# Soret Effect on Unsteady MHD Flow through Porous Medium Past an Oscillating Inclined Plate with Variable Wall Temperature and Mass Diffusion

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**Abstract -** *In the present paper, we study the effects of Soret on unsteady flow of a viscous, incompressible and electrically conducting fluid past an oscillating inclined plate through a porous medium with variable wall temperature and mass diffusion in the presence of transversely applied uniform magnetic field. The plate temperature and the concentration level near the plate increase linearly with time. The exact solutions of momentum, energy and concentration equations are obtained in closed form by the Laplace transform technique. The velocity and concentration profile is discussed with the help of graphs drawn for different parameters like thermal Grashof number, mass Grashof number, Prandtl number, Soret parameter, phase angle, permeability parameter, the magnetic field parameter and Schmidt number.*

***Key Words***: MHD, Soret effect, oscillating inclined plate, porous medium, variable temperature, mass diffusion.

## 1. INTRODUCTION

The application of incompressible viscous flow through a porous medium with involving heat and mass transfer under the influence of transversely applied uniform magnetic field is of great importance in many areas of science and engineering. Soret effect on MHD flow is also significant in many cases. Some such problems already studied are mentioned here. Ahmed and Sinha[10] have analyzed Soret effect on an oscillatory MHD mixed convective mass transfer flow past an infinite vertical porous plate with variable suction. Bhavana et al[11] have considered the Soret effect on free convective unsteady MHD flow over a vertical plate with heat source. Heat and mass transfer on unsteady MHD free convective flow past a semi-infinite vertical plate with Soret effect was analyzed by Anuradha and Priyadharshini[13]. Influence of a magnetic field on heat and mass transfer by natural convection from a vertical surface in porous media considering Soret and Dufour effects was considered by Postelnicu[2]. Further Postelnicu[3] has studied Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Beg et al[5] have worked on numerical study of free convection magnetohydrodynamic heat and mass trartsfer from a stretching surface to a saturated porous medium with Soret and Dufour effects. Oscillatory MHD free convective flow through a porous medium with mass transfer, Soret effect and chemical reaction was investigated by Ahmed and Kalila[9]. Analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and Hall effect was investigated by Ibrahim[4]. Hassan[6] has considered Soret and Dufour effects on natural convection flow past a vertical surface in a porous medium with variable surface temperature. Makinde[8] has worked on MHD mixed convection with Soret and Dufour effects past a vertical plate embedded in a porous medium. Saraswat and Srivastava[12] have investigated MHD flow past an oscillating infinite vertical plate with variable temperature through porous media. Murali et al[14] has investigated Soret and Dufour effects on unsteady hydromagnetic free convective fluid flow past an infinite vertical porous plate in the presence of chemical reaction. Anghel et al[1] have considered Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in porous medium. Srinivasacharya and Reddy[7] has investigated Soret and Dufour effects on mixed convection from an exponentially stretching surface. We are considering Soret effect on unsteady MHD flow through porous medium past an oscillating inclined plate with variable temperature and mass diffusion. The results are shown with the help of graphs.

## 2. MATHEMATICAL ANALYSIS

The x axis is taken along the vertical plane and $y$ normal to it. The plate is inclined at angle $\alpha$ from vertical. The magnetic field $B_0$ of uniform strength is applied perpendicular to the flow. Initially the plate as well as the fluid is at the same temperature $T_\infty$. The species concentration in the fluid is taken as $C_\infty$. At time t > 0, the plate starts oscillating in its own plane with frequency $\omega$ and temperature of the plate is raised to $T_w$. The concentration $C$ near the plate is raised linearly with respect to time. So, under the above assumptions, the governing equations are as follows:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta Cos\alpha\,(T - T_\infty) + g\beta^* Cos\alpha\,(C - C_\infty) - \frac{\sigma B_0^2 u}{\rho} - \frac{\upsilon u}{K}, \quad (1)$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} + D_m \frac{K_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \quad (2)$$

