Multi Objective Dynamic Economic Dispatch With Cubic Cost Functions

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Abstract - *Economic dispatch (ED) is the most important problem in the power system operation and control. Optimization of power system operation is generally assumed to be a smooth static economic dispatch (SED) modeled as a quadratic cost function. However this model makes many assumptions which are impractical to real systems. Consideration of such assumptions in ED formulation leads to the dynamic economic dispatch (DED) formulation. When more than two objectives are taken in to consideration a multi objective dynamic economic dispatch (MODED) is formed. The MODED has been considered on a quadratic cost function which is less accurate. In this paper we consider the higher order cost functions for example cubic order cost functions to model MODED problem. This higher order cost functions are more accurate and realistic than the traditional quadratic cost functions. In this paper unit commitment (UC) also considered along with MODED problem with four objectives. Dynamic programming method was used for UC and Lagrange's method was used for MODED.*

Key Words: Power system optimization, economic dispatch, multi objective dynamic economic dispatch problem, unit commitment, valve point effect loading, Lagrange's method and dynamic programming method.

1. INTRODUCTION

Today the main aim of electric power utilities is to provide high-quality reliable power supply to the consumers at the lowest possible cost while operating to meet the limits and constraints imposed on the generating units and environmental considerations. These constraints formulates the economic load dispatch (ELD) problem for finding the optimal combination of the output power of all the online generating units that minimizes the total fuel cost while satisfying an equality constraints and a set of inequality constraints. Economic dispatch is the short term determination of the optimal output of the generators, to meet the system load, at the lowest possible cost, subjected to different transmission and operational constraints.

DED problem is one of the most important problem which must be taken into consideration in power systems planning and operation. DED is aimed at planning the power output for each devoted generator unit in such a way that the operating cost is minimized and simultaneously, matching the load demand, operating limits and above all maintaining the system stability.

The Thermal, Emissions, reserve cost functions and Transmission Loss cost functions when considered individually, results in a Single Objective DED (SODED).Where more than two objectives are taken into consideration, a MODED problem results. The solution accuracy of economic dispatch problems is associated with the accuracy of the fuel cost curve parameters. In most studies, the generation cost function is considered to be quadratic function, but a cubic cost function more closely conforms to the generation cost. Therefore, the use of a cubic cost function leads to more accurate modeling of power plant costs.

This research paper aims at addressing the MODED problem formulation including with unit commitment and its solution. In this paper Lagrange's method was used for MODED problem and dynamic programming method was used for unit commitment problem.

2. MATHEMATICAL MODELLING

2.1. UNIT COMMITMENT PROBLEM FORMULATION

The objective function of UC problem is the minimization of the total cost which is the sum of the fuel cost and the start up cost of individual units for the given period subject to various constraints. Mathematically, the UC problem model can be formulated as [6]

$$\min F_t = \sum_{t=1}^{T}\sum_{i=1}^{N} U_{it}\begin{bmatrix} F_i(p_i(t))+ \\ SUC_i(1-U_{i(t-1)}) \end{bmatrix} \qquad \text{Eq. 1}$$

where F_t is the total operating cost in \$, $F_i(p_i(t))$ is the fuel cost of unit i at hour t $p_i(t)$ is the output power of ith unit at hour t, U_{it} is the on/off status of ith unit at hour t.

The major component of the operating cost, for thermal units, is the power production cost of the committed units. This can be calculated using the economic dispatch problem. In this paper MODED with four objectives was considered.

2.2. MODED PROBLEM FORMULATION

The proposed MODED problem is to minimize four objective functions namely fuel cost with valve point effect, reserve cost, transmission losses cost and emission, while satisfying a set of equality and inequality constraints. The mathematical formulation of this MODED problem is described as follows.

Fuel cost function - The cost function is generally considered to be a square (Quadratic) cost function . However, a cubic cost function is more appropriate and accurate. So, the proposed total generation cost can be expressed as follows:

$$\min F = \sum_{i=1}^{N} F_i(p_i) = \sum_{i=1}^{N}(a_i p_i^3 + b_i p_i^2 + c_i p_i + d_i) \qquad \text{Eq. 2}$$

where

F - Total fuel cost

$F_i(p_i)$ - Fuel cost of the ith generator

p_i - real power generation of unit i

a_i, b_i, c_i, d_i - cost coefficients

N   - Total number of generators

Thermal power plants have multiple steam valves. To accurately evaluate the fuel cost function, the valve point effect loading is considered as follows:

