Numerical Analysis of Laminar flow of Viscous Fluid Between Two Porous Bounding walls

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Abstract: *This problems deals the study of velocity profile of viscous fluid between two parallel porous walls, when the fluid is being withdrawn both the walls of a channel at the same rate. A solution for suction Reynolds Number & Large Reynolds Number is discussed in this paper. Expressions for the velocity components are obtained. The governing differential equation is solved by using perturbation method and the graphs of axial and radial velocity profiles have been drawn.*

Key Words: - *Viscous fluid, Suction Reynolds number, Parallel Porous medium, Slip coefficient, Laminar flow.*

Nomenclature

x The axial distance from the channel entrance;
y The coordinate axis perpendicular to the channel walls measured from the Non-porous wall;
u Velocity component in the x-direction;
u_0 Average velocity over the channel at channel inlet;
Re_w Wall Reynolds number, $\frac{v_w h}{\nu}$;
Re_{ent} Reynolds Number for flow entering the channel, $u0\, h/\nu$;
v Velocity component in the y- direction;
v_w Velocity of the fluid through the membrane;

Greek Symbols

ρ Solution density;
λ Dimensionless variable in the $y-$ direction, y/h;
μ Viscosity;
ν Kinematic viscosity;
Δp Dimensionless pressure drop, $2[p(0,\lambda) - p(x,\lambda)]/(\rho {u_0}^2)$;
ϕ Slip coefficient, $\sqrt{k}/\alpha h$;
ψ Stream Function;

1. Introduction

The Flow of viscous fluid with two porous bounding walls is very important prevalent in nature as well as practical interest in view of their varied applications in different fields of engineering science and technology such as extraction of plastic sheets, cooling of infinite metallic plates in a cool bath, extractions of plastic sheets, liquid film condensation process and in major fields of glass and polymer industry. In view of this application, a series of investigations were made to study the flow of past a vertical wall. Hasimoto (1997), has discussed the boundary layer growth on a flat plate with suction or injection. Avramenko et. al (2005), has studied, Investigation of stability of a laminar flow in a parallel-plate channel filled with a saturated porous medium. Kuznetsov, A.V., (1997), has studied the analytical investigation of the fluid flow in the interface region between a porous medium and a clear fluid in channels partially filled with a porous medium. Raptis, A and C. Perdikis (1987), has studied Hydromagnetic free-convective flow through porous media. Beavers G.S. and Joseph D.D. (1967), has studied Boundary conditions at a naturally permeable wall. Beavers et. al (1970),has studied the Experiments on coupled parallel Flows in a Channel and a Bounding porous medium. Bujurke et. al (2010), has studied Analysis of Laminar flow in a channel with one porous bounding wall. Nield D. A. (1994) has studied modeling high speed flow of an incompressible fluid in a saturated porous medium. Ochoa-Tapia J. A. and S. Whitaker (1995a & 1995b) have studied Momentum transfer at the boundary between a porous medium and a homogeneous fluid I & II.

The study of flow of viscous fluid through porous channel with rectangular cross section, when the Reynolds number is low was studied and a perturbation solution assuming normal wall velocities to be equal was obtained Berman (1953), Lage J.L. (1992), has studied Effect of the convective inertia term on Bonnard convection in a porous medium. Lage, J.L. (1998), has studied the fundamental theory of flow through permeable media from Darcy to turbulence. Vafai K and S. J. Kimi (1990), has studied Fluid mechanics of an interface region between a porous medium and a fluid layer-an exact solution. Makinde and E. Osalusi (2006), has studied MHD Steady flow in a channel with slip at the permeable boundaries. Robinson (1976), has studied the Existence of Multiple Solutions for the Laminar Flow in a Uniformly Porous Channel with Suction at Both Walls. Saffman P. G. (1971), has studied On the Boundary condition at the Surface of a Porous medium. Das S.S. and U.K. Tripathy (2010), has studied Effect of Periodic suction on three dimensional flow and heat transfer past a vertical porous plate embedded in a porous medium. Terrill (1964), has studied Laminar Flow in a Uniformly Porous Channel with Large injection. Terrill (1968), has studied Laminar Flow with Large Injection Through parallel and Uniformly Porous Walls of Different Permeability. Takatsu Y and T. Masuka (1998), has studied turbulent phenomena in flow through porous media. In this analysis we have study the effect of Reynolds number on velocity of fluid and considered as the fluid is moving in a channel with two porous bounding walls in the presence viscous fluid, in different Reynolds Number.