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2}. \quad (3)$$

The initial and boundary conditions are:

$$\left.\begin{array}{l} t \le 0: u = 0,\ T = T_\infty,\ C = C_\infty,\ \forall y, \\ t > 0: u = u_0 Cos\omega t, \\ T = T + (T_w - T_\infty)\dfrac{u_0^2 t}{\upsilon}, \quad \text{at } y=0, \\ C = C_\infty + (C_w - C_\infty)\dfrac{u_0^2 t}{\upsilon}, \\ u \to 0,\ T \to T_\infty,\ C \to C_\infty \text{ as } y \to \infty. \end{array}\right\} \quad (4)$$

Here u is the velocity of the fluid,
$g$ - The acceleration due to gravity,
$\beta$- Volumetric coefficient of thermal expansion,
t- Time,
T- Temperature of the fluid,
$\beta^*$ - Volumetric coefficient of concentration expansion,
$C$ -Species concentration in the fluid,
$\nu$- The kinematic viscosity,
$\rho$ - The density,
$C_p$- The specific heat at constant pressure,
$K$- The permeability parameter,
$k$ - Thermal conductivity of the fluid,
$D$ - The mass diffusion coefficient,
$D_m$-The effective mass diffusivity rate,
$T_w$-Temperature of the plate at y= 0,
$C_w$- Species concentration at the plate $y= 0$,
$B_0$- The uniform magnetic field,
$\sigma$ – The electrically conductivity.

To obtain the governing equations in dimensionless form the following nondimensionl variables were introduced.

$$\left.\begin{array}{l} \bar{y} = \dfrac{y u_0}{\upsilon},\ \bar{u} = \dfrac{u}{u_0},\ \bar{v} = \dfrac{v}{u_0},\ \theta = \dfrac{(T - T_\infty)}{(T_w - T_\infty)}, \\ S_c = \dfrac{\upsilon}{D},\ \mu = \rho\upsilon,\ S_t = \dfrac{D_m K_t T_\infty}{\upsilon T_\infty (C_w - C_\infty)}, \\ G_r = \dfrac{g\beta\upsilon(T_w - T_\infty)}{u_0^3}, G_m = \dfrac{g\beta^*\upsilon(C_w - C_\infty)}{u_0^3}, \\ M = \dfrac{\sigma B_0^2 \upsilon}{\rho u_0^2},\ \bar{C} = \dfrac{(C - C_\infty)}{(C_w - C_\infty)},\ \bar{t} = \dfrac{t u_0^2}{\upsilon}, \\ \bar{K} = \dfrac{u_0}{\upsilon^2} K,\ P_r = \dfrac{\mu c_p}{k}. \end{array}\right\} \quad (5)$$

Where $\bar{u}$ is the dimensionless velocity,
$\bar{t}$ - The dimensionless time,
$\theta$ - The dimensionless temperature,
$\bar{C}$ - The dimensionless concentration,
$G_r$ -Thermal Grashof number,
$G_m$ -Mass Grashof number,
$\mu$ - The coefficient of viscosity,
$P_r$ -The Prandtl number,
$S_c$ - The Schmidt number,
$\bar{K}$ - The dimensionless permeability parameter,
$S_r$ - Soret number,
$K_T$ -Thermal diffusion ratio,
$T_m$ -The mean fluid temperature,
$M$ - The magnetic field parameter.