$$F_{vp} = \sum_{i=1}^{N} F_i(p_i) = \sum_{i=1}^{N}[(a_i p_i^3 + b_i p_i^2 + c_i p_i + d_i)+$$

$$\left| e_i \sin(f_i(p_{i,\min} - p_i)) \right|] \qquad \text{Eq.3}$$

where

F_{vp} -Total fuel cost with valve point effect

e_i, f_i -valve point effect coefficients of unit i

Power plant spinning reserve cost function - Plants should have enough spinning reserve to provide energy for the customers without interruption. This reserve provides cost for the system. The determination of spinning reserve values to minimize the total reserve cost function is one of the main objective in power system operation. Therefore,

$$\min FR_{\cos t} = \sum_{i=1}^{N} FR_i(R_i) = \sum_{i=1}^{N} (a_{ri}R_i^2 + b_{ri}R_i + c_{ri}) \qquad \text{Eq. 4}$$

where $FR_{\cos t}$ is the total reserve cost of the whole system, R_i is the reserve for the ith unit and a_{ri}, b_{ri}, c_{ri} are the coefficients of the reserve cost of the ith generator.

Transmission line losses cost function - Generally the generating centers and the connected load exist in geographically distributed scenario. So, the transmission network losses must be taken into account to achieve true economic dispatch. Network loss is a function of power injection at each node. Where the real power system transmission loss, PL, is expressed using B-coefficients as follows:

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{0i} P_i + B_{00} \qquad \text{Eq. 5}$$

Where i is the number of generators , j is the number of buses in the system, B_{ij} is the ij^{th} element of the loss coefficient Square matrix, B_{0i} is the ith element of the loss coefficient matrix and B_{00} is the constant loss coefficient .

The cost of transmission line losses between plants are accounted with the actual fuel cost function by using a price factor g .This factor is defined as the ratio between the fuel cost at its maximum power output to the maximum power output .That is for this multi objective case

$$g = \frac{\sum_{i=1}^{N} F(p_{i,\max})}{p_{i,\max}} \qquad \text{Eq. 6}$$

Thus, the cost function for the losses at a particular time becomes

$$F(p_{L,i}) = \sum_{i=1}^{N} g_i(p_L) \qquad \text{Eq. 7}$$

Emission dispatch formulation - The emission function of economic load dispatch problem is defined as follows [3]:

$$E(p_{ij}) = \alpha_i P_i^3 + \beta_i P_i^2 + \chi_i P_i + \delta_i \qquad \text{Eq. 8}$$

where $\alpha_i, \beta_i, \chi_i and \delta_i$ are coefficients of ith generator emission characteristics.

The proposed objective function - The proposed MODED problem can be mathematically formulated as follows [1], [2]:

$$\min F = \min(W_1\{F_{vp} + F(p_{L,i}) + FR_{\cos t}\} + W_2\{E(p_{ij})\})$$

Eq. 9

W_1, W_2 are non-negative weights used to make tradeoff between emission security and total fuel cost such that $W_1 + W_2 = 1$.

2.3. CONSTRAINTS

system power balance - $\displaystyle\sum_{i=1}^{N} U_i p_i(t) = p_d(t)$

Eq. 10

where $p_d(t)$ is the power demand at i^{th} interval.

Spinning Reserve Constraints - The sum of the maximum power generating capacities of all the committed units at a time instant should be at least equal to the sum of the known power demand and minimum spinning reserve requirement at that time instant [8], *i.e.*

$$\sum_{i=1}^{N} U_{it} p_{i(\max)} \geq p_d(t) + R_t$$

Eq. 11

where $p_{i(\max)}$ is the maximum power that can be generated by unit i and R_t is the minimum spinning reserve requirement at time t.

Generation Limit Constraints - $\begin{array}{ll} p_{i(\min)} \leq p_i(t) \leq p_{i(\max)} & when U_{it} = 1 \\ p_i(t) = 0 & when U_{it} = 0 \end{array}$

Eq. 12

where $p_{i(\min)}$ and $p_{i(\max)}$ represents the minimum and maximum generation limits of thermal units.

Unit Minimum up/down Time Constraints -

$$\left[X_{i(t-1)}^{on} - T_i^{on}\right] * \left[U_{i(t-1)} - U_{it}\right] \geq 0$$
$$\left[X_{i(t-1)}^{off} - T_i^{off}\right] * \left[U_{it} - U_{i(t-1)}\right] \geq 0$$

Eq. 13

where $X_{i(t)}^{on}$ and $X_{i(t)}^{off}$ is the time duration for unit i has been on and off respectively at hour t.

