huang transform which uses the IMFs, residue to obtain an analytical signal by taking the Hilbert transform on these signals. Mathematically it can be written as:

$$Z_i(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{C_i(\tau) d\tau}{t-\tau} \tag{2}$$

Hilbert Transform helps in generating an analytic signal by providing 90° phase shift to the input signal.

Mathematically analytical signal is written as:

$$V_i(t) = C_i(t) + Z_i(t) = a(t)e^{-i\theta t} \tag{3}$$

The analytical signal formed is a complex quantity and hence the amplitude and its phase could be found as below,

$$a(t) = \sqrt{C_i(t)^2 + Z_i(t)^2}, \theta_i(t) = \tan^{-1} \frac{Z_i(t)}{C_i(t)} \tag{4}$$

The Instantaneous frequency of the analytical signal is,

$$\omega_i(t) = \frac{d\theta_i(t)}{dt} \tag{5}$$

This final value of instantaneous frequency along with the time period of the input signal helps in constructing the time-frequency plot, which is regarded as HHT.

2.3 Algorithm of EMD in terms of flowchart

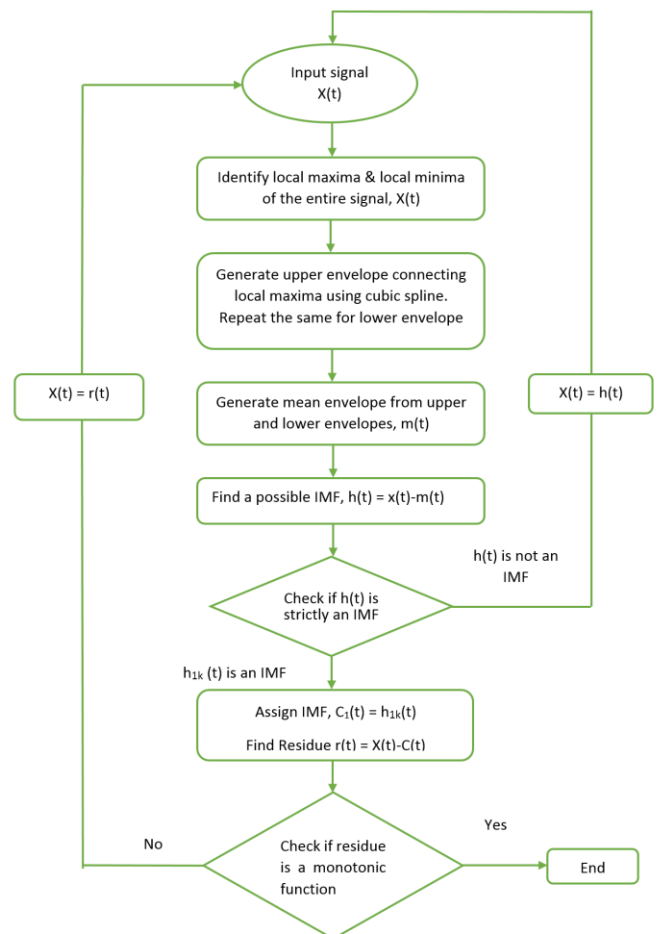


Figure 1: Flowchart of EMD

3. RESULTS AND DISCUSSION

A non-stationary signal is simulated and generated using MATLAB and the signal is analyzed using conventional methods like Fourier transform (F.T), Short Fourier Transform (STFT) and Wavelet Transform.

The mathematical representation of the synthesized non-stationary signal is shown in the below equation:

$$Sig = \begin{cases} Sin(2 * \pi * f * t) & \text{for } 0 < t < 1 \text{ sec} \\ & \text{where } f = 5 + (10 * t) \end{cases} \quad (6)$$

This signal linearly increases in frequency with time as shown in the figure 2. The frequency is offset to 5 Hz and scaled with a multiplying factor of 10 as shown in the equation. The duration of the signal under consideration is for 1 sec. The frequency of the signal would be initially 5Hz and gradually increases to 15 Hz at time, t=1 sec.

