On Direct Sum of Two Intuitionistic Fuzzy Graphs

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Abstract: In this paper, the direct sum $G_A \oplus G_B$ of two Intuitionistic Fuzzy Graphs (IFGs) G_A and G_B is defined. It is proved that when two IFGs are effective then their direct sum need not be effective. The degrees of the vertices in the direct sum $G_A \oplus G_B$ of two IFGs G_A and G_B in terms of degrees of the vertices in the IFGs G_A and G_B are obtained.

Key words: Fuzzy Graph, Effective Fuzzy Graph, IFG, Effective IFG, Degree of an IFG

1. Introduction

K.T. Atanassov (4) introduced the concept of IFGs. R.Parvathi and M.G.Karunambigai (3) gave the definition of IFG. A.Nagoorgani and S.Shajitha Begum (2) gave the various types of degree in IFGs. Dr.K.Radha and Mr.S.Arumugam (1) defined the direct sum of two fuzzy graphs.

An Intuitionistic Fuzzy Graph is of the form G= (V, E), where

(i) $V = \{v_1, v_2, ..., v_n\}$ **such that** $\mu_1 : V \to [0,1]$ **and** $\gamma_1 : V \to [0,1]$ **denote the degree of membership and non-membership of the element** $v_i \in V$ **respectively and** $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$**, for every** $v_i \in V$ **(i=1, 2,3,....,n)**

(ii) $E \subset VXV$ **where** $\mu_2 : VXV \to [0,1]$ **and** $\gamma_2 : VXV \to [0,1]$ **are such that** $\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)]$

$\gamma_2(v_i, v_j) \le \max[\gamma_1(v_i), \gamma_1(v_j)]$ **and** $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ **for every** $(v_i, v_j) \in V$

Here the triple $(v_i, \mu_{1i}, \gamma_{1i})$ **denotes the degree of membership and degree of non-membership of the vertex** v_i**. The triple** $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ **denotes the degree of membership and degree of non-membership of the edge** $e_{ij} = (v_i, v_j)$ **on V.**

Let $G = \langle V, E \rangle$ **be an IFG. Then the degree of a vertex v is defined by** $d(v) = (d\mu(v), d\gamma(v))$ **where**

$d\mu(v) = \sum_{u \ne v} \mu_2(v,u)$ **and** $d\gamma(v) = \sum_{u \ne v} \gamma_2(v,u)$

2. Direct Sum

Definition 2.1

Let $G_A : ((v_i, \mu_{1iA}, \gamma_{1iA}), (e_{ij}, \mu_{2ijA}, \gamma_{2ij}))$ **and** $G_B : ((v_i, \mu_{1iB}, \gamma_{1iB}), (e_{ij}, \mu_{2ijB}, \gamma_{2ijB}))$ **denote two IFGs with underlying crisp graphs** $G_A^* : (V_1, E_1)$ and $G_B^* : (V_2, E_2)$ **respectively. Let** $v \in V_1 \cup V_2$ **and let** $E = \{uv / u, v \in V, \text{uv} \in \text{E}_1 \text{ or } \text{uv} \in \text{E}_2 \text{ but not both}\}$. **Define** $G = G_A \oplus G_B$ **by**

$$(\mu_1, \gamma_1)(u) = \begin{cases} (\mu_{1A}, \gamma_{1A}), & \text{if } u \in V_1 \\ (\mu_{1B}, \gamma_{1B}), & \text{if } u \in V_2 \\ ((\mu_{1A} \vee \mu_{1B}), (\gamma_{1A} \wedge \gamma_{1B})), & \text{if } u \in V_1 \cap V_2 \end{cases}$$

and

$$(\mu_2, \gamma_2)(uv) \leq \begin{cases} (\mu_{1A}(u) \wedge \mu_{1A}(v) \ , \ \gamma_{1A}(u) \vee \gamma_{1A}(v)) & \text{if } uv \in E_1 \\ (\mu_{1B}(u) \wedge \mu_{1B}(v) \ , \ \gamma_{1B}(u) \vee \gamma_{1B}(v)) & \text{if } uv \in E_2 \end{cases}$$

This G is the direct sum of two IFGs G_A and G_B.

Example 2.2

The following Fig 1 gives the example of the direct sum of two IFGs which have distinct edge sets.

