On Solving Fuzzy Rough Linear Fractional Programming Problem

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Abstract *In this paper, introduce algorithm for solving fuzzy rough linear fractional programming (FRLFP) problem. All variables and coefficients of the objective function and constraints are fuzzy rough number. The FRLFP problem can be reduced as to the multi objective fuzzy linear fractional programming (FLFP) problems, where all variables and coefficients of the objective function and* constraint are fuzzy number. Further, using the decomposition algorithm to obtain an optimal fuzzy rough solution. A numerical example is given for the sake of illustration.

***Key Words*:Triangular fuzzy number, Fuzzy rough interval, Multi objective linear fractional programming**

1.INTRODUCTION

We need to fractional linear programming in many real-world problems such as production and financial planning and institutional planning and return on investment, and others. Multi objective Linear fractional programming problems useful targets in production and financial planning and return on investment. Charnes and Cooper, Used variable transformation method to solve linear fractional programming problems [1]. Tantawy, Proposes a new method for solving linear fractional programming problems [2]. Jayalakshmi and Pandian, Proposed a new method namely, denominator objective restriction method for finding an optimal solution to linear fractional programming problems [3]. Moumita and De, Study of the fully fuzzy linear fractional programming problem using graded mean integration representation method [4]. Ezzati et al, Used a new algorithm to solve fully fuzzy linear programming problems using the multi objective linear programming problem [5]. Haifang et al, Solving a fully fuzzy linear programming problem through compromise programming [6].Pawlak, Used a rough set theory a new mathematical approach to imperfect knowledge [7]. Kryskiewice, Used a rough set theory to incomplete has found many interesting applications [8]. Pal, The rough set approach seems to be of fundamental importance to cognitive sciences, especially in the areas of machine learning, decision analysis, and expert systems [9]. Pawlak, Rough set theory, introduced by the author, expresses vagueness, not by means of membership, but employing a boundary region of a set. The theory of rough set deals with the approximation of an arbitrary subset of a universe by two definable or observable subsets called lower and upper approximations [10]. Tsumoto, Used the concept of lower and upper approximation in rough sets theory, knowledge hidden in information systems may be unraveled and expressed in the form of decision rules [11]. Lu and Huang, The concept of rough interval will be introduced to represent dual uncertain information of many parameters, and the associated solution method will be presented to solve rough interval fuzzy linear programming problems [12].
In this paper, we propose algorithm for solve fuzzy rough linear fractional programming problem where all variables and coefficients are fuzzy rough number. Use the decomposition to the fully fuzzy linear fractional programming problem for obtaining an optimal fuzzy rough solution, based on the variable transformation method.

2. Preliminaries

2.1. Triangular Fuzzy Number

Definition 1. For any a fuzzy set $\hat{A}$the membership function of $\hat{A}$is written asn$A(x)$, a fuzzy set $\hat{A}$ is defined by:

$\tilde{A} = \{(x, A(x)) : \ x\epsilon\mathcal{R}^n \ , A(x)\epsilon[0,1]\}$

The $(\alpha - cut)$of fuzzy set $\hat{A}$ defined as:

$A_\alpha = \{x: \ A(x) \geq \alpha ,\} = [A_\alpha^L, \ A_\alpha^U], \ \alpha\epsilon[0,1]$ Where

$A_\alpha^L = inf\{x: A(x) \geq \alpha \}$and

$A_\alpha^U = sup\{x: A(x) \geq \alpha \}, \ \alpha\epsilon[0,1]$.The support set of a fuzzy set $\hat{A}$defined as:

$\sup(\tilde{A}) = \{x\epsilon\mathcal{R} : A(x) > 0 \}$.

Definition 2.A fuzzy set $\widetilde{A}$ is convex if for any $x_1, x_2 \in \mathcal{R}^n \ and \ \omega \in [0,1]$, we have:

$A(\omega x_1 + (1-\omega)x_2) \geq min\{A(x_1), A(x_2)\}$.

Definition 3. A fuzzy set $\hat{A}$ is called normal if A_1=$\{x: A(x) = 1 \} \neq \varphi$. The set of all points $x\epsilon\mathcal{R}^n$ with $A(x) = 1$ is called core of a fuzzy set$\hat{A}$.

Definition 4. Let there exists
$a_1, a_2, a_3 \in \mathcal{R}$

such that:

$$A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{x-a_3}{a_2-a_3}, & a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases}$$

then we say that $\hat{A}$ is triangular fuzzy number, written as: $\widetilde{A} = (a_1, a_2, a_3)$

In this paper the class of all triangular fuzzy number is called Triangular fuzzy number space, which is denoted by $TF(N)$.

Definition 5. For any triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$, for all $\alpha \epsilon [0,1]$ we get a crisp

interval by $(\alpha - cut)$ operation defined as:

$\widetilde{A}_\alpha = [a_1 + (a_2 - a_1)\alpha \ , \ a_3 + (a_2 - a_3)\alpha]$

$\widetilde{A}_\alpha = [a_1^L(\alpha), \ a_3^U(\alpha)]$

Definition 6. A positive triangular fuzzy number $\widetilde{A}$ is denoted as

$\widetilde{A} = (a_1, a_2, a_3)$ where $a_1 > 0$.

And we say that the fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$ is negative where $a_3 < 0$.

Definition 7. For any two fuzzy numbers $\widetilde{A}$, $\widetilde{B} \in TF(N)$ we say that $\tilde{A} \tilde{\subseteq} \tilde{B}$ iff $A(x) \lesssim B(x)$ for

all $x \epsilon \mathcal{R}$.

2.2 Basic Operation of Triangular Fuzzy Number:

Let $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. Then

i. $\widetilde{A} + \widetilde{B} = (a_1 + b_1 \ , \ a_2 + b_2 \ , \ a_3 + b_3)$

ii. $\widetilde{A} - \widetilde{B} = (a_1 - b_3 \ , \ a_2 - b_2 \ , \ a_3 - b_1)$,

iii. $k\widetilde{A} = (k\,a_1\,,\ ka_2\,,\ ka_3),\quad for\,k \geq 0.$

iv. $k\widetilde{A} = (k\,a_3\,,\ ka_2\,,\ ka_1),\quad for\,k < 0.$

v. $$\widetilde{A} \times \widetilde{B} = \begin{cases} (a_1b_1\,,\ a_2b_2\,,\ a_3b_3) a_1 \geq 0 \\ (a_1b_3\,,\ a_2b_2,\ a_3b_3) a_1 < 0\,, a_3 \geq 0 \\ (a_1b_3\,,\ a_2b_2\,,\ a_3b_1) a_3 < 0 \end{cases}$$

vi. $If\ 0 \notin \widetilde{B} = (b_1, b_2\,, b_3)\quad then \widetilde{A} \div \widetilde{B} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right).$

