Representation of Data Association Problems using Temporal Relational Probability Models: An Overview

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Abstract - *Keeping track of dynamic changing complex world is one of the biggest challenges in developing automated systems because of the data association problems. In dynamic world, data association problems arise due to two types of uncertainties i.e. (i) Existence Uncertainty where it is very difficult to predict the actual existence of the objects in the world, (ii) Identity uncertainty in which it becomes very difficult to correlate the identity of an object to its data values. In the present scenario most of the research is being carried out in developing automated systems based on prepositional reasoning (as in case all types of Bayesian networks). These types of prepositional models are not capable enough to represent the complexities of the real world as these fail to establish relations among various objects of real world existence. To represent objects and their relations with other objects there is a need to define models that are capable of defining relations among real world objects and also its associated data values. This paper defines a novel relational model that combines the probability theory with expressive power of first order logic to represent data association problems of complex world which in turn helps to make sound inference in uncertain environment.*

Key Words: ***first order logic, tuple,uncertainity, object, data assocition***

1.INTRODUCTION

Planning and acting in an uncertain environment is quite a bit challenging where an agent has to keep track of a belief state i.e. a set of all possible world states that it might be in for some time. As world is not static over time, belief states must be updated regularly according to the information provided by the sensors or percepts. Sometimes sensors only give partial information about the current state of the world thereby inducing more uncertainty [1]. An agent will never know for certain what would be its next course of actions that leads to robust or rational decisions. Moreover if agent is logical agent, it has to take decisions considering every logically possible explanation of the received perceptions. This results in possibly infinite belief state domain.

2. Probability Theory

When it is impossible to make logical decision due lack of knowledge or due to impossible large belief state representations in uncertain environment, an agent can act only when it is provided with some "**degree of belief"** pertaining to its future actions [7][2]. Here Probability theory provides a way to deals with uncertainty. Contrary to logical agent whose opinion takes the form of true, false or no opinion, a probabilistic agent may have numerical value of degree of belief ranging between 0 and 1 as describe as:

$$0 \le P(w) \le 1 \text{ for every } w \text{ and } \sum_w P(w) = 1 \qquad (1)$$

Assertions used in probability theory tell about the how probable the possible states of the world are whereas logical assertions strictly rules out the possible world when information given by percepts are uncertain or reported to be false. In probable world we cannot rule out uncertain information [7]. To deal with uncertain or absence of information we assume prior probabilities of that world state. Those probabilities are called *unconditional probabilities*. Sometimes however we always have some information or evidence about some state information. In this situation we assign posterior probabilities to world state given evidence. This form of probabilities is called *conditional probabilities*. Conditional probabilities can always be described in terms of unconditional probabilities as follows:

$$P(x|y) = \frac{P(x,y)}{P(y)} \quad \text{Where } P(y)>0 \qquad (2)$$

Here probability of world state x , given evidence y can be describe in terms of unconditional probabilities of x and y. the above equation is also called ***Bay's Rule.***

3. Relational theory and first order logic

Relational theory can be thought of as framework to understand the real world as association of different objects due to their position or properties. In this context, a space cannot exist unless the presence of objects in it and a time has no value until some event occur [5]. Much of every day's existence can be thought of dealing with objects and relations among them.

First order logic can also express facts about some of all objects in the universe. It contains domains of elements called objects and every possible world must contain at least one object at a particular time. Models in first order require total functions [4] i.e. there must be value for every input tuple. A tuple can be considered as a collection of related objects arranged in some order.

4. relational probability model for temporal reasoning

Relational probability models use expressive and declarative power of first order logic with slight changes. First order logic assumes assignment to variable as true, false or indifferent and can discard the value where assignments are false [2], whereas in relational probability models, one cannot rule out false or indifference. There is always a belief state of the possible assignment. In fact RPMs use database semantics where each symbol refers to different object called unique name assumption. Like First Order Logic, RPMs also have constants, predicate and function symbols and also a type signature which is used to identify the object types. Predicate symbols are used to define the relations among objects and also act as the functions to some input value. Functions can be used in when an object or objects make transitions from one state to another and their old values need to be updated. These functions return joint probability distributions over all possible values rather than returning merely true or false.

Given the constants and their types along with functions and their type signatures, the variables of RPM can be obtained by passing arguments to each function with each possible combination of objects and defined their prior probabilities as described below:

$$\boldsymbol{P}\,(R\,(a_1, a_2)|F(e_1, e_2)\,) \qquad (3)$$

The above representation describes probability of the relation R between two objects a_1 and a_2 where function F defines the calculated values of given evidences or observations on e1 and e2. In the language of first order logic R and F are just predicates. If there are number of objects to deal with, above representation can be easily generalized to n objects and n function arguments as follows

$$\boldsymbol{P}\,(\,R\,(a_1, a_2, \dots, a_n)\,|\,F(e_1, e_2, \dots, e_n)\,) \qquad (4)$$

The above two equations can be used for probabilistic reasoning in the context of static world in which each relation variable has a fixed value. But actual world cannot remain same over the period of time. It changes rapidly over a span of time. In dynamic environment we view the world as a series of snapshots each of which objects have certain properties or values, some are observable and some not. Representing a dynamic world is somewhat cumbersome task as compare to static world representation.