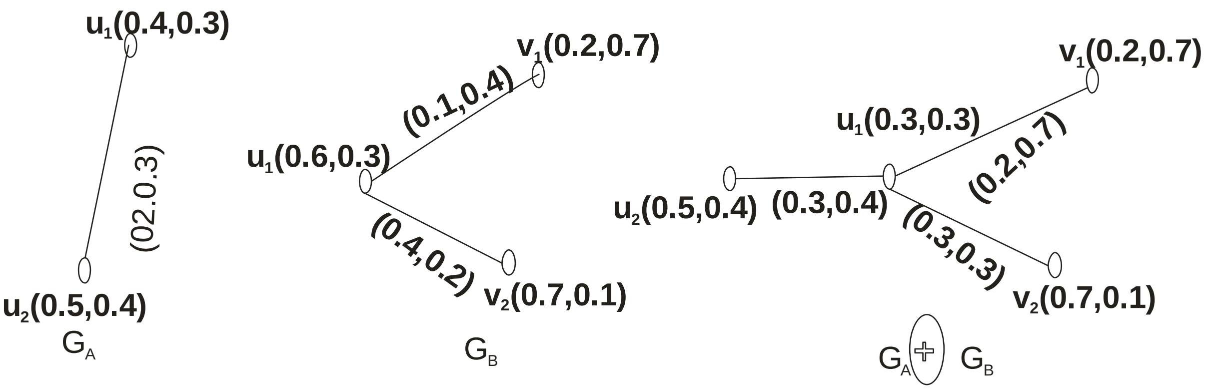
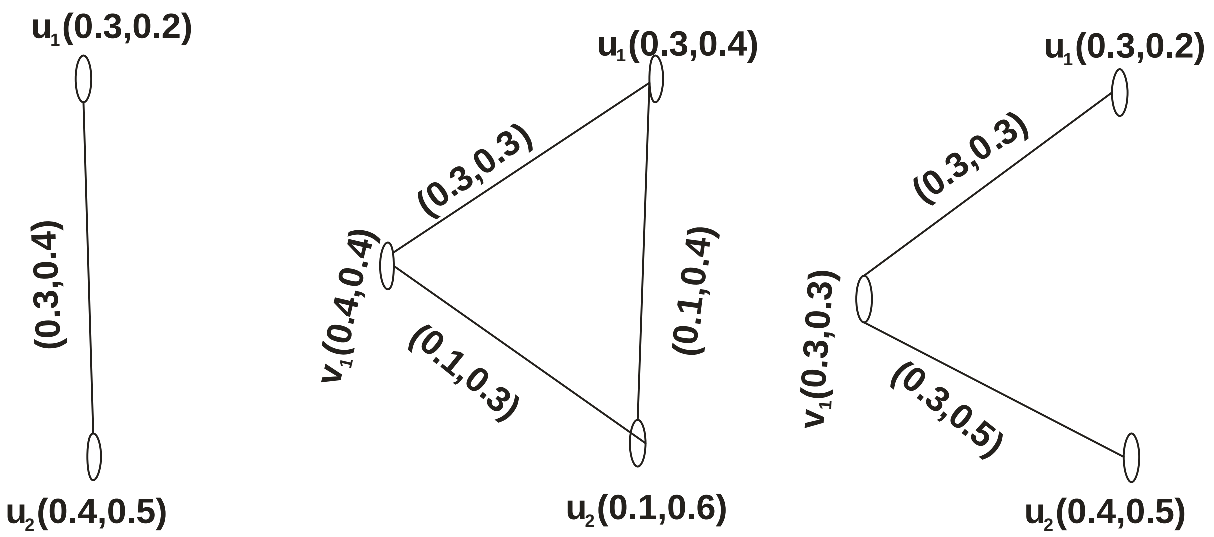


Fig 1: Direct Sum of two IFGs with disjoint edge sets.

Example 2.3

The following Fig 2 gives the example of the direct sum of two IFGs in which the edge sets are not disjoint.



3 Direct sum of effective IFGs

Definition 3.1: An IFG G is an effective IFG if

$\mu_2(uv) = \mu_1(u) \wedge \mu_1(v)$ and $\gamma_2(uv) = \gamma_1(u) \vee \gamma_1(v)$ for all $u, v \in E$.

Example 3.2

If G_A and G_B are two effective IFGs, their direct sum $G_A \oplus G_B$ need not be an effective IFG which can be seen from the example in Fig 2.