Definition 8. Let $\widetilde{A} = (a_1, a_2\,, a_3)$ and $\widetilde{B} = (b_1, b_2\,, b_3)$be two triangular fuzzy numbers, and then greater-than and less-than operations can be defined as follows [12]:

$$\widetilde{A} \geq \widetilde{B} \Leftrightarrow a_1 \geq b_1\,,\ a_2 \geq b_2\ ,\ a_3 \geq b_3$$

$$\widetilde{A} \leq \widetilde{B} \Leftrightarrow a_1 \leq b_1\,,\ a_2 \leq b_2\ ,\ a_3 \leq b_3$$

2.3 Fuzzy Rough Interval

Definition 9. Let X be denote a compact set of real numbers. A fuzzy rough interval $\widetilde{X}^R$ is defined as: $\widetilde{X}^R = [\widetilde{X}^L : \widetilde{X}^U]$ where $\widetilde{X}^L$ and $\widetilde{X}^U$ are fuzzy sets called lower and upper approximation fuzzy of $\widetilde{X}^R$ with $\widetilde{X}^L \tilde{\subseteq} \widetilde{X}^U$.

In this paper we denote by $\tilde{I}^R$ the set of all fuzzy rough with triangular fuzzy number in the real number$\mathcal{R}$.

Suppose $\widetilde{A}^R, \widetilde{B}^R \in \tilde{I}^R$ we can write

$\widetilde{A}^R = [\widetilde{A}^L : \widetilde{A}^U]$, $\widetilde{B}^R = [\widetilde{B}^L : \widetilde{B}^U]$,

Where$\widetilde{A}^L, \widetilde{A}^U$, $\widetilde{B}^L$ and $\widetilde{B}^U$triangular fuzzy numbers defined as:

$\tilde{A}^L = (a_1^L, a_2, a_3^L), \tilde{A}^U = (a_1^U, \quad a_2, a_3^U),$

$\tilde{B}^L = (b_1^L, b_2, b_3^L),$ and $\tilde{B}^U(b_1^U, b_2, b_3^U)$

Where $a_1^U \le a_1^L \le a_2 \le a_3^L \le a_3^U$ *and*

$b_1^U \le b_1^L \le b_2 \le b_3^L \le b_3^U.$

Proposition 1. For the fuzzy rough $\tilde{X}^R$ the following holds:

i. $\tilde{X}^R \succcurlyeq \tilde{0}^R$, iff $\tilde{X}^L \gtrsim \tilde{0}$ and $\tilde{X}^U \gtrsim \tilde{0}$

ii. $\tilde{X}^R \preccurlyeq \tilde{0}^R$, iff $\tilde{X}^L \lesssim \tilde{0}$ and $\tilde{X}^U \lesssim \tilde{0}$.

Definition 9. A fuzzy rough interval $\tilde{A}^R = [\tilde{A}^L : \tilde{A}^U]$ is said to be normalized if $\tilde{A}^L$ *and* $\tilde{A}^U$ are normal.

Definition 10. Let $\tilde{A}^R = [\tilde{A}^L : \tilde{A}^U]$ and $\tilde{B}^R = [\tilde{B}^L : \tilde{B}^U]$ be two triangular fuzzy rough numbers and then

greater-than and less-than operations can be defined as follows:

$\tilde{A}^R \ge \tilde{B}^R \Leftrightarrow \tilde{A}^L \ge \tilde{B}^L \text{ and } \tilde{A}^U \ge \tilde{B}^U$

$\tilde{A}^R \le \tilde{B}^R \Leftrightarrow \tilde{A}^L \le \tilde{B}^L \text{ and } \tilde{A}^U \le \tilde{B}^U$

Definition 11. Let $\tilde{A}^R = [\tilde{A}^L : \tilde{A}^U]$ and

$\tilde{B}^R = [\tilde{B}^L : \tilde{B}^U]$

be two fuzzy rough intervals in $\mathcal{R}$. We write $\tilde{A}^R \cong^R \tilde{B}^R$ if and only if $\tilde{A}^L \cong \tilde{B}^L$ and $\tilde{A}^U \cong \tilde{B}^U$.

2.4 Basic Operation for Fuzzy Rough Interval

For any fuzzy rough $\tilde{A}^R$, $\tilde{B}^R$ when $\tilde{A}^R \gtrsim^R \tilde{0}^R$ *and* $\tilde{B}^R \gtrsim^R \tilde{0}^R$ we can defined the operation for fuzzy rough as follows:

1) $\tilde{A}^R + \tilde{B}^R = [(\tilde{A}^L + \tilde{B}^L) : (\tilde{A}^U + \tilde{B}^U)]$

2) $\tilde{A}^R - \tilde{B}^R = [(\tilde{A}^L - \tilde{B}^L) : (\tilde{A}^U - \tilde{B}^U)]$

3) $\tilde{A}^R \times \tilde{B}^R = [(\tilde{A}^L \times \tilde{B}^L) : (\tilde{A}^U \times \tilde{B}^U)]$

4) $\tilde{A}^R \div \tilde{B}^R = [(\tilde{A}^L \div \tilde{B}^L) : (\tilde{A}^U \div \tilde{B}^U)]$.

2.5 Linear Fractional Programming Problem

The general linear fractional programming (LFP) problem is defined as follows:

$$Max \ \left\{ z = \frac{N(x)}{D(x)} = \frac{c^T x + \alpha}{d^T x + \beta} \right\}$$

Subject to:
$x \in S = \{ x \in \mathcal{R}^n : \ Ax \leq b, x \geq 0, \ \}$

Where, $c^T,\ d^T \in \mathcal{R}^n,\ \alpha,\ \beta \in \mathcal{R},\ b \in \mathcal{R}^m,\ A \in \mathcal{R}^{m \times n}, d^T x + \beta \neq 0 \ for\ all\ \ x \in S$

Variable Transformation Method

In this method for solving linear fractional programming, we as usual assume that the denominator is positive everywhere in S and make the variable change

$$w = \frac{1}{d^T x + \beta}$$

With this change, the objective function becomes

$$\sum_{i=1}^{n}(c_i x_i w) + \alpha w$$

If we make the additional variable changes $y_i = x_i w$ For all $i = 1,2, \ldots n$. The linear

fractional programming problem with variable changes as formulation

$$Max \quad \{z = c^T y + \alpha w\}$$

Subject to: $Ay - bw \leq 0$,

$$(d^T x + \beta)w = 1$$

$$0 \leq y \in \mathcal{R}^n, \qquad 0 \leq w \in \mathcal{R}$$

With $m+1$ constraints and $n+1$ variables that can be solved by the simplex method.

