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Mathematical Model for Analysing a Tapered Optical Coupler

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Abstract – *The mathematical model introduced in this paper is based on a new concept of intrinsic mode. This model could be applied to a physical structure which is the tapered wave guide used in integrated optics communication; also known as the tapered optical coupler. We shall show how one can introduce an efficient computer program for the determination of the steepest descent path contour, in order to be able to evaluate numerically an Intrinsic Field Integral applied to our structure. Consequently, we shall show that computer results agree, to a certain extent, with the theory of functions of complex variables.*

Key Words: Modelling, Integrated Optics, waveguides, Optical Communication

1.INTRODUCTION

Let us reconsider very briefly the main structure of the Tapered wave guide as encountered in Integrated Optics applications [1]. Fig 1 shows a film of range dependent thickness and uniform refractive index n_1, which is sandwiched between an air-cover of uniform refractive index n_3 (air) and another infinite medium of uniform refractive index n_2.

At a given observation point characterised by a local thickness T and an angular angle X, corresponds one and only one saddle point θ_q (which will be defined further). In order to analyse the electromagnetic field distribution inside the guide [2] [3], and also to be able to predict the performances of our structure as encountered in integrated optics communication, one has introduced the concept of an Intrinsic Integral $I(X,\theta_q)$ as follows :

$$\mathbf{I}\,(\mathbf{X},\theta_q) = A\int_{c} \exp\left[\mathbf{j}\,\mathbf{k}\,\mathbf{S}(\mathbf{X},\theta)\right]\mathbf{d}\,\theta \qquad (1)$$

The contour of integration (C) can be any arbitrary contour as depicted in fig. 2 and 'A' is a constant depending on the wedge angle 'a'.

Physically $I(X,\theta_q)$ describes a local mode generated by integrations over any angular plane waves spectrum. Such a source-free mode (labelled q) is defined at an observation point (X,θ) and propagates smoothly along the tapered wave guide with a wave number k. The imaginary number j is such as:

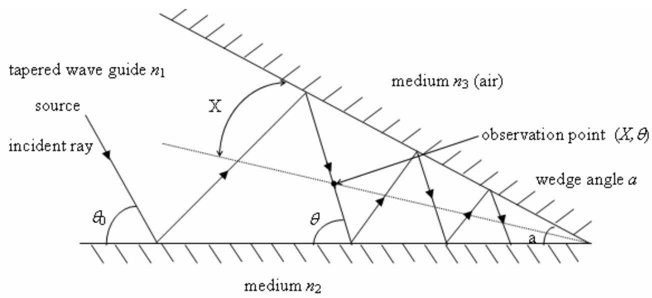

Fig -1: Configuration of the Tapered Waveguide.

$j^2 = -1$. The angle θ remains the incident angle of plane wave with respect to bottom boundary of the tapered wave guide.

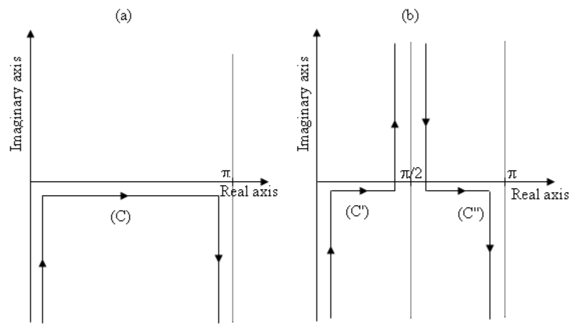

Fig -2: (a) Arbitrary contour of integration C in the complex θ-plane. (b) Contour C decomposed into contours C' and C''.

The phase function $S(X,\theta)$ could be any phase among the four species of rays involved in the propagation process; they are fully developed in [3]. In this case, the constructed plane wave spectrum maintains itself self-consistently with no effect from the source. In order to be able to evaluate systematically (eq.1), using the saddle point method, one has to know exactly the steepest descent path (SDP) contour, because [4-7] :

$$\mathbf{I}\,(\mathbf{X},\theta_q) = A\int_{SDP} \exp\left[\mathbf{j}\,\mathbf{k}\,\mathbf{S}(\mathbf{X},\theta)\right]\mathbf{d}\,\theta \qquad (2)$$

We remind that integrating along the contour (C) is synonymous to integrating along the (SDP) contour at θ_q (observation point). As long as the observation point is located far from singularity θ_c . But, once one approaches singularity, integration along (BC) need to be taken into consideration, according to some properties of special asymptotic complex function in cited references above.