Ramp Rate Limits - $p_i(t) - p_i(t-1) \leq UR_i$

$$p_i(t-1) - p_i(t) \leq DR_i$$

Eq. 14

where *UR* and *DR* represents the generator ramp up and ramp down limits.

3. MODED BY LAGRANGE'S METHOD

The MODED problem is solved using Lagrange's method by introduction of the Lagrange's variables λ, ϕ and formulation of a Lagrange's function:

$$L = F_T - \lambda(\sum_{i=1}^{N} p_i - p_d - p_{loss}) - \phi(\sum_{i=1}^{N} R_i - SP)$$

Eq. 15

where λ, ϕ are the Lagrange multipliers, R_i is the reserve capacity of ith generator, SP is the spinning reserve.

$$SP = \max(0.1 * load, \max(p_i))$$

Algorithm for this method is as follows:

Step 1:- Read the data.

Step 2:- Calculate the $g_i = \dfrac{\sum_{i=1}^{N} F(p_i^{\max})}{p_i^{\max}}$

Step 3:- Equation $\dfrac{\partial L}{\partial p_i} = 0$ is solved and p_i^k was determined. Check the obtained vector   is fit to the constraints.

Step 4:- Equation $\dfrac{\partial L}{\partial R_i} = 0$ is solved and R_i was determined and check for the spinning reserve constraints.

Step 5:- Calculate the losses using equation(5).

Step 6:- Equation $\dfrac{\partial L}{\partial \lambda} = 0$ is solved and $\Delta\lambda_0^k$ was calculated.

Step 7:- Check the condition $\Delta\lambda^k \leq \varepsilon$

Step 8:- If the condition is fulfilled the calculations are stopped, if not, improve the value of $\lambda^k \rightarrow \lambda^{k+1}$.

Step 9:- Calculations of the improved values of $p_i^k \rightarrow p_i^{k+1}$ and repeat the steps 3,4,5and 6.

Step 10:- Iterations were continued until the condition $\Delta\lambda^k \leq \varepsilon$ was satisfied.

Step 11:- Optimal solution is used to calculate the total production cost.

4. UC BY DYNAMIC PROGRAMMING METHOD

The forward dynamic programming method has the following advantages:

1) Startups and shutdowns are not a problem;

2) Well-established theory;

3) Fast execution time.

This model is based on the priority list method but accounts for start-up and no-load costs, and minimum up and down times. A priority list [13] contains all state combinations of generators. The combinations or states are then listed from highest maximum net output power to lowest. When given the total load demand for any particular hour the list is used to determine which combinations are feasible. The feasible states include all combinations, which have a maximum net output power greater than or equal to the load demand. The costs of all feasible states are calculated and the lowest cost combination is stored. Usually in the forward-dynamic programming method the cost function is assumed to be linear, however, in this case the function is a cubic. For any given state, the cost for that state is given by the following expression:

F(P) = No-load cost + Incremental fuel cost × Power + Constraint costs.

In addition to thermal constraints, several other constraints must be considered when choosing generator combinations. The recursive algorithm is used to compute the minimum cost in hours 't' with feasible state 'L' is [10]:

$$F_{\cos t}(j,k) = \min[P_{\cos t}(j,k) + S_{\cos t}(j-1,L:j,k)$$
$$+ F_{\cos t}(j-1,L)]$$

Fig. 1 represents the flow chart for the unit commitment problem with dynamic programming method.