The dimensionless form of the equations (1), (2) and (3) are as follows:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + G_r Cos\alpha\theta + G_m Cos\alpha\bar{C} - M\bar{u} - \frac{\bar{u}}{\bar{K}}, \quad (6)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{S_c}\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + S_r \frac{\partial^2 \theta}{\partial \bar{y}^2}, \quad (7)$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \bar{y}^2}. \quad (8)$$

The corresponding boundary conditions are

$$\left.\begin{array}{l} \bar{t} \le 0: \bar{u} = 0,\ \theta = 0,\ \bar{C} = 0,\ \forall \bar{y}, \\ \bar{t} > 0: \bar{u} = Cos\varpi \bar{t},\quad \theta = \bar{t},\ \bar{C} = \bar{t}, \text{ at } \bar{y} = 0, \\ \bar{u} \to 0,\quad \theta \to 0,\ \bar{C} \to 0, \text{ as } \quad \bar{y} \to \infty. \end{array}\right\} \quad (9)$$

Dropping bars in the above equations, we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r Cos\alpha\,\theta + G_m Cos\alpha\, C - Mu - \frac{u}{K}, \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} + S_r \frac{\partial^2 \theta}{\partial y^2}, \quad (11)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2}. \quad (12)$$

The boundary conditions become:

$$\left.\begin{array}{l} t \le 0: u = 0, \theta = 0,\ C = 0, \quad \forall y, \\ \bar{t} > 0: u = Cos\omega t, \theta = t,\ C = t, \quad \text{at } \ y = 0, \\ u \to 0,\ \theta \to 0,\ C \to 0, \text{ as } \ y \to \infty. \end{array}\right\} \quad (13)$$

The dimensionless governing equations (10) to (12), subject to the boundary conditions (13), are solved by using the

Laplace - transform technique. The solutions derived are as under:

$$u = \frac{1}{4}e^{-it\omega}(A_{21} + A_{22} - A_{23} - A_{24},) + \frac{1}{2A^2}Cos\alpha \Big[(2G_r + \xi G_m)(2e^{-\sqrt{A}y}A_1 + 2(At + P_r)A_2e^{-\sqrt{A}y} + y\sqrt{A}e^{-\sqrt{A}y}A_3 + \frac{(4-P_r)}{2}A_4A_7) + 2(\xi G_m - G_r)(A_{11} + A_9(Ay^2P_r + 2At + 2P_r - 2) + A_7(A_5 - 1) - \frac{A_5P_r}{\sqrt{\pi}}) - 2G_m(1+\xi)(A_{12} + A_{10}(Ay^2S_c + 2At + 2S_c - 2) + A_6A_8(S_c - 1))\Big]$$

$$C = (1+\xi)(\frac{2t + y^2S_c}{2}Erfc[\frac{y\sqrt{S_c}}{2\sqrt{t}}] - \frac{A_{12}}{2A}) + \xi(\frac{2t + y^2P_r}{2}Erfc[\frac{y\sqrt{P_r}}{2\sqrt{t}}] - \frac{A_{11}}{2A})$$

$$\theta = t\{(1 + \frac{y^2P_r}{2t})erfc[\frac{\sqrt{P_r}}{2\sqrt{t}}] - \frac{y\sqrt{P_r}}{\sqrt{\pi\sqrt{t}}}e^{-\frac{y^2}{4t}}P_r\}$$

For making the solution concise, the symbols and expressions for the constants involved in the above equations are defined in the appendix.

## 3. RESULT AND DISCUSSIONS

The velocity and temperature profile for different parameters like, mass Grashof number *(Gm)*, thermal Grashof number *(Gr)*, magnetic field parameter *(M)*, permeability parameter *(K)*, Soret number *(Sr)*, Prandtl number *(Pr)* and time *(t)* are shown in figures 1 to 14. It is observed from figure1, that the velocity of fluid decreases when the angle of inclination *(α)* is increased. It is observed from figure 2 and 3, that the mass and thermal Grashof numbers increase the velocity. It is observed from figure 4, that the effect of increasing values of the parameter *(M)* results in decreasing u. Further, it is observed that velocity is increases when Prandtl number increased (figure 5). When the Schmidt number is increased then the velocity gets decreased (figure 6). If Soret number is increased then the velocity is increased (figure 7). It is deduced that when phase angle increased then the velocity gets decreased (figure 8). When the permeability parameter is increase then the velocity is increased (figure 9). Further, from figure 10, it is observed that velocities increase with time. It is observed that the concentration increases when Soret number, Prandtl number and time are increased (figures 11, 13, 14). However, concentration decreases when Schmidt number is increased (figure 12).