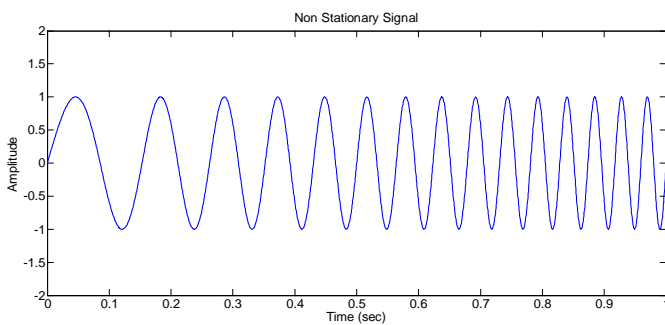


Figure 2 : Synthesized Non-Stationary Signal

The traditional approach to understand and visualize the frequencies present in the signal under consideration described by equation (6), is by Fourier Transform. Upon application of Fourier transform, which is shown in figure 3, it can be noticed that, it has detected the frequencies with reasonably good amplitude between 5-25 Hz. However, F.T. was not accurate in locating the changes that are happening in time, i.e. linear increase in frequency with time.

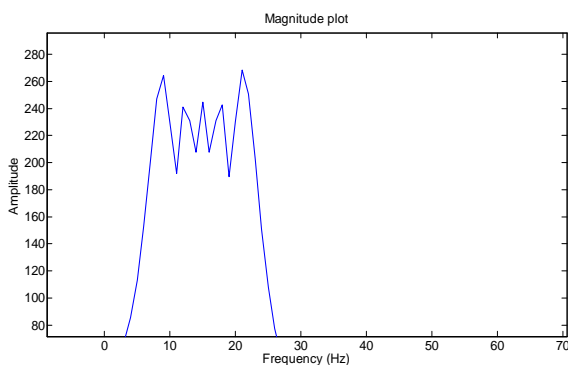


Figure 3 : F.F.T Magnitude Spectrum of Non-Stationary signal

To visualize and understand the dynamics of the signal with respect to time, as well in frequency domains, the commonly used method, viz., Short Time Fourier Transform (STFT) is applied. The STFT reveals the frequency information with respect to the time domain, but with some limitations. The STFT using hamming window, of size 100 samples and overlap of 50 samples, gives a glimpse of time content of the signal. The spectrogram with short window shown in the figure 4, has poor frequency resolution and decent time resolution.

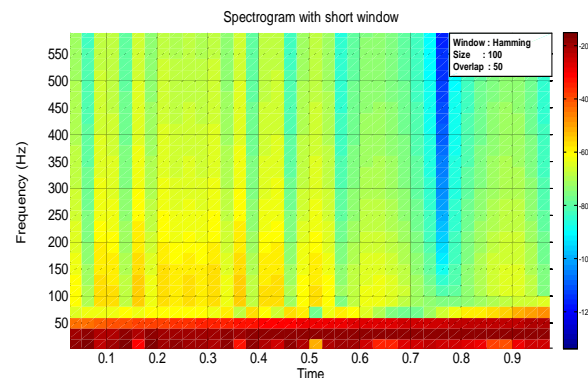


Figure 4 : Spectrogram of non-stationary signal (Short Window)

Similar approach is followed by increasing the window size. Upon increasing the window size to 400 samples with an overlap of 300 samples, there is an improvement in frequency resolution, but time resolution is degraded as shown in figure 5. Therefore, there is an interplay between the window size and the resolutions of time and frequency, which is an inherent drawback in STFT.

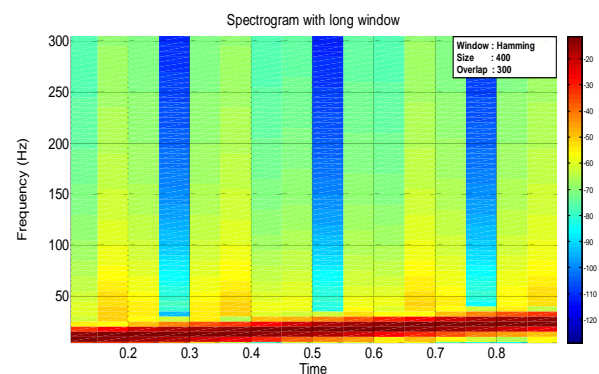


Figure 5 : Spectrogram of Non-Stationary signal (Long Window)

The vast majority of the physiological signals are characterized by low frequency signals that vary slowly and high frequency signals that vary fast. Wavelets were developed keeping this in mind and hence is generally accepted for a wide variety of physiological signals. The upper portion of figure 6 shows the non-stationary signal to which scalogram and CWT are applied.

The scalogram indicated in figure 6, reveals the percentage of energy for each computed wavelet coefficients along Y-axis. This scalogram was calculated for the synthesized signal under consideration using MATLAB. The scalogram shows reduction in scale values with increase in time. Initially, the signal has a low frequency which is depicted by higher scale value and as time approaches to 1 sec, the frequency of the synthesized signal would have increased linearly. This is indicated by lower scales than previous ones, as time approaches 1 second. Therefore as the frequency of non-stationary signal indicated in the upper graph of figure 6 is increasing, the scale value in scalogram is decreasing.