Definition12. A point $x^* \in \mathcal{R}^n$is said to be optimal solution of the linear fractional programming

problem if there does not exist

$x \in \mathcal{R}^n$such that $\frac{N(x^*)}{D(x^*)} \leq \frac{N(x)}{D(x)}$.

2.6. Multiobjective Linear Fractional Programming Problem

The general multi objective linear fractional programming (MOLFP) problem may be written as:

$$Max \ z(x) = \{z_1(x), z_2(x), \ldots \ldots z_k(x)\}$$

Subject to:

$$x \in S = \{x \in \mathcal{R}^n : \ Ax \leq b, x \geq 0\}$$

Where $z_i(x) = \frac{\sum_{i=1}^{n} c_i x + \alpha_i}{\sum_{i=1}^{n} d_i x + \beta_i} = \frac{N_i(x)}{D_i(x)}$,

$c_i, d_i \in \mathcal{R}^n, \alpha_i, \beta_i \in \mathcal{R}$

$D_i(x) > 0,\ for\ all\ i = 1,2,\dots,K$

Definition 13. A point $x^* \in$

3. Problem Formulation

The fuzzy rough linear fractional programming (FRLFP) problem is defined as follows:

$$Max\left\{\tilde{Z}^R(x) = \frac{\tilde{N}^R(x)}{\tilde{D}^R(x)} = \frac{\sum_{j=1}^{m} \tilde{c}_j^R \tilde{x}_j^R + \tilde{\alpha}^R}{\sum_{j=1}^{m} \tilde{d}_j^R \tilde{x}_j^R + \tilde{\beta}^R}\right\}, \quad (1)$$

Subject to:

$$\sum_{j=1}^{m} \tilde{A}_{ij}^R \tilde{x}_j^R \leq \tilde{B}_i^R \quad i = 1,2,\dots,n\ .$$

$$\tilde{x}_j^R \geq 0$$

Where $\tilde{c}_j^R$, $\tilde{d}_j^R$, $\tilde{\alpha}_j^R, \tilde{\beta}_j^R$ and $\tilde{B}_i^R \in \tilde{I}^R$ defined as:

$\tilde{C}_j^R = [\tilde{C}_j^L : \tilde{C}_j^U]$, $\tilde{d}_j^R = [\tilde{d}_j^L : \tilde{d}_j^U]$

$\tilde{\alpha}^R = [\tilde{\alpha}^L : \tilde{\alpha}^U]$, $\tilde{\beta}^R = [\tilde{\beta}^L : \tilde{\beta}^U]$

$\tilde{B}_i^R = [\tilde{B}_i^L : \tilde{B}_i^U]$

Also $\tilde{A}_{ij}^R$ is an $m \times n$ constraint matrix defined as: $\tilde{A}_{ij}^R = [\tilde{A}_{ij}^L : \tilde{A}_{ij}^U]$.

This problem can be writing as the form:

$$\text{Max}\left\{\tilde{Z}^R(x) = \frac{[\sum_{j=1}^{m} \tilde{c}_j^L \tilde{x}_j^L + \tilde{\alpha}^L : \sum_{j=1}^{m} \tilde{c}_j^U \tilde{x}_j^U + \tilde{\alpha}^U]}{[\sum_{j=1}^{m} \tilde{d}_j^L \tilde{x}_j^L + \tilde{\beta}^L : \sum_{j=1}^{m} \tilde{d}_j^U \tilde{x}_j^U + \tilde{\beta}^U]}\right\} \quad (2)$$

Subject to:

$$\sum_{j=1}^{m}[\tilde{A}_{ij}^{L}\tilde{x}_j^{L} : \tilde{A}_{ij}^{U}\tilde{x}_j^{U}] \leq [\tilde{B}_i^{L} : \tilde{B}_i^{U}]$$

$$i = 1,2, \dots, n \ .$$

$$[\tilde{x}_j^{L} : \tilde{x}_j^{U}] \geq 0$$

Now using the operation of the rough interval we have :

Max$\left\{\tilde{Z}^{R}(x) = \left[\frac{\sum_{j=1}^{m}\tilde{c}_j^{L}\tilde{x}_j^{L}+\tilde{\alpha}^{L}}{\sum_{j=1}^{m}\tilde{d}_j^{L}\tilde{x}_j^{L}+\tilde{\beta}^{L}} : \frac{\sum_{j=1}^{m}\tilde{c}_j^{U}\tilde{x}_j^{U}+\tilde{\alpha}^{U}}{\sum_{j=1}^{m}\tilde{d}_j^{U}\tilde{x}_j^{U}+\tilde{\beta}^{U}}\right]\right\}$ (3)

Subject to:

$$\sum_{j=1}^{m}[\tilde{A}_{ij}^{L}\tilde{x}_j^{L} : \tilde{A}_{ij}^{U}\tilde{x}_j^{U}] \leq [\tilde{B}_i^{L} : \tilde{B}_i^{U}]$$

$$i = 1,2, \dots, n \ .$$

$$[\tilde{x}_j^{L} : \tilde{x}_j^{U}] \geq 0$$

Now the fuzzy rough linear fractional programming problem (3) can be reduced as the multi objective fuzzy linear fractional programming problem as follows:

$$Max\ \tilde{Z}^{U}(x) = \frac{\sum_{j=1}^{m}\tilde{C}_j^{U}\tilde{x}_j^{U} + \tilde{\alpha}^{U}}{\sum_{j=1}^{m}\tilde{d}_j^{U}\tilde{x}_j^{U} + \tilde{\beta}^{U}}$$

$$Max\ \tilde{Z}^{L}(x) = \frac{\sum_{j=1}^{m}\tilde{c}_j^{L}\tilde{x}_j^{L}+\tilde{\alpha}^{L}}{\sum_{j=1}^{m}\tilde{d}_j^{L}\tilde{x}_j^{L}+\tilde{\beta}^{L}} (4)$$

Subject to:

$$\sum_{j=1}^{m}\tilde{A}_{ij}^{U}\tilde{x}_j^{U} \leq \tilde{B}_i^{U}$$

$$\sum_{j=1}^{m}\tilde{A}_{ij}^{L}\tilde{x}_j^{L} \leq \tilde{B}_i^{L}$$

$\tilde{x}_j^{U}, \tilde{x}_j^{L} \geq 0$ and $\tilde{x}_j^{L} \tilde{\subseteq} \tilde{x}_j^{U}$, $i = 1,2, \dots, n$