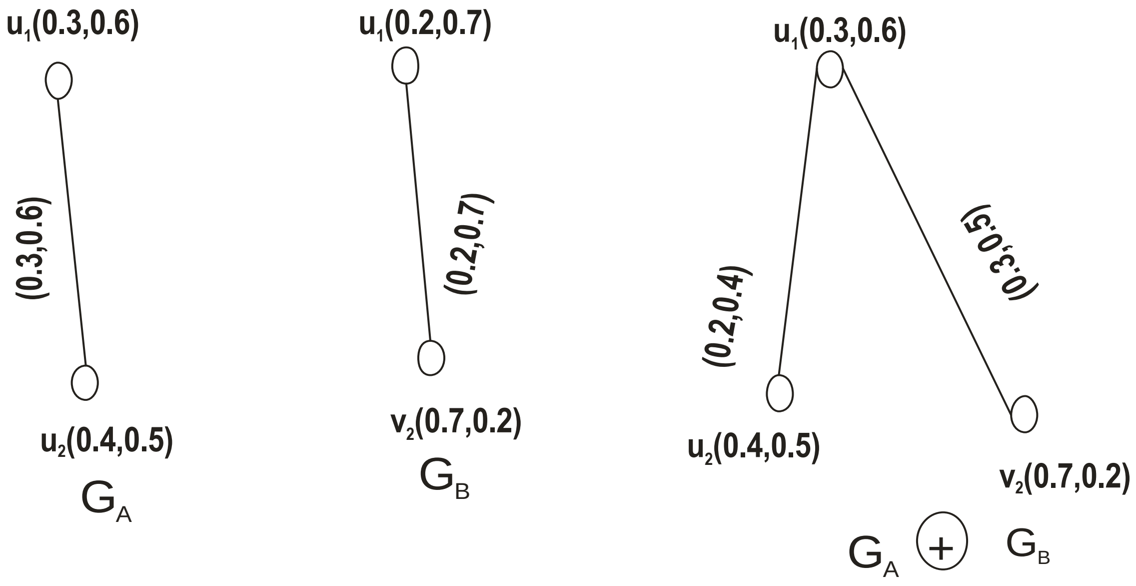


Fig 2. Direct Sum of two effective IFGs

Theorem 3.3

If G_A and G_B are two effective IFGs such that no edge of $G_A \oplus G_B$ has both ends in $V_1 \cap V_2$ and every edge uv of $G_A \oplus G_B$ with one end $u \in V_1 \cap V_2$ and $uv \in E_1$ (or E_2) is such that

$\mu_{1A}(u) \geq \mu_{1A}(v)$ or $\gamma_{1A}(u) \geq \gamma_{1A}(v)$ [(or) $\mu_{1B}(u) \geq \mu_{1B}(v)$ or $\gamma_{1B}(u) \geq \gamma_{1B}(v)$], then $G_A \oplus G_B$ is an effective IFG.

Proof:

Let uv be an edge of $G_A \oplus G_B$. We have two case to consider.

Case (i): Given $u, v \notin V_1 \cap V_2$

Then $u, v \in V_1$ or V_2 but not both.

Suppose $u, v \in V_1$, then $uv \in E_1$

$$\therefore \mu_1(u) = \mu_{1A}(u), \quad \mu_1(v) = \mu_{1A}(v) \text{ and } \mu_2(uv) = \mu_{2A}(uv), \; \gamma_2(uv) = \gamma_{2A}(uv)$$

Since G_A is an effective IFG,

$$\begin{aligned} \mu_2(uv) &= \mu_{2A}(uv) \\ &= \mu_{1A}(u) \wedge \mu_{1A}(v) \\ &= \mu_1(u) \wedge \mu_1(v) \end{aligned} \qquad \begin{aligned} \gamma_2(uv) &= \gamma_{2A}(uv) \\ &= \gamma_{2A}(u) \vee \gamma_{2A}(v) \\ &= \gamma_2(u) \vee \gamma_2(v) \end{aligned}$$

The proof is similar for $u, v \in V_2$.

Case (ii):

If $u \in V_1 \cap V_2$, $v \notin V_1 \cap V_2$ (or vice-versa), without loss of generality, assume that $v \in V_1$

Then $\mu_1(v) = \mu_{1A}(v)$.

By hypothesis, $\mu_{1A}(u) \geq \mu_{1A}(v)$, $\gamma_{1A}(u) \geq \gamma_{1A}(v)$

$$\begin{aligned} Now \quad \mu_1(u) &= \mu_{1A}(u) \vee \mu_{1A}(v) \\ &\geq \mu_{1A}(u) \\ &\geq \mu_{1A}(v) \\ &\geq \mu_1(v) \end{aligned} \qquad \begin{aligned} \gamma_1(u) &= \gamma_{1A}(u) \vee \gamma_{1A}(v) \\ &\geq \gamma_{1A}(u) \\ &\geq \gamma_{1A}(v) \\ &\geq \gamma_1(v) \end{aligned}$$

$$\text{So } \mu_1(u) \wedge \mu_1(v) = \mu_1(v) \qquad \gamma_1(u) \vee \gamma_1(v) = \gamma_1(v)$$

$$\begin{aligned} \textit{Hence}\ \ \mu_2(uv) &= \mu_{2A}(uv) & \gamma_2(uv) &= \gamma_{2A}(uv) \\ &= \mu_{1A}(u)\wedge\mu_{1A}(v) & &= \gamma_{1A}(u)\vee\gamma_{1A}(v) \\ &= \mu_{1A}(\mathrm{v}) & &= \gamma_{1A}(\mathrm{u}) \\ &= \mu_1(\mathrm{v}) & &= \gamma_1(\mathrm{u}) \\ &= \mu_1(\mathrm{u})\wedge\mu_1(\mathrm{v}) & &= \gamma_1(\mathrm{u})\vee\gamma_1(\mathrm{v}) \end{aligned}$$

Therefore $\mathrm{G_A} \oplus G_B$ is an effective IFG.