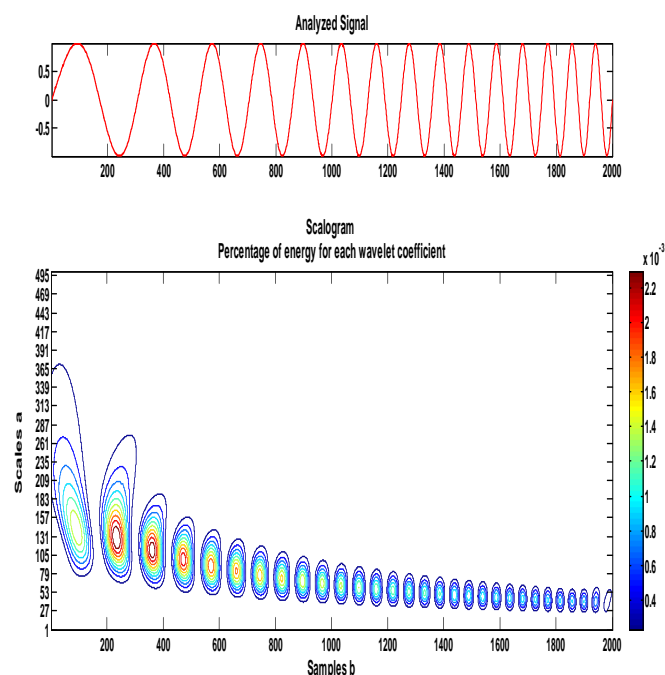


Figure 6 : Scalogram of Non-stationary Signal

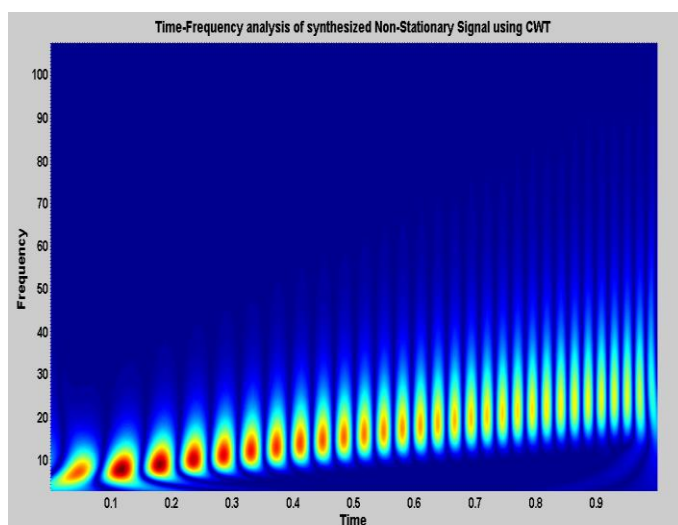


Figure 7 : Time-Frequency distribution using CWT of Non-stationary Signal

Figure 7, shows the time-frequency graph of the synthesized nonstationary signal using continuous wavelet transform (CWT). It shows that the energy of the signal initially is high at frequency, 5 Hz (Y-axis) and gradually increases to approximately 15 Hz, clearly following the dynamics of the considered signal. In spite of the fact that the wavelet analysis may provide agreeable results in cases of non-stationary signals, yet while considering the combined signals, which are non-stationary and non-linear, it envisages a couple of drawbacks.

A non-stationary as well as non-linear signal is synthesized using MALAB, and its mathematical representation is given in the equation below:

$$Sig = \begin{cases} \{ \sin(2 * \pi * f * t) + \sin(4 * \pi * f * t) \} . \exp(t); & \text{for } 0 < t < 1 \text{ sec} \\ \text{where } f = 50 + (10 * t) & \end{cases} \quad (7)$$

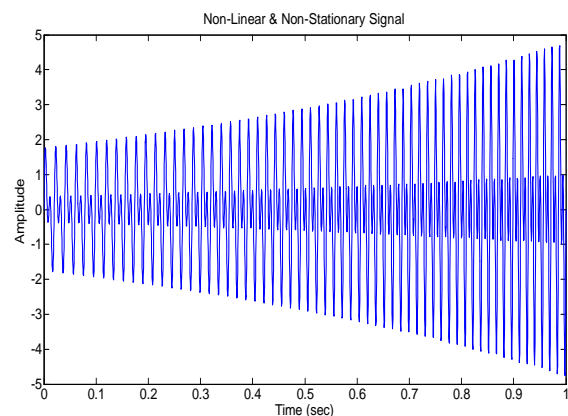


Figure 8 : Synthesized signal with nonstationary and nonlinearity characteristics

This synthesized signal represented in equation (7), is a combination of two sine waves, the frequency of which are linearly dependent on time, with one of the sine waves having double the frequency of the other. Here, the frequency is offset by 50 Hz and multiplied by scaling factor of 10 with respect to time. Figure 8 depicts the nonlinear and nonstationary signal under discussion.