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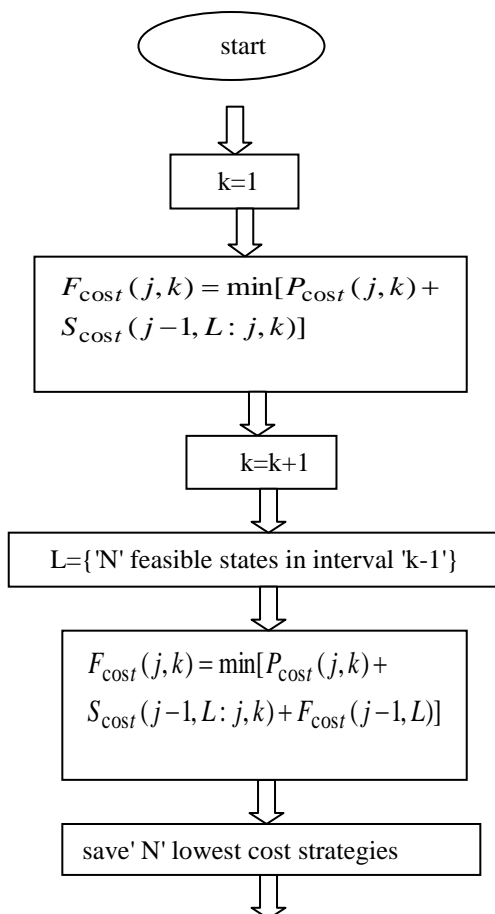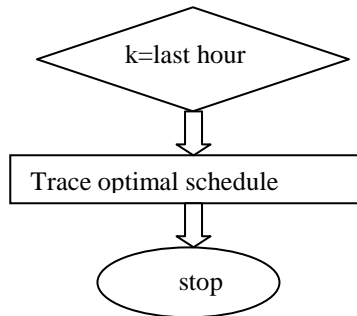
Fig – 1: Flowchart for UC by dynamic programming method

5. SIMULATION RESULTS OF MODED WITH UC

The four generator system is represented by fuel cost with valve point effect, reserve coefficients, emission coefficients and real power limits as given in Table 1.Unit characteristics and load characteristics were given in Table 2 and 3. The problem is solved in Mat lab environment. The Lagrange's method is used to obtain the solution of the dispatch problem. Dynamic programming method is used for UC problem. The solution of this problem for 8 hours is shown in Table 4.

Table 1. Characteristics of the four unit system

PARAMETERS	Gen-1	Gen-2	Gen-3	Gen-4
a($)	4.1*10^-8	8.1*10^-8	8.1*10^-7	8.2*10^-8
b($/MW)	0.00028	0.00056	0.00056	0.00056
c($/MW^2)	0.0081	0.0081	0.0081	0.0081
d($/MW^3)	35	30.9	30.9	30.9
e	300	200	200	200
f	0.035	0.042	0.042	0.042
α	75	63	63	63
β	-5.76	-5.46	-5.46	-5.46
χ	0.09	0.093	0.093	0.092
a_{ri}	45	52	52	52
b_{ri}	0.09	0.12	0.12	0.12

c_{ri}	0.0044	0.0056	0.0056	0.0056
pg_{min}	0	0	0	60
pg_{max}	680	360	360	400

Table 2. Unit characteristics

Unit	No load cost Rs/h	Full load avg. cost Rs/MWh	Initial condition	Incremental heat rate Btu/KWh	Min up time	Min down time	Cold start up cost	Hot start up cost
1	213	23.54	-5	10440	4	2	350	150
2	585.62	20.34	8	9000	5	3	400	170
3	684.74	19.74	8	8730	5	4	1100	500
4	252	28	-6	11900	1	1	0.02	0

Table 3.Load demand for three units.

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Demand	450	1530	600	540	400	1280	290	1500	1100	1221	390	490
Hour	13	14	15	16	17	18	19	20	21	22	23	24
Demand	450	569	890	1000	980	1100	560	452	780	600	400	395

Table 4. UC Results for the 4 generator test system

Hour	Demand	Unit Status				Power generation				Production cost	Transition cost+ previous state cost	Total cost
		G1	G2	G3	G4	Pg1	Pg2	Pg3	Pg4			
1	600	0	1	0	1	0	310	0	304.6	737.7	0	737.7
2	500	0	1	0	1	0	250.8	0	253.3	605.1	737.7	1342.7

3	1000	1	0	0	1	607.6	0	0	400	874.2	1532.4	2406.6
4	900	1	0	0	1	505.1	0	0	400	994.8	2406.6	3401.4
5	800	1	0	0	1	408.6	0	0	394.7	1163.2	3401.4	4564.6
6	1050	1	0	0	1	659	0	0	400	838.7	4564.6	5403.3
7	1000	1	0	0	1	607.6	0	0	400	874.2	5403.3	6277.5
8	700	1	0	0	1	357.9	0	0	344.9	959.7	6277.5	7237.1

6. CONCLUSION

The multi objective dynamic economic dispatch problem with four objectives (fuel cost including with valve point effect, reserve cost, losses cost and emission) including unit commitment was formulated. The Lagrange's algorithm is developed for the solution of the MODED problem, dynamic programming method for UC.

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