Let the parameters $\tilde{C}_j^L, \tilde{C}_j^U, \tilde{d}_j^L, \tilde{d}_j^U, \tilde{\alpha}^L, \tilde{\alpha}^U, \tilde{\beta}^L, \tilde{\beta}^U, \tilde{A}_{ij}^L, \tilde{A}_{ij}^U, \tilde{B}_i^L, \tilde{B}_i^U, \tilde{x}_j^L$ *and* $\tilde{x}_j^U$ are the triangular fuzzy numbers then we have:

$\tilde{C}_j^R = [(p_j^{LL}, q_j, r_j^{UL}) : (p_j^{LU}, q_j, r_j^{UU})]$, $\tilde{d}_j^R = [(m_j^{LL}, n_j, l_j^{UL}) : (m_j^{LU}, n_j, l_j^{UU})]$

$\tilde{B}_i^R = [(b_i^{LL}, g_i, h_i^{UL}) : (b_i^{LU}, g_i, h_i^{UU})]$, $\tilde{A}_{ij}^R = [(a_{ij}^{LL}, b_{ij}, c_{ij}^{UL}) : (a_{ij}^{LU}, b_{ij}, c_{ij}^{UU})]$

$\tilde{\alpha}^R = [(\alpha_1^{LL}, \alpha_2, \alpha_3^{UL}) : (\alpha_1^{LU}, \alpha_2, \alpha_3^{UU})]$,

$\tilde{\beta}^R = [(\beta_1^{LL}, \beta_2, \beta_3^{UL}) : (\beta_1^{LU}, \beta_2, \beta_3^{UU})]$

Suppose that $\tilde{x}_j^R$ defined as:

$\tilde{x}_j^R = [\tilde{x}_j^L : \tilde{x}_j^U]$ thus writing as

$\tilde{x}_j^R = [(x_j^{LL}, y_j, t_j^{UL}) : (x_j^{LU}, y_j, t_j^{UU})]$

Also

$\tilde{Z}^U(x) = (Z_1^{LU}, Z_2, Z_3^{UU})$

$\tilde{Z}^L(x) = (Z_1^{LL}, Z_2, Z_3^{UL})$

Now the problem (4) can be written as follows:

$$Max\ (Z_1^{LU}, Z_2, Z_3^{UU}) = \frac{\sum_{j=1}^{m}\left((p_j^{LU}, q_j, r_j^{UU}) \times (x_j^{LU}, y_j, t_j^{UU})\right) + (\alpha_1^{LU}, \alpha_2, \alpha_3^{UU})}{\sum_{j=1}^{m}\left((m_j^{LU}, n_j, l_j^{UU}) \times (x_j^{LU}, y_j, t_j^{UU})\right) + (\beta_1^{LU}, \beta_2, \beta_3^{UU})}$$

$$Max\ (Z_1^{LL}, Z_2, Z_3^{UL}) = \frac{\sum_{j=1}^{m}\left((p_j^{LL}, q_j, r_j^{UL}) \times (x_j^{LL}, y_j, t_j^{UL})\right) + (\alpha_1^{LL}, \alpha_2, \alpha_3^{UL})}{\sum_{j=1}^{m}\left((m_j^{LL}, n_j, l_j^{UU}) \times (x_j^{LL}, y_j, t_j^{UL})\right) + (\beta_1^{LL}, \beta_2, \beta_3^{UL})} \quad (5)$$

Subject to:

$$\sum_{j=1}^{m}((a_{ij}^{LU}, b_{ij}, c_{ij}^{UU}) \times (x_j^{LU}, y_j, t_j^{UU}) \precsim (b_i^{LU}, g_i, h_i^{UU})$$

$$\sum_{j=1}^{m}((a_{ij}^{LL}, b_{ij}, c_{ij}^{UL}) \times (x_j^{LL}, y_j, t_j^{UL}) \precsim (b_i^{LL}, g_i, h_i^{UL})$$

Where $(x_j^{LU}, y_j, t_j^{UU}), (x_j^{LL}, y_j, t_j^{UL}) \gtrsim 0$

Now since
$\tilde{x}_j^R = [\tilde{x}_j^L : \tilde{x}_j^U] = [(x_j^{LL}, y_j, t_j^{UL}) : (x_j^{LU}, y_j, t_j^{UU})]$

is a fuzzy rough with triangular fuzzy number, then $(x_j^{LL}, y_j, t_j^{UL}) \subseteqq (x_j^{LU}, y_j, t_j^{UU})$ this

implies:
$x_j^{LU} \leq x_j^{LL} \leq y_j \leq t_j^{UL} \leq t_j^{UU}$(6)

The relation (6) is called bounded constraints.
Now using the arithmetic operations and partial ordering relations we decompose the problem (5) as the follows:
$Max\ (Z_1^{LU}) = lower\ value\ of$

$$\left\{\frac{\sum_{j=1}^{m}\left((p_j^{LU}, q_j, r_j^{UU}) \times (x_j^{LU}, y_j, t_j^{UU})\right) + (\alpha_1^{LU}, \alpha_2, \alpha_3^{UU})}{\sum_{j=1}^{m}\left((m_j^{LU}, n_j, l_j^{UU}) \times (x_j^{LU}, y_j, t_j^{UU})\right) + (\beta_1^{LU}, \beta_2, \beta_3^{UU})}\right\}$$

$Max\ Z_2 = Middle\ value\ of$

$$\left\{\frac{\sum_{j=1}^{m}\left((p_j^{LU}, q_j, r_j^{UU}) \times (x_j^{LU}, y_j, t_j^{UU})\right) + (\alpha_1^{LU}, \alpha_2, \alpha_3^{UU})}{\sum_{j=1}^{m}\left((m_j^{LU}, n_j, l_j^{UU}) \times (x_j^{LU}, y_j, t_j^{UU})\right) + (\beta_1^{LU}, \beta_2, \beta_3^{UU})}\right\}$$

$Max\ (Z_3^{UU}) = Upper\ value\ of$

$$\left\{\frac{\sum_{j=1}^{m}\left((p_j^{LU}, q_j, r_j^{UU}) \times (x_j^{LU}, y_j, t_j^{UU})\right) + (\alpha_1^{LU}, \alpha_2, \alpha_3^{UU})}{\sum_{j=1}^{m}\left((m_j^{LU}, n_j, l_j^{UU}) \times (x_j^{LU}, y_j, t_j^{UU})\right) + (\beta_1^{LU}, \beta_2, \beta_3^{UU})}\right\}$$

$Max\ (Z_1^{LL}) = Lower\ value\ of$

$$\left\{\frac{\sum_{j=1}^{m}\left((p_j^{LL}, q_j, r_j^{UL}) \times (x_j^{LL}, y_j, t_j^{UL})\right) + (\alpha_1^{LL}, \alpha_2, \alpha_3^{UL})}{\sum_{j=1}^{m}\left((m_j^{LL}, n_j, l_j^{UU}) \times (x_j^{LL}, y_j, t_j^{UL})\right) + (\beta_1^{LL}, \beta_2, \beta_3^{UL})}\right\}$$