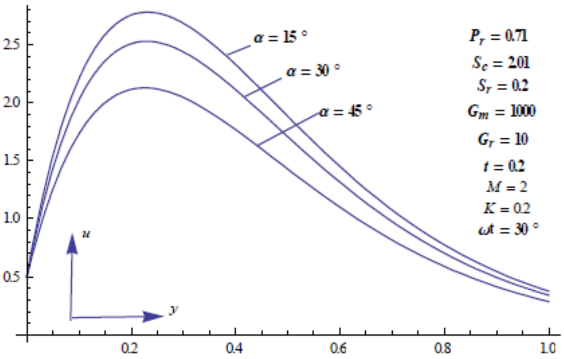

Figure 1: Velocity u for different values of *α*

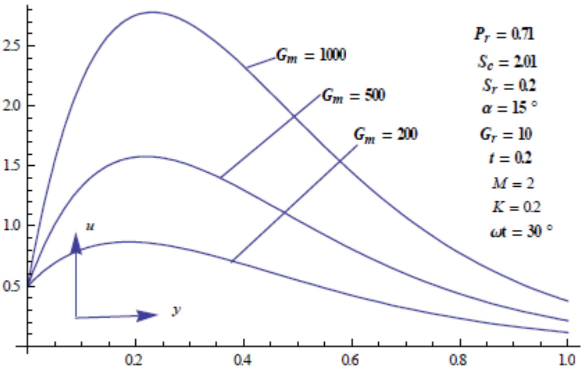

Figure 2: Velocity u for different values of *Gm*

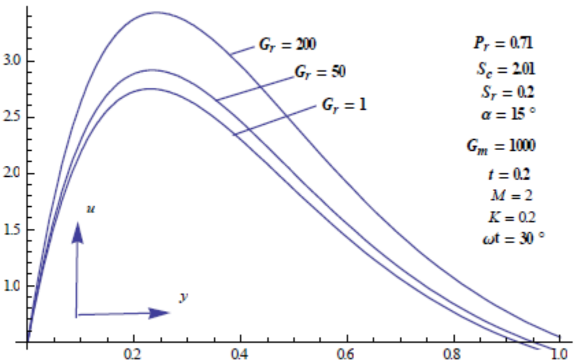

Figure 3: Velocity u for different values of *Gr*

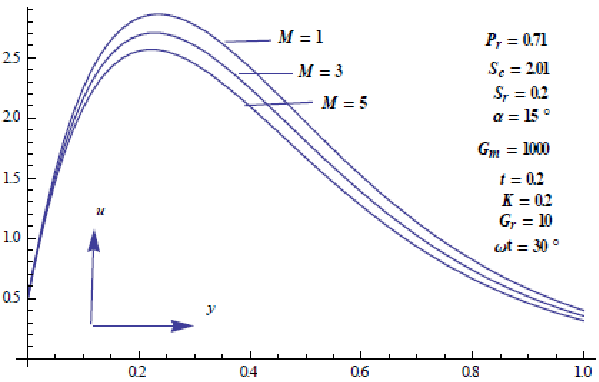

Figure 4: Velocity u for different values of *M*

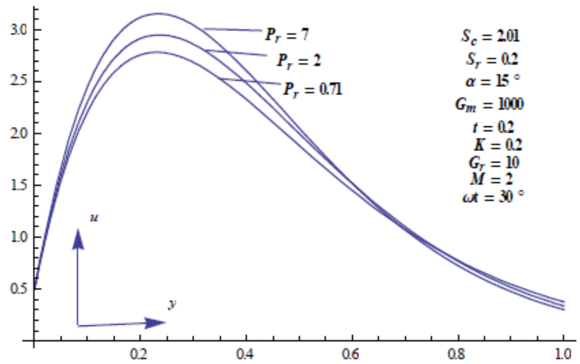


Figure 5: Velocity u for different values of *Pr*

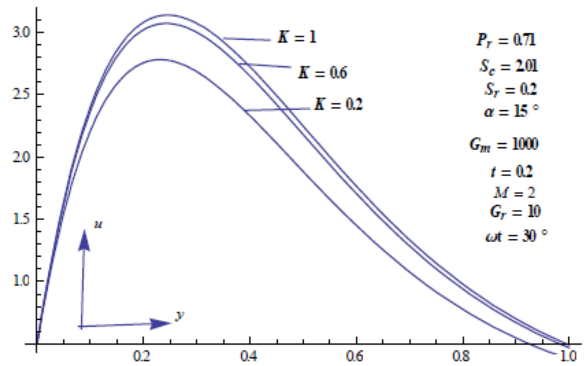


Figure 9: Velocity u for different values of *K*

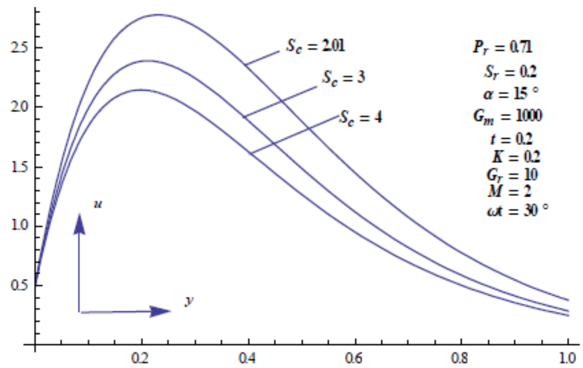


Figure 6: Velocity u for different values of *Sc*