2. CONTOUR CONSTRUCTION OF THE STEEPEST DESCENT PATH

The asymptotic evaluation of the Integral in (eq. 2) by the steepest descent path method requires the exact location of saddle point θ_q at any arbitrary observation point (X,θ) along the tapered wave guide, as well as the locus of the steepest descent path (SDP) contour. At a given local thickness T, in order to investigate the saddle point θ_q involved in $I(X,\theta_q)$; it is necessary to find the zero's of the derivative of the phase $S(X,\theta)$ in (eq. 2), that is to say :

$$\frac{\mathbf{dS(X,}\theta)}{\mathbf{d}\theta} = 0 \tag{3}$$

Because the function $S(X,\theta)$ is simply the θ-dependent part of the phase in (eq. 2). The equation obtained, allows us to investigate the saddle point θ_q just by a simple numerical method such as the Newton-Raphson's. Once the saddle point θ_q has been located, the burden of the integration in (eq. 2) lies in finding the steepest descent path (SDP) contour. For a specified observation point (X,θ), as well as for a given mode number q, the steepest descent path contour can be constructed analytically via the following equation:

$$\mathrm{Im}\,[j\,k\,S\,(X,\theta)] = \mathrm{Im}\,[j\,k\,S\,(X,\theta_q)\,] \tag{4}$$

Along the steepest descent path contour, the major contribution in (eq. 2) is dominated by the angles θ in the vicinity of the saddle point θ_q. The computer program which has been developed to implement the (SDP) contour; consists of mainly, in determining first θ_q at each observation point (X,θ) by using the Newton-Raphson numerical method and then, having (eq. 4) satisfied. The appendix gives a brief explanation about the computer program developed to implement ''SADDLE POINT-SDP''. In addition, one has to declare very clearly in the computer program, the principal branch corresponding to different Riemann-sheet of all complex square roots involved in the calculation of (eq. 2). Because of the multiple-valued function $\phi(\theta)$ indirectly involved in (eq. 2), through $S(X,\theta)$. We recall that $\phi(\theta)$ is the phase of the Fresnel reflection coefficient at the bottom and top boundaries of fig. 1:

$$\phi(\theta) = \arctan\left[\frac{n_{2,3}^2 - n_1^2\cos^2\theta}{n_1^2 - n_1^2\cos^2\theta}\right]^{-1/2} \tag{5}$$

3. COMPUTATIONAL RESULTS AND CONCLUSION

Figs. 3 depict the computational results of the steepest descent path contour for three different cases related to the position of the saddle point θ_q with respect to the branch point θ_c. We recall the singularity θ_c, defined as follows:

$$\theta_c = \arccos\left(\frac{n_2}{n_1}\right) \qquad \text{(eq. 6)}$$

Bearing in mind that the singularity θ_c represents only the root of $\phi(\theta)$ and it also represents physically, the critical angle of the structure given in fig. 1.

Fig. 3a corresponds to the (SDP) contour in the guided wave region, that is to say in region where $\theta_q<\theta_c$. One notices that the (SDP) contour crosses the real axis at an angle $\pi/4$; which agrees with the theory of function of complex variables in region free of singularity[4] [5]. In this region, the real saddle point θ_q denoted by 'x' sign in figure is such that $\theta_q< \theta_c$. In this case, the major contribution of integral in (eq. 2) is dominated by the portion of (SDP) near the saddle point θ_q. The branch cut contour starting from the branch point θc, denoted by the '*' sign in figure, does not contribute to the integration in (eq. 2); for in this region, the (SDP) contour does not cross the branch point θ_c yet.