$Max\ (Z_3^{UL}) = Upper\ value\ of$

$$\left\{\frac{\sum_{j=1}^{m}\left(\left(p_j^{LL}, q_j, r_j^{UL}\right) \times \left(x_j^{LL}, y_j, t_j^{UL}\right)\right) + \left(\alpha_1^{LL}, \alpha_2, \alpha_3^{UL}\right)}{\sum_{j=1}^{m}\left(\left(m_j^{LL}, n_j, l_j^{UU}\right) \times \left(x_j^{LL}, y_j, t_j^{UL}\right)\right) + \left(\beta_1^{LL}, \beta_2, \beta_3^{UL}\right)}\right\}$$

Subject to:

$$\sum_{j=1}^{m} Lower\ value\ of$$

$$\left\{\left(a_{ij}^{LU}, b_{ij}, c_{ij}^{UU}\right) \times \left(x_j^{LU}, y_j, t_j^{UU}\right)\right\} \lesssim b_i^{LU}$$

$$\sum_{j=1}^{m} Middle\ value\ of$$

$$\left\{\left(a_{ij}^{LU}, b_{ij}, c_{ij}^{UU}\right) \times \left(x_j^{LU}, y_j, t_j^{UU}\right)\right\} \lesssim g_i$$

$$\sum_{j=1}^{m} Upper\ value\ of$$

$$\left\{\left(a_{ij}^{LU}, b_{ij}, c_{ij}^{UU}\right) \times \left(x_j^{LU}, y_j, t_j^{UU}\right)\right\} \lesssim h_i^{UU}$$

$$\sum_{j=1}^{m} Lower\ value\ of$$

$$\left\{\left(a_{ij}^{LL}, b_{ij}, c_{ij}^{UL}\right) \times \left(x_j^{LL}, y_j, t_j^{UL}\right)\right\} \lesssim b_i^{LL}$$

$$\sum_{j=1}^{m} Upper\ value\ of$$

$$\left\{\left(a_{ij}^{LL}, b_{ij}, c_{ij}^{UL}\right) \times \left(x_j^{LL}, y_j, t_j^{UL}\right)\right\} \lesssim h_i^{UL}$$

and all decision variables are non-negative.
From the above decomposition problem we construct the following five crisp linear fractional programming problems namely, Middle Level problem (MLP), Upper Upper Level problem (UULP), Lower Upper Level problem (LULP), Upper Lower Level problem (ULLP) and Lower Lower Level problem (LLLP) as follows:
(MLP): $Max\ Z_2 = Middle\ value\ of$

$$\left\{\frac{\sum_{j=1}^{m}\left(\left(p_j^{LU},\ q_j,\ r_j^{UU}\right)\times\left(x_j^{LU},y_j,\ t_j^{UU}\right)\right)+\left(\alpha_1^{LU},\ \alpha_2,\ \alpha_2^{UU}\right)}{\sum_{j=1}^{m}\left(\left(m_j^{LU},\ n_j,\ l_j^{UU}\right)\times\left(x_j^{LU},y_j,\ t_j^{UU}\right)\right)+\left(\beta_1^{LU},\ \beta_2,\ \beta_2^{UU}\right)}\right\}$$

Subject to: $\sum_{j=1}^{m} \mathit{Middle\ value\ of}$

$$\{(a_{ij}^{LU}, b_{ij}, c_{ij}^{UU})\times(x_j^{LU}, y_j,\ t_j^{UU})\} \lesssim g_i$$

constraints in the decomposition problem in which at last one decision variable of the MLP occurs and all decision variables are non-negative.

UULP: $Max\ (Z_2^{UU}) = \mathit{Upper\ value\ of}$

$$\left\{\frac{\sum_{j=1}^{m}\left(\left(p_j^{LU},\ q_j,\ r_j^{UU}\right)\times\left(x_j^{LU},y_j,\ t_j^{UU}\right)\right)+\left(\alpha_1^{LU},\ \alpha_2,\ \alpha_2^{UU}\right)}{\sum_{j=1}^{m}\left(\left(m_j^{LU},\ n_j,\ l_j^{UU}\right)\times\left(x_j^{LU},y_j,\ t_j^{UU}\right)\right)+\left(\beta_1^{LU},\ \beta_2,\ \beta_2^{UU}\right)}\right\}$$

Subject to: $\mathit{Upper\ value\ of}$

$$\left\{\frac{\sum_{j=1}^{m}\left(\left(p_j^{LU},\ q_j,\ r_j^{UU}\right)\times\left(x_j^{LU},y_j,\ t_j^{UU}\right)\right)+\left(\alpha_1^{LU},\ \alpha_2,\ \alpha_2^{UU}\right)}{\sum_{j=1}^{m}\left(\left(m_j^{LU},\ n_j,\ l_j^{UU}\right)\times\left(x_j^{LU},y_j,\ t_j^{UU}\right)\right)+\left(\beta_1^{LU},\ \beta_2,\ \beta_2^{UU}\right)}\right\} \geq Z_2^*$$

$$\sum_{j=1}^{m} \mathit{Upper\ value\ of}$$

$$\{(a_{ij}^{LU}, b_{ij}, c_{ij}^{UU})\times(x_j^{LU}, y_j,\ t_j^{UU})\} \lesssim h_i^{UU}$$

$$\sum_{j=1}^{m} \mathit{Lower\ value\ of}$$

$$\{(a_{ij}^{LU}, b_{ij}, c_{ij}^{UU})\times(x_j^{LU}, y_j,\ t_j^{UU})\} \lesssim b_i^{LU}$$

and all variables in the constraints and objective function in UULP must satisfy the bounded constraints replacing all values of the decision variables which are obtained in MLP and all decision variables are non-negative.

Where Z_2^* is the optimal objective value of MLP.