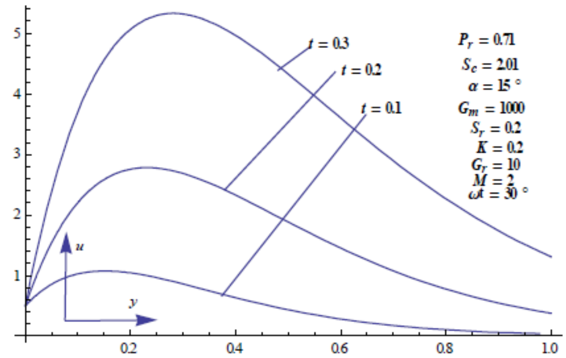


Figure 10: Velocity u for different values of *t*

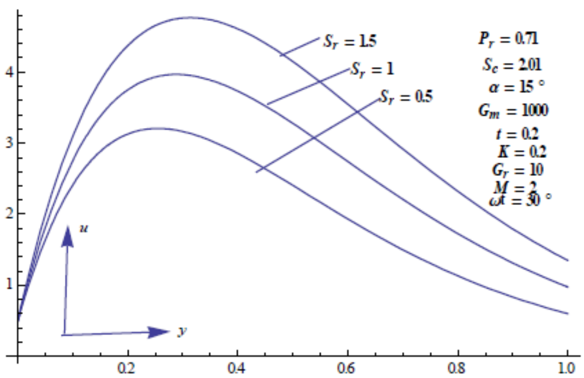


Figure 7: Velocity u for different values of *Sr*

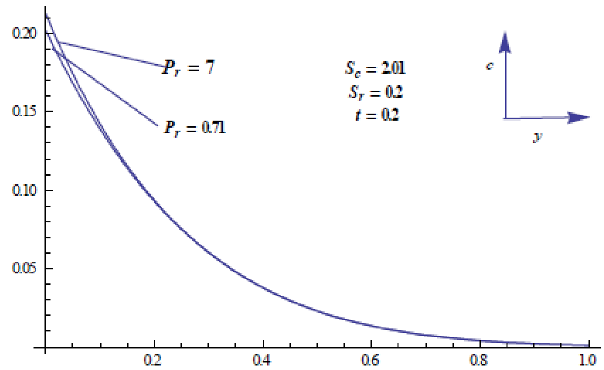


Figure 11: Concentration c for different values of *Pr*

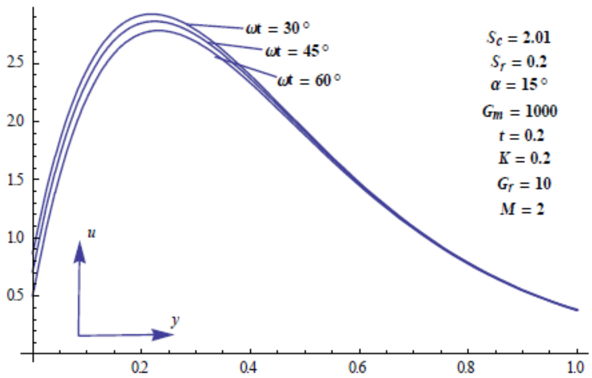


Figure 8: Velocity u for different values of *ωt*

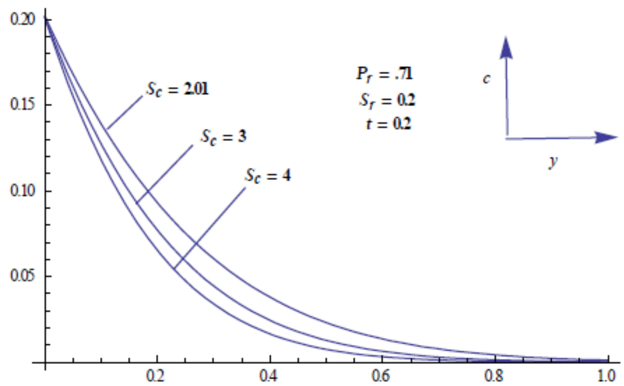


Figure 12: Concentration c for different values of *Sc*

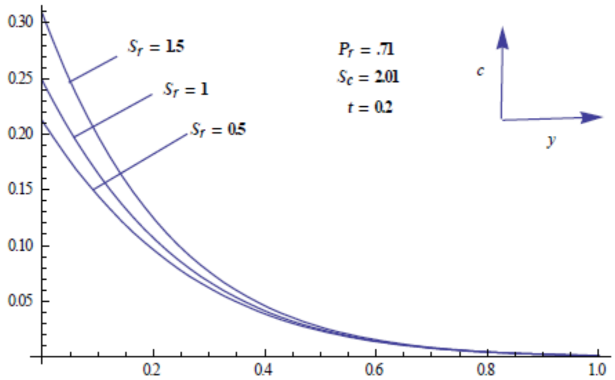


Figure 13: Concentration c for different values of *Sr*

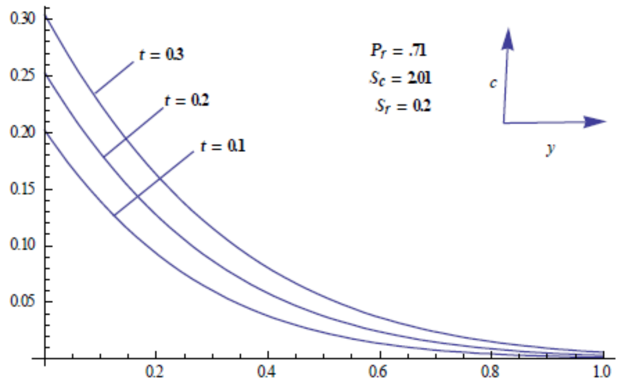


Figure 14: Concentration c for different values of *t*

## 4. CONCLUSION

The conclusions of the study are as follows:

- Velocity increases with the increase in thermal Grashof number *($G_r$)*, mass Grashof number *($G_m$)*, Prandtl number *($P_r$)*, Soret number *($S_r$)*, permeability parameter *(K)* and time *(t)*.
- Velocity decreases when the angle of inclination of the plate *(α)*, the magnetic field *(M)* and Schmidt number *($S_c$)* are increased.
- Concentration increases with the increase in Prandtl number *($P_r$)*, Soret number *($S_r$)* and time *(t)*.
- Concentration decreases when Schmidt number *($S_c$)* is increased.