Fig. 3b represents physically the (SDP) contour near the transition region, where $\theta_q\approx\theta_c$. The contour tends to surround the branch point θc. In this case, saddle point and branch point are confluents and further move of θ_q towards θ_c will make the (SDP) contour cross θ_c. We then physically enter a new region known as leaky wave region. In this case, calculations of (eq. 2) start becoming difficult, because of the contributions to the (SDP) contour of the branch cut (BC) contour [3].

Fig. 3c corresponds physically to the (SDP) contour in the leaky wave region which locates a saddle point θ_q beyond the branch point θ_c, that is to say $\mathrm{Real}(\theta_q)>\theta_c$. This region has a saddle point with a complex imaginary part. It is this very imaginary part that is responsible for the decaying of waves as they propagate along the tapered wave guide [3]. It is also noticed from figure, that the (SDP) contour crosses the line $\mathrm{Real}(\theta)=\theta_c$ and it is continuous. Beside, the branch cut contour is asymptotic to the lower part of the (SDP) contour. These are also standards properties of complex variables functions [4] [5], in region where $\mathrm{Real}(\theta_q)>\theta_c$.

The integration of (eq. 2) is theoretically supposed to be accomplished by the (SDP) method as well as the branch cut contribution. However, in practice, it is difficult to separate the two contributions and numerical problems arise. This is a major limitation of the saddle point method in region beyond singularity. In addition, in such a region, it is computationally

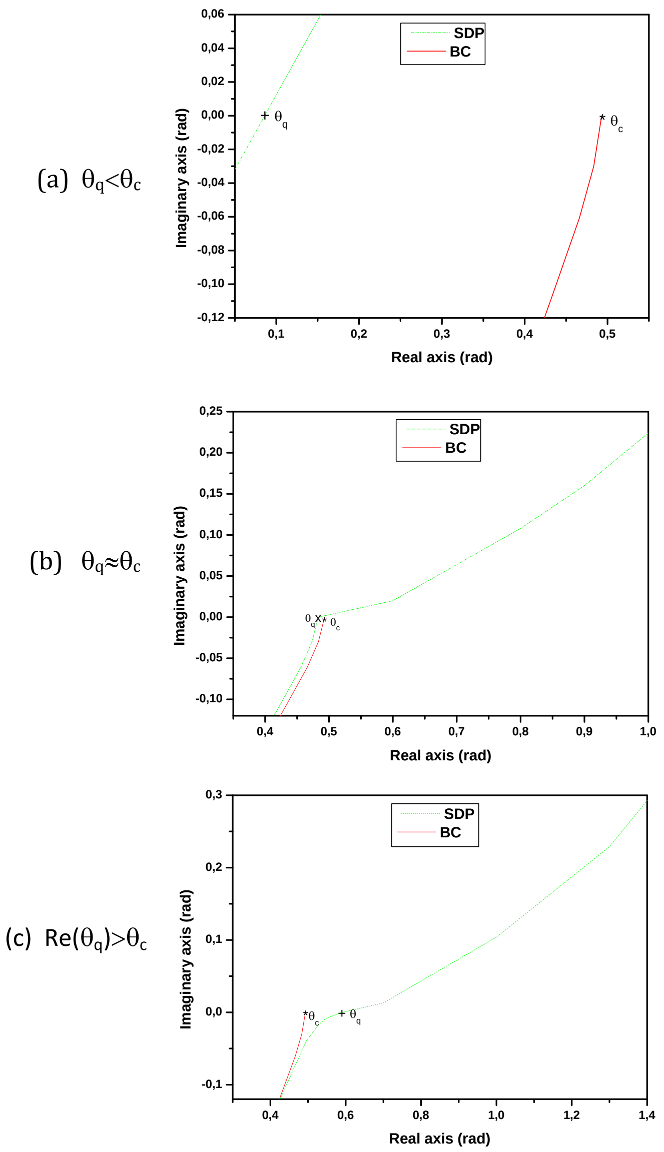

Fig -3: Plot of computed steepest descent path (SDP) and branch cut (BC) contours, for calculations of the Intrinsic Field Integral, in the complex θ-plane. Three different positions of observation point are considered; (a) Saddle point θ_q located in the guided wave region; (b) Saddle point θ_q located near transition region; (c) Saddle point θ_q located in the leaky wave region. (x denotes the saddle point θ_q; * denotes the branch point θ_c). n_1=2 , n_2=1.76 , n_3=1 , a =0.027 rad.

very cumbersome to pick-up the (SDP) contour. For at vicinity of saddle point θ_q, the steepest descent path (SDP) and the steepest ascent path (SAP) contours become very hard to distinguish.