LULP: $Max\ (Z_1^{LU}) = \mathit{lower\ value\ of}$

$$\left\{\frac{\sum_{j=1}^{m}\left(\left(p_j^{LU},\ q_j,\ r_j^{UU}\right)\times\left(x_j^{LU},y_j,\ t_j^{UU}\right)\right)+\left(\alpha_1^{LU},\ \alpha_2,\ \alpha_2^{UU}\right)}{\sum_{j=1}^{m}\left(\left(m_j^{LU},\ n_j,\ l_j^{UU}\right)\times\left(x_j^{LU},y_j,\ t_j^{UU}\right)\right)+\left(\beta_1^{LU},\ \beta_2,\ \beta_2^{UU}\right)}\right\}$$

Subject to: $\mathit{Lower\ value\ of}$

$$\left\{\frac{\sum_{j=1}^{m}\left(\left(p_j^{LU},\ q_j,\ r_j^{UU}\right)\times\left(x_j^{LU},y_j,\ \ t_j^{UU}\right)\right)+\left(\alpha_1^{LU},\ \alpha_2,\ \alpha_3^{UU}\right)}{\sum_{j=1}^{m}\left(\left(m_j^{LU},\ n_j,\ l_j^{UU}\right)\times\left(x_j^{LU},y_j,\ \ t_j^{UU}\right)\right)+\left(\beta_1^{LU},\ \beta_2,\ \beta_3^{UU}\right)}\right\}\leq\ Z_2^{*},$$

$$\sum_{j=1}^{m} Upper\ value\ of$$

$$\{(a_{ij}^{LU},b_{ij},c_{ij}^{UU})\times\ (x_j^{LU},y_j\ ,\ \ t_j^{UU})\}\lesssim h_i^{UU}$$

$$\sum_{j=1}^{m} Lower\ value\ of$$

$$\{(a_{ij}^{LU},b_{ij},c_{ij}^{UU})\times\ (x_j^{LU},y_j\ ,\ \ t_j^{UU})\}\lesssim b_i^{LU}$$

and all variables in the constraints and objective function in LULP must satisfy the bounded constraints replacing all values of the decision variables which are obtained in MLP, UULP and all decision variables are non-negative.

ULLP: $Max\ (Z_2^{UL}) = Upper\ value\ of$

$$\left\{\frac{\sum_{j=1}^{m}\left(\left(p_j^{LL},q_j\ ,r_j^{UL}\right)\ \times\left(x_j^{LL},y_j\ ,\ \ t_j^{UL}\right)\right)\ +\ \left(\alpha_1^{LL},\ \alpha_2\ ,\ \alpha_3^{UL}\right)}{\sum_{j=1}^{m}\left(\left(m_j^{LL},\ n_j,\ l_j^{UU}\right)\ \times\left(x_j^{LL},y_j\ ,\ \ t_j^{UL}\right)\right)\ +\ \left(\beta_1^{LL},\ \beta_2\ ,\ \beta_3^{UL}\right)}\right\}$$

Subject to:

$$Z_2^{*}\leq upp.\ val.\left\{\frac{\sum_{j=1}^{m}\left(\left(p_j^{LL},q_j\ ,r_j^{UL}\right)\times\left(x_j^{LL},y_j\ ,\ \ t_j^{UL}\right)\right)+\left(\alpha_1^{LL},\alpha_2\ ,\alpha_3^{UL}\right)}{\sum_{j=1}^{m}\left(\left(m_j^{LL},\ n_j,\ l_j^{UU}\right)\times\left(x_j^{LL},y_j\ ,\ \ t_j^{UL}\right)\right)+\left(\beta_1^{LL},\beta_2\ ,\beta_3^{UL}\right)}\right\}\leq\qquad (Z_3^{UU})^{*},$$

$$\sum_{j=1}^{m} Lower\ value\ of$$

$$\{(a_{ij}^{LL},b_{ij},c_{ij}^{UL})\times(x_j^{LL},y_j\ ,\ \ t_j^{UL})\}\lesssim b_i^{LL}$$

$$\sum_{j=1}^{m} Upper\ value\ of$$

$$\{(a_{ij}^{LL},b_{ij},c_{ij}^{UL})\times(x_j^{LL},y_j\ ,\ \ t_j^{UL})\}\lesssim h_i^{UL}$$

and all variables in the constraints and objective function in ULLP must satisfy the bounded constraints replacing all values of the decision variables which are obtained in MLP, UULP, LULP and all decision variables are non-negative. Where $(Z_3^{UU})^{*}$ is the optimal objective value of **UULP**.

LLLP: $Max\ (Z_1^{LL}) = Lower\ value\ of$

$$\left\{\frac{\sum_{j=1}^{m}\left((p_j^{LL}, q_j, r_j^{UL}) \times (x_j^{LL}, y_j, t_j^{UL})\right) + (\alpha_1^{LL}, \alpha_2, \alpha_2^{UL})}{\sum_{j=1}^{m}\left((m_j^{LL}, n_j, l_j^{UU}) \times (x_j^{LL}, y_j, t_j^{UL})\right) + (\beta_1^{LL}, \beta_2, \beta_2^{UL})}\right\}$$

Subject to:

$$(Z_1^{LU})^* \leq Low.Val.\left\{\frac{\sum_{j=1}^{m}\left((p_j^{LL}, q_j, r_j^{UL}) \times (x_j^{LL}, y_j, t_j^{UL})\right) + (\alpha_1^{LL}, \alpha_2, \alpha_2^{UL})}{\sum_{j=1}^{m}\left((m_j^{LL}, n_j, l_j^{UU}) \times (x_j^{LL}, y_j, t_j^{UL})\right) + (\beta_1^{LL}, \beta_2, \beta_2^{UL})}\right\}$$

$\leq Z_2^*$,

$$\sum_{j=1}^{m} Lower\ value\ of$$

$$\{(a_{ij}^{LL}, b_{ij}, c_{ij}^{UL}) \times (x_j^{LL}, y_j, t_j^{UL})\} \lesssim b_i^{LL}$$

$$\sum_{j=1}^{m} Upper\ value\ of$$

$$\{(a_{ij}^{LL}, b_{ij}, c_{ij}^{UL}) \times (x_j^{LL}, y_j, t_j^{UL})\} \lesssim h_i^{UL}$$

and all variables in the constraints and objective function in LLLP must satisfy the bounded constraints replacing all values of the decision variables which are obtained in MLP, UULP, LULP, ULLP and all decision variables are non-negative.

Definition14. A point $x^* \in \tilde{I}^R \subseteq \mathcal{R}$ is said to be optimal fuzzy rough solution of the fuzzy rough linear fractional programming problem if there does not exist $x \in \tilde{I}^R$ such that $\frac{\tilde{N}^R(x^*)}{\tilde{D}^R(x^*)} \leq \frac{\tilde{N}^R(x)}{\tilde{D}^R(x)}$.