## APPENDEX

$$A_1 = (1 + A_{13} + e^{2\sqrt{A}y}(1 - A_{14})),\ A_2 = -A_1,$$

$$A_3 = (1 + A_{13} + e^{2\sqrt{A}y}(A_{14} - 1)),$$

$$A_4 = (-1 + A_{15} + A_{19}(A_{16} - 1)),$$

$$A_5 = (-1 + A_{17} + A_{19}(A_{18} - 1)),$$

$$A_6 = (1 + A_{17} + A_{20}(1 - A_{18})),$$

$$A_7 = \exp\left(\frac{At}{-1+P_r} - y\sqrt{\frac{At}{-1+P_r}}\right),$$

$$A_8 = \exp\left(\frac{At}{-1+S_c} - y\sqrt{\frac{At}{-1+S_c}}\right),$$

$$A_9 = \left(-1 + Erf\left[\frac{y\sqrt{P_r}}{2\sqrt{t}}\right]\right),$$

$$A_{10} = \left(-1 + Erf\left[\frac{y\sqrt{S_c}}{2\sqrt{t}}\right]\right),$$

$$A_{11} = \frac{2Ay\exp\left(-\frac{y^2P_r}{4t}\right)\sqrt{tP_r}}{\sqrt{\pi}},$$

$$A_{12} = \frac{2Ay\exp\left(-\frac{y^2S_c}{4t}\right)\sqrt{tS_c}}{\sqrt{\pi}},$$

$$A_{13} = Erf\left[\frac{2\sqrt{At} - y}{2\sqrt{t}}\right],$$

$$A_{14} = Erf\left[\frac{2\sqrt{At} + y}{2\sqrt{t}}\right],$$

$$A_{15} = Erf\left[\frac{y - 2t\sqrt{\frac{AP_r}{-1+P_r}}}{2\sqrt{t}}\right],$$

$$A_{16} = Erf\left[\frac{y + 2t\sqrt{\frac{AP_r}{-1+P_r}}}{2\sqrt{t}}\right],$$

$$A_{17} = Erf\left[\frac{2t\sqrt{\frac{A}{-1+P_r}} - y\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_{18} = Erf\left[\frac{2t\sqrt{\frac{A}{-1+P_r}} + y\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_{19} = \exp\left(2y\sqrt{\frac{AP_r}{-1+P_r}}\right),$$

$$A_{20} = \exp\left(2y\sqrt{\frac{AS_c}{-1+S_c}}\right),$$

$$A_{21} = \exp(-y\sqrt{A - iw}) + \exp(y\sqrt{A - iw}),$$

$$A_{22} = \exp(-y\sqrt{A + iw} + 2itw) + \exp(y\sqrt{A + iw} + 2itw),$$

$$\zeta = \frac{S_cP_rS_r}{P_r - S_c},$$

$$A_{23} = \exp(-y\sqrt{A-iw})erf[\frac{y-2t\sqrt{A-iw}}{2\sqrt{t}}]$$
$$+\exp(y\sqrt{A-iw})erf[\frac{y+2t\sqrt{A-iw}}{2\sqrt{t}}],$$
$$A_{24} = \exp(-y\sqrt{A+iw}+2itw)erf[\frac{y-2t\sqrt{A+iw}}{2\sqrt{t}}]$$
$$+\exp(y\sqrt{A+iw}+2itw)erf[\frac{y+2t\sqrt{A+iw}}{2\sqrt{t}}].$$

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