Equation (2) can further be modified as follows for temporal relation probability models where time is the major concern and observations are generated by many objects.

$$\boldsymbol{P}\,(R_t(a_1, a_2, \dots, a_n)|F_{0:t-1}(e_1, e_2, \dots, e_n)\,) \qquad (5)$$

Equation (3) describes the probability P of relation *R* among n objects at a particular time *t*, given *computed* functional values of *F* upto *t-1* time steps. Strictly speaking the association of observations with the objects is a collection of probabilities of relations among objects at each time step. As more and more evidences appear, probabilities of relation among objects will go on higher. Since $F_{0:t-1}$ would be unbounded and becomes very large as *t* increases. One cannot compute the observations from scratch. To overcome this problem our relational probability model uses **Morkov Process** or **Markov Chain Rule** which assumes that the value of current state can be determined by only a few numbers of previous states [8]. Hence we can further modify our model according to this assumption as:

$$\boldsymbol{P}\,(R_t(a_1, a_2, \dots, a_n)|F_{t-1}(e_1, e_2, \dots, e_n)) \qquad (6)$$

$\mathbf{P}(R_t(a_1, a_2, \dots, a_n) | F_{t-1}(e_1, e_2, \dots, e_n))$=

$$\frac{P(R_t(a_1, a_2, \dots, a_n), F_{t-1}(e_1, e_2, \dots, e_n))}{P(F_{t-1}(e_1, e_2, \dots, e_n))} \text{ (Bay's Rule)}$$

$P(R_t(a_1, a_2, \dots, a_n), F_{t-1}(e_1, e_2, \dots, e_n))$ =

$\mathbf{P}\ (R_t(a_1, a_2, \dots, a_n) | F_{t-1}(e_1, e_2, \dots, e_n))$

$P(F_{t-1}(e_1, e_2, \dots, e_n))$ (8)

Equation (6) is refinement of (5) according first order Markov assumption where current states depends only on its previous states. We can also refine our model using higher order Markov assumptions. Equation (8) can be thought of probability of association objects at time *t* along with given observations perceived as evidences at *t-1.*

As for all probability models, inference can be drawn by summing all variables other than query and evidence. If we have very large domain of variable to calculate at each time step, it become very slow to draw the inference [9]. As real world is impatience and has no place for sluggishness, it is imperative to employ efficient algorithms that speed up the calculations. In RPMs, modern approximation methods such as **particle filtering** or **Monte Carlo Markov Chain (MCMC)** are best to work

5. CONCLUSIONS

Keeping Track of many objects of a complex world is very hectic and tedious task. Real world can be perceived as dynamically changing with time. In such a complex scenario there is a need to develop such automated system models that are sound enough to represent the world thoroughly. Data association is the only way to combine multiple observations of any given object at a specific time slice. Since objects interact with each other in every complex activities of the world, developing relational models that use the expressiveness of first order logic together with probability theory that is powerful enough to deal with any type of uncertainties are the prime thought of this paper.

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REFERENCES

[1] Kaelbling, L.P. Litman. M.L. Cassandra, A.R. (1998), "Planning and acting in a partially observablestochastic domain, AIJ, 101, 99-134.

[2] Jones T. "Artificial Intelligence: A moderm Approach. Infinity Science Press.

[3] Kersting, K. Raedt, L.D. and Kramer S., "Interpreting baysian logic programs", AIII workshop on learning satstical models from relational data.

[4] Halpern, J.Y. and Weissman V. (2008), "Using first order logic to reason about policies". ACM Transectionson information and system security, 11(4).

[5] Halevy,A.,Norvig,P. and Pereira, F. (2009), "The unresasonable effectiveness of data". IEEE Intelligent Systems, march, 8-12.

[6] Darwiche, A. (2009)"Modeling and reasonong with baysian network". Cambridge University Press.

[7] Koller, D., Friedman, N.,"Probablistic Graphical Models:Principles and Techniques" MIT Press.

[8] Levin, D.A., Peres, Y. and Wilmer, E.L. (2008) "Markov Chain and mixing Times" American Mathematical Society.

[9] Oh, S., Russell, S.J., Sastry, S. (2009),"Markov Chain Monte Carlo data association for multi target tracking", IEEE Transections on Automatic Control. 54(3), 481-497.

[10] Schulz, D.,Burgard, W. Fox, D., Cramer, A.B.(2003). People Tracking with mobile robotsusing sample based joint probablistic data association filters", Int. J. Robotic Research 22(2), 99-116