Theorem 1. Let $[y_M^*] = \{y_M^* : y_M^* \in M\}$ be an optimal solution of (MLP), $[t_{UU}^*] = \{t_{UU}^* : t_{UU}^* \in UU\}$ be an optimal solution of (UULP), $[x_{LU}^*] = \{x_{LU}^* : x_{LU}^* \in LU\}$ be an optimal solution of (LULP), $[t_{UL}^*] = \{t_{UL}^* : t_{UL}^* \in UL\}$ be an optimal solution of (ULLP), and $[x_{LL}^*] = \{x_{LL}^* : x_{LL}^* \in LL\}$ be an optimal solution of (LLLP), where M, UU, LU, UL and LL sets of decision variables in the (MLP), (UULP), (LULP), (ULLP) and (LLLP) respectively.
Then $\{(\tilde{x}_j^R)^* = [(x_j^{LL*}, y_j^*, t_j^{UL*}) : (x_j^{LU*}, y_j^*, t_j^{UU*})]\}$ is an optimal fuzzy rough solution to the given FRLFP problem.

The proof of this theorem is much similar to the proof theorem (4-1) given by Pandian and Jayalakshmi [3].

Algorithm Solution for FRLFP problem

The propose algorithm for solving FRLFP problem (1) as follows:

Step1. Construct five crisp linear fractional programming (LFP) problems namely Middle Level problem, Upper Upper Level problem, Lower Upper Level problem, Upper Lower Level problem and Lower Lower Level problem from the given FRLFP problem.
Step2. Solve the Middle Level problem (MLP) by the variable transformation method.
Step3. Using the result of step2 and the variable transformation method, solve the Upper Upper Level problem.
Step4. Using the result of step2, step3 and the variable transformation method, solve the Lower Upper Level problem.
Step5. Using the result of step2, step3, step4 and the variable transformation method, solve the Upper Lower Level problem.
Step6. Using the result of step2, step3, step4, step5 and the variable transformation method, solve the Lower Lower Level problem.
Step7. Using the result of step2, step3, step4, step5, step6, thus obtains an optimal solution to the given FRLFP problem by theorem (1).

Numerical example:

Consider the following FRLFP problem

$$Max\ \tilde{Z}^R(x) = \frac{\tilde{6}^R\tilde{x}_1^R + \tilde{5}^R\tilde{x}_2^R}{\tilde{2}^R\tilde{x}_1^R + \tilde{7}^R}$$

Where $\tilde{6}^R = [(5.5,\ 6,\ 6.5):(5,\ 6,\ 7)]$,

$\tilde{5}^R = [(4.5,\ 5,\ 5.5):(3,\ 5,\ 6)]$,

$\tilde{2}^R = [(1.5,\ 2,\ 2.5):(1,\ 2,\ 3)]$,

$\tilde{7}^R = [(6,\ 7,\ 8):(5.5,\ 7,\ 8.5)]$,

Subject to:
$\tilde{1}^R\tilde{x}_1^R + \tilde{2}^R\tilde{x}_2^R \precsim \tilde{3}^R, \tilde{3}^R\tilde{x}_1^R + \tilde{2}^R\tilde{x}_2^R \precsim \tilde{6}^R$,where

$\tilde{1}^R = [(0.5, 1,\ 1.25):(0.5, 1, 1.5)]$,

$\tilde{2}^R = [(1.75,\ 2,\ 2.25):(1.5, 2, 2.5)]$,

$\tilde{3}^R = [(2.5,\ 3, 9):(2, 3, 15)]$,

$\tilde{3}^R = [(2.5,\ 3, 3.5):(2, 3, 4)]$,

$\tilde{2}^R = [(1.5,\ 2,\ 2.5):(1, 2, 3)]$,

$\tilde{\delta}^R = [(5.5\ , 6\ ,\ 15\ ) : (5\ , 6, 21)],$

$$\tilde{x}_1^R \ and \ \tilde{x}_2^R \gtrsim 0$$

Let $\tilde{x}_1^R = [\tilde{x}_1^L : \tilde{x}_1^U] = [(x_1^{LL}, y_1\ , t_1^{UL}) : (x_1^{LU}, y_1, t_1^{UU})]$

$\tilde{x}_2^R = [\tilde{x}_2^L : \tilde{x}_2^U] = [(x_2^{LL}, y_2\ , t_2^{UL}) : (x_2^{LU}, y_2, t_2^{UU})]$ and

$\tilde{Z}^R = [(Z_1^{LL}, Z_2\ , Z_3^{UL}) : (Z_1^{LU}, Z_2, Z_3^{UU})]$

Now, the decomposition problems of the given FRLFP problem are given below:

$$\text{Max } Z_1^{LL} = \frac{5.5x_1^{LL} + 4.5x_2^{LL}}{2.5t_1^{UL} + 8}$$

$$\text{Max } Z_2 = \frac{6y_1 + 5y_2}{2y_1 + 7}$$

$$\text{Max } Z_3^{UL} = \frac{6.5t_1^{UL} + 5.5t_2^{UL}}{1.5x_1^{LL} + 6}$$

$$\text{Max } Z_1^{LU} = \frac{5x_1^{LU} + 3x_2^{LU}}{3t_1^{UU} + 8.5}$$

$$\text{Max } Z_3^{UU} = \frac{7t_1^{UU} + 6t_2^{UU}}{x_1^{LU} + 5.5}$$

Subject to:

$$0.5x_1^{LL} + 1.75x_2^{LL} \leq 2.5$$

$$y_1 + 2y_2 \leq 3$$

$$1.25t_1^{UL} + 2.25t_2^{UL} \leq 9$$

$$0.5x_1^{LU} + 1.5x_2^{LU} \leq 2$$

$$1.5t_1^{UU} + 2.5t_2^{UU} \leq 15$$

$$2.5x_1^{LL} + 1.5x_2^{LL} \leq 5.5$$

$$3y_1 + 2y_2 \leq 6$$

$$3.5t_1^{UL} + 2.5t_2^{UL} \leq 15$$

$$2x_1^{LU} + x_2^{LU} \leq 5$$

$$4t_1^{UU} + 3t_2^{UU} \leq 21$$

$$x_1^{LU} \leq x_1^{LL} \leq y_1 \leq t_1^{UL} \leq t_1^{UU} ,$$

$$x_2^{LU} \leq x_2^{LL} \leq y_2 \leq t_2^{UL} \leq t_2^{UU}$$

$$x_1^{LU}, x_1^{LL}, y_1, t_1^{UL} \text{ and } t_1^{UU} \geq 0 \quad x_2^{LU}, x_2^{LL}, y_2, t_2^{UL} \text{ and } t_2^{UU} \geq 0$$

Now, the Middle Level problem is given below:

(MLP): Max $Z_2 = \frac{6y_1+5y_2}{2y_1+7}$

Subject to:

$$y_1 + 2y_2 \leq 3 \; ; \qquad 3y_1 + 2y_2 \leq 6 \; ;$$

$$y_1 , \quad y_2 \geq 0 .$$

Now, solving the problem (MLP) by the Variable Transformation Method and using solver parameters excel, we attain an optimal solution

$y_1 = 1.5, \quad y_2 = 0.75 \text{ and } \quad Z_2 = 1.28$.