The above complications may render the asymptotic evaluation of I(X,θ_q) impractical; consequently, one may require direct numerical method of evaluation for (eq. 2).

Bearing in mind that physically all incident rays in fig. 1 have an incident angle θ less than the critical angle θ_c which itself is less than $\pi/2$. We remind that physically the contribution of rays having an incident angle θ higher than $\pi/2$ have no significant mathematical contribution in (eq. 2).

As a matter of facts and besides all difficulties already mentioned, one has computed (eq. 2) using the (SDP) method for the three lowest modes [3]. Those results are compared with the field distributions of (eq. 2) calculated along the real axis ($0<\theta<\pi/2$). Very good agreements were obtained between the fields distributions calculated separately via different contour of integration [2]. In this way, restrictions mentioned in this paper will be overcome by suggesting another simpler contour of integration, which is the real axis, more precisely, the interval $0<\theta<\pi/2$.

In conclusion one can say that the computer plots of figs. 3 have all been carried out for a mode number q=1 propagating along the structure of fig. 1 and for $X=a$; which corresponds to an observation point (X,θ) situated on top interface of the tapered wave guide. For others X, as well as for higher modes, the contours obtained (though not represented) are qualitatively similar to the plots of figs. 3 but different quantitatively.

IV. APPENDIX

This appendix gives the brief development of the implementation of "SADDLE POINT METHOD" to compute the steepest descent path (SDP) contour of any phase function S(X,θ) of (eq. 2). The procedure locates first the saddle point θ_q for a certain observation point (X,θ), by using the Newton Raphson numerical algorithm. θ_q could be real or imaginary, depending physically on which region with respect to the critical angle θ_c one is dealing with.

The computation of the (SDP) contour is via (eq. 4). It implements the "FALSE POSITION METHOD" algorithm on each complex variable θ, defined with respect to the origin (0,0). Initially, one fixes a_{inc} (abitrary initial value equals to the wedge angle a for instance) positive so as to depict the upper part of the (SDP) contour, and chooses two arbitrary points θ_{01} and θ_{02} as initial guesses, whose values are very close to $\theta_{q's}$. Iterating θ_{01} and θ_{02} so as to have (eq. 4) satisfied, to a certain approximation (less than 10^{-8}), leads to a root of (eq. 4). Incrementing a_{inc} and starting again from θ_{01} and θ_{02} close to θ (previous root), we repeat the procedure until the upper part of the (SDP) contour is completed. That is to say when the total number of points θ is equal to N, where N is an arbitrary specified number of points in each part of the (SDP) contour. For computation of the lower part of the (SDP) contour, we repeat the same procedure, bearing in mind that a_{inc} is negative. Of course, the running time will automatically depend on the number of points N making up the (SDP) contour. For a quite small number of points N; one can get away very efficiently with a good approximation of integral I(X,θ_q).

As a matter of fact, only few points near saddle points θ_q contribute significantly to the asymptotic evaluation of any Integral I(X,θ_q) by the saddle point technique. The drawback of the algorithm, however, is that it works perfectly well in regions where $\theta_q<\theta_c$ (guided wave region); but beyond any singularity where Real(θ_q)>θ_c (leaky wave region), it becomes very difficult to compute the (SDP) contour. For past the transition region, the (SDP) contour tends to surround the singularity. As one can see it in figs. 3. Consequently, the computer program depicts points on the steepest ascent path instead of points on the steepest descent path. For this reason, one must resort to another method of evaluation of Integral I(X,θ_q) in region past the singularity. This was dealt with in some papers as in [8] and it will be done in our others works in near future.

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