4. Degree of a vertex in the direct sum

In this section, we find the degrees of the vertices in the direct sum $G_A \oplus G_B$ of two IFGs G_A and G_B in terms of degrees of the vertices in the IFGs G_A and G_B .

Theorem 4.1

The degree of a vertex in $G_A \oplus G_B$ in terms of the degree of the vertices in G_A and G_B is given by

$$d_{G_A \oplus G_B}(u) = \begin{cases} d_{G_A}(u) & \text{if } \mathrm{u} \in \mathrm{V}_1 - V_2 \\ d_{G_B}(u) & \text{if } \mathrm{u} \in \mathrm{V}_2 - V_1 \\ d_{G_A}(u) + d_{G_B}(u) & \text{if } \mathrm{u} \in \mathrm{V}_1 \cap V_2 \ \text{ and } \mathrm{E}_1 \cap E_2 = \phi \end{cases}$$

Proof:

For any vertex in the direct sum $G_A \oplus G_B$, we have two cases to consider.

Case (i):

Either $u \in V_1$ or $\mathrm{u} \in \mathrm{V}_2$ but not both. Then no edge incident at u lies in $E_1 \cap E_2$

If $u \in V_1$,then

$$d_{G_A \oplus G_B}(u) = \left(\mathrm{d}\mu_A(u), d\gamma_A(u)\right)$$
$$= \mathrm{d}_{G_A}(u) \qquad \text{where} \quad \mathrm{d}\mu_A(u) = \sum_{u \neq v} \mu_2(u,v) \ \text{ and } \ \mathrm{d}\gamma_A(u) = \sum_{u \neq v} \gamma_2(u,v)$$

Similarly for $\mathrm{u} \in \mathrm{V}_2$

Case (ii):

If $u \in V_1 \cap \mathrm{V}_2$ but no edge incident at u lies in $E_1 \cap E_2$.

Then any edge incident at u is either in E_1 or E_2 but not in $E_1 \cap E_2$.

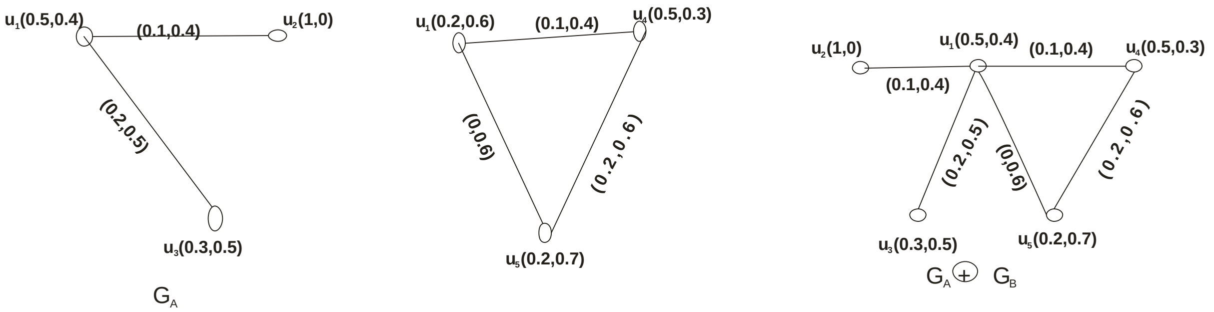
Also all these edges are included in $G_A \oplus G_B$ is given by

$$d_{G_A \oplus G_B}(u) = (\mathrm{d}\mu_A(u), d\gamma_A(u)) + (\mathrm{d}\mu_B(u), d\gamma_B(u))$$
$$= \mathrm{d}_{G_A}(u) + \mathrm{d}_{G_B}(u)$$

Hence the theorem is proved.

Example 4.2

Consider the two IFGs G_A and G_B in which the edge sets are disjoint and their direct sum is $G_A \oplus G_B$.



$$d_{G_1 \oplus G_2}(u_1) = d_{G_1}(u_1) + d_{G_2}(u_2)$$
$$(i.e) \quad (0.1,0.4) + (0.1,0.4) + (0.2,0.5) + (0,0.6) = (0.1,0.4) + (0.1,0.4) + (0,0.6) + (0.2,0.5)$$
$$(0.4,1.9) = (0.4,1.9)$$
$$\text{Now } d_{G_1 \oplus G_2}(u_2) = d_{G_1}(u_2)$$
$$(0.1,0.4) = (0.1,0.4)$$

Similarly for all other vertices, the theorem can be verified.

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