Now, the Upper Upper Level problem is given below:

(UULP): Max $Z_3^{UU} = \frac{7t_1^{UU}+6t_2^{UU}}{x_1^{LU} + 5.5}$

Subject to:

$$\frac{7t_1^{UU} + 6t_2^{UU}}{x_1^{LU} + 5.5} \geq 1.28$$

$$0.5x_1^{LU} + 1.5x_2^{LU} \leq 2$$

$$1.5t_1^{UU} + 2.5t_2^{UU} \leq 15$$

$$2x_1^{LU} + x_2^{LU} \leq 5$$

$$4t_1^{UU} + 3t_2^{UU} \leq 21$$

$$t_1^{UU} \geq 1.5\,, \qquad t_2^{UU} \geq 0.75\,, \qquad x_1^{LU} \leq 1.5, \qquad x_2^{LU} \leq 0.75$$

$$t_1^{UU}\,,\ t_2^{UU}\,, \quad x_1^{LU}\ and\ \ x_2^{LU} \geq 0$$

Now, solving the (UULP) by the Variable Transformation Method and using solver parameters excel, we attain an optimal solution
$x_1^{LU} = 0, x_2^{LU} = 0$, $t_1^{UU} = 1.5$, $t_2^{UU} = 5$ and $Z_3^{UU} = 7.36$.

Now, the Lower Upper Level problem is given below:

(LULP): Max $Z_1^{LU} = \frac{5x_1^{LU} + 3x_2^{LU}}{3t_1^{UU} + 8.5}$

Subject to:

$$\frac{5x_1^{LU} + 3x_2^{LU}}{3t_1^{UU} + 8.5} \leq 1.28$$

$$0.5x_1^{LU} + 1.5x_2^{LU} \leq 2$$

$$1.5t_1^{UU} + 2.5t_2^{UU} \leq 15$$

$$2x_1^{LU} + x_2^{LU} \leq 5$$

$$4t_1^{UU} + 3t_2^{UU} \leq 21$$

$$t_1^{UU} \geq 1.5 \quad ; \quad t_2^{UU} \geq 0.75 \quad ; \quad x_1^{LU} \leq 1.5 \quad ; \quad x_2^{LU} \leq 0.75 \ ;$$

$$t_1^{UU}\,,\ t_2^{UU}\,,\ x_1^{LU}\ and\ x_2^{LU} \geq 0\,.$$

Now, substituting $x_1^{LU} = 0$, $x_2^{LU} = 0$, $t_1^{UU} = 1.5$, $t_2^{UU} = 5$

in the problem (LULP), the optimal solution is
$x_1^{LU} = 0$, $x_2^{LU} = 0$, $t_1^{UU} = 1.5$, $t_2^{UU} = 5$ and $Z_1^{LU} = 0$.

Now, the Upper Lower Level problem is given below:

(ULLP): Max $Z_3^{UL} = \frac{6.5t_1^{UL}+5.5t_2^{UL}}{1.5x_1^{LL}+6}$

Subject to:

$$1.28 \leq \frac{6.5t_1^{UL} + 5.5t_2^{UL}}{1.5x_1^{LL} + 6} \leq 7.36$$

$$0.5x_1^{LL} + 1.75x_2^{LL} \leq 2.5$$

$$1.25t_1^{UL} + 2.25t_2^{UL} \leq 9$$

$$2.5x_1^{LL} + 1.5x_2^{LL} \leq 5.5$$

$$3.5t_1^{UL} + 2.5t_2^{UL} \leq 15$$

$$0 \leq x_1^{LL} \leq 1.5 \ , \ \ 0 \leq x_2^{LL} \leq 0.75$$

$$1.5 \leq t_1^{UL} \leq 1.5 \ , \ \ 0.75 \leq t_2^{UL} \leq 5$$

$$t_1^{UL} , \quad t_2^{UL} , x_1^{LL} \ and \ \ x_2^{LL} \geq 0$$

Now, solving the problem (ULLP) by the Variable Transformation Method and using solver parameters excel the optimal solution is:
$t_1^{UL} = 1.5,\ t_2^{UL} = 3.17\ ,\ x_1^{LL} = 0\ ,\ x_2^{LL} = 0$ and $Z_3^{UL} = 4.53.$

Now, the Lower Lower Level problem is given below:

(LLLP): Max $Z_1^{LL} = \frac{5.5x_1^{LL}+4.5x_2^{LL}}{2.5t_1^{UL}+8}$

Subject to:

$$0 \leq \frac{5.5x_1^{LL} + 4.5x_2^{LL}}{2.5t_1^{UL} + 8} \leq 1.28$$

$$0.5x_1^{LL} + 1.75x_2^{LL} \leq 2.5$$

$$1.25t_1^{UL} + 2.25t_2^{UL} \leq 9$$

$$2.5x_1^{LL} + 1.5x_2^{LL} \leq 5.5$$

$$3.5t_1^{UL} + 2.5t_2^{UL} \leq 15$$

$$0 \leq x_1^{LL} \leq 1.5\ ,\ 0 \leq x_2^{LL} \leq 0.75\ ,$$

$$1.5 \ \leq\ t_1^{UL} \leq 1.5 \qquad , \qquad 0.75 \ \leq\ t_2^{UL} \leq 5$$

$$x_1^{LL}, x_2^{LL}, t_1^{UL} \text{ and } t_2^{UL} \geq 0$$

Now, substituting $x_1^{LL} = 0$, $x_2^{LL} = 0$, $t_1^{UL} = 1.5$,

$t_2^{UL} = 3.17$

in the problem (LLLP), the optimal solution is
$t_1^{UL} = 1.5$, $t_2^{UL} = 3.17$, $x_1^{LL} = 0$, $x_2^{LL} = 0$ and $Z_1^{LL} = 0$

Therefore, the optimal fuzzy rough solution to the given FRLFP problem is:
$(\widetilde{x}_1^{\,R})^* = [(0, 1.5, 1.5) : (0, 1.5, 1.5)]$

$(\widetilde{x}_2^{\,R})^* = [(0, 0.75, 3.17) : (0, 0.75, 5)]$

with the maximum objective value
$\tilde{Z}^R = [(0, 1.28, 4.53) : (0, 1.28, 7.36)]$

Conclusion

In this paper, we propose algorithm for solve fuzzy rough linear fractional programming problem, where all variables and coefficentics are fuzzy rough number . Use the decomposition to the fuzzy linear fractional programming problem for obtaining an optimal fuzzy rough solution, based on the variable transformation method. Further the proposed approach can be extended for solving FRLFP problem where all coefficentics are trapezoidal fuzzy numbers.

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