After the synthesis of the non-stationary and non-linear signal, the scalogram is applied to know the wavelet coefficients having highest energies. The scalogram for the signal under consideration is shown in figure 9. It may be noticed that within the scale limit of 2000, identification of proper scales having accurate energy distribution, following the input signal characteristics is not possible.

The time-frequency graph is constructed using continuous wavelet transform, as shown in figure 10. It shows the presence of energy along the 50 Hz frequency, which initially started with diminishing energy and missed the presence of another sine wave having 100Hz frequency at time, t=0 sec. But the synthesized signal of equation (7), has one sine wave

with 50 Hz frequency and another sine wave with 100 Hz frequency at time, $t=0$ sec. At time, $t=1$ sec, the frequencies would have reached 60 Hz and 120 Hz respectively, due to linear increase in frequency with time. This change in frequency content of the nonstationary and nonlinear signal has not been properly depicted in the scalogram and in the time-frequency distribution using CWT.

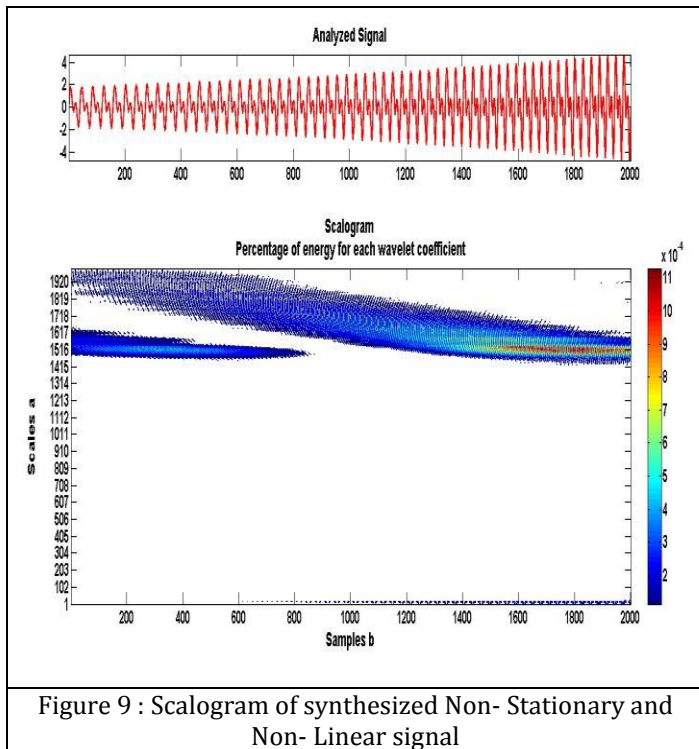


Figure 9 : Scalogram of synthesized Non- Stationary and Non- Linear signal

Dr. Huang et.al [2] developed a novel technique viz., Hilbert-Huang transform for analyzing such signals having the non-stationary as well as non-linear characteristics. In this technique, the first step is to apply Empirical Mode Decomposition (EMD). Upon application of EMD, the signal is decomposed into a set of sub component monotonic signals called as Intrinsic Mode Functions (IMF). After obtaining the IMFs from the result of first stage, viz., EMD, Hilbert transformation is applied to every IMF to obtain the time-frequency plot, viz. HHT.

The EMD was applied to the above considered nonstationary and nonlinear signal (Equation 7), and two IMFs were obtained as shown in figure 11. These two IMFs are the two sine wave components, which form the synthesized signal. Each of these 2 IMFs, extracted via the EMD operation, represents the respective frequency components of the individual sine waves forming the synthetic signal, i.e., the 50 Hz sine wave and 100 Hz sine waves at time, $t=0$ sec.

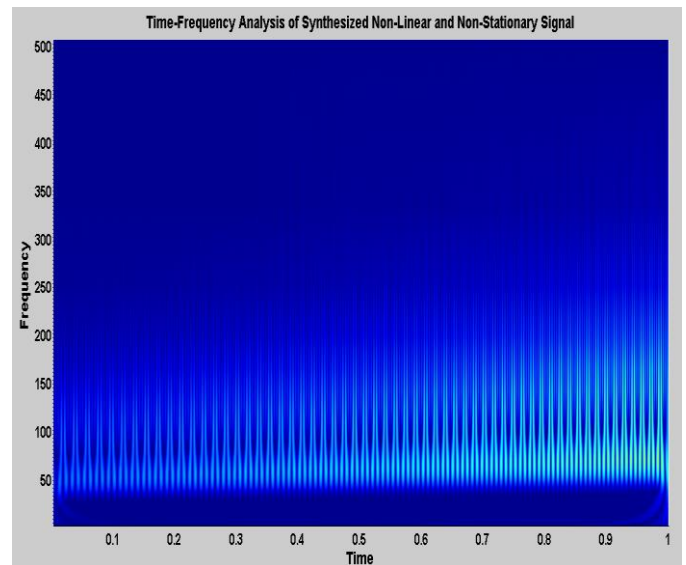


Figure 10 : Time-Frequency distribution using CWT of non-stationary as well as non-linear Signal

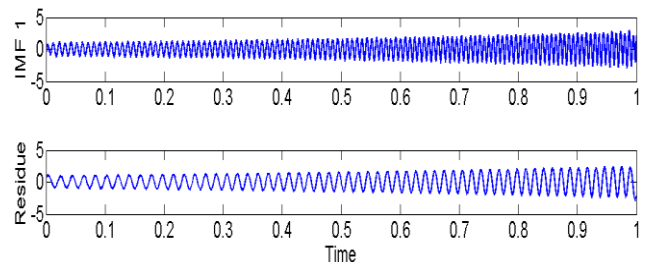


Figure 11 : IMFs obtained for synthesized nonstationary and nonlinear signal
Hilbert-Huang Transform

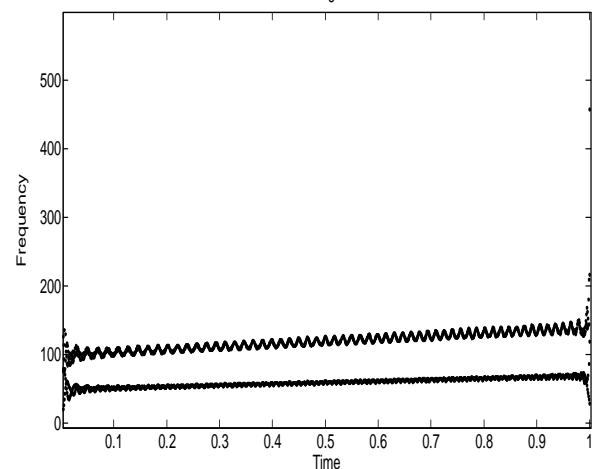


Figure 12 : HHT of Non-Stationary and Non-Linear signal

The HHT is a time-frequency plot, constructed with the help of application of Hilbert Transform on the IMFs obtained.

The time-frequency plot using HHT, clearly shows the dynamics of the considered signal. At initial time, i.e. $t=0$ sec, there are two signals starting from 50 Hz and 100Hz, and gradually grows to 60 Hz and 120 Hz, respectively, at time $t=1$ sec. The frequency increases linearly with time, as shown in figure 12. This clearly matches with the synthetic signal shown in equation (7) that was generated using MATLAB.

4. CONCLUSION

The conventional signal processing methods like FFT, STFT are not effective enough to showcase the changes in frequency contents of the non-stationary signal with good accuracy. The Wavelet Transform is capable of analyzing and following the frequency changes of the non-stationary signals. The Wavelet Transform may also be helpful in locating the time at which the changes in frequency may occur. However, wavelet transform was not effective in analyzing the synthesized signal having non-stationary and non-linear characteristics. In the scalogram, there were no scales having proper energy distribution which followed the input signal characteristics. Also, CWT is not capable of detecting one of the sine wave components present in the synthesized input signal, in the time-frequency plot. Hilbert-Huang transform technique proved to be effective to study the changes in frequencies of the non-linear as well as non-stationary signal. The EMD analysis is also efficient to decompose the input signal appropriately into its individual components having their respective frequencies. The HHT is also adaptive to the data, i.e. the prior nature of the signal was not necessary to be known. Hence, HHT is the most effective tool for analyzing signals having non-stationary as well as non-linear characteristics.

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