2. Mathematical Formulation

Let us consider the viscous flow of Non-Newtonian fluid between two parallel porous plates, the length of the channel is assumed to be L and 2h is the distance between the two plates, u and v be the velocity components in the x and y directions respectively.

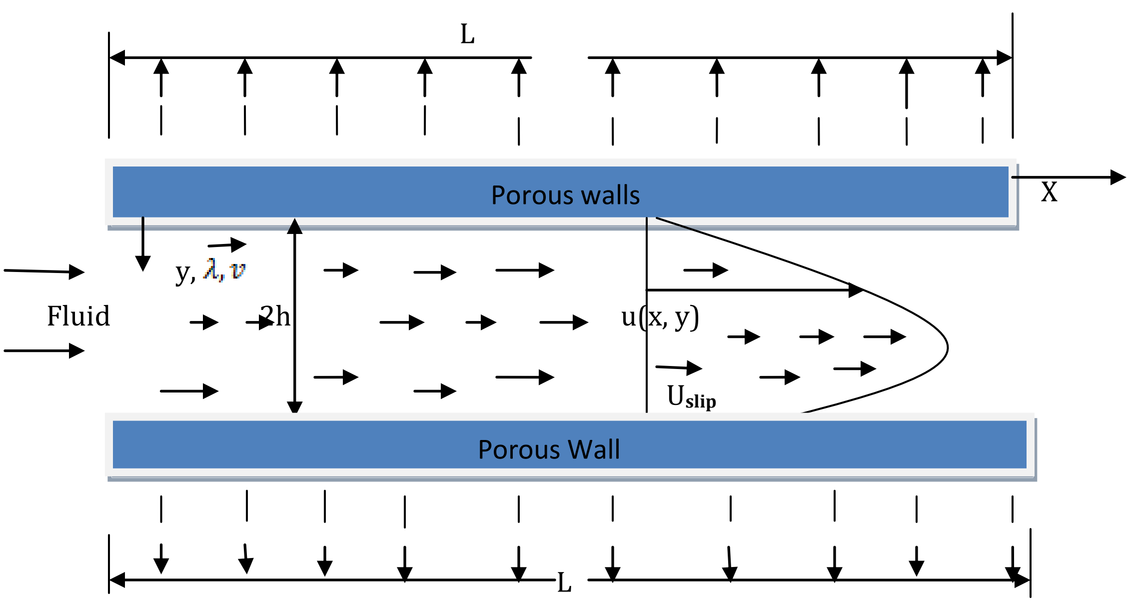


Fig. 1. The coordinate system used in the solution of the two dimensional steady State flow of Non-Newtonian fluid.

The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

The equation of momentum is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (3)$$

The boundary conditions are $u(x,h)=0, u(x,-h)=0, v(x,h)=V \text{ and } v(x,-h)=-V$

Where V is the velocity of suction at the walls of the channel. Let the dimensionless variable be $\lambda=\frac{y}{h}$ and then the equation 1 -3 becomes

$$\frac{\partial u}{\partial x}+\frac{1}{h}\frac{\partial v}{\partial \lambda}=0 \qquad (4)$$

$$u\frac{\partial u}{\partial x}+\frac{v}{h}\frac{\partial u}{\partial \lambda}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 u}{\partial \lambda^2}\right) \qquad (5)$$

$$u\frac{\partial v}{\partial x}+\frac{v}{h}\frac{\partial v}{\partial \lambda}=-\frac{1}{\rho h}\frac{\partial p}{\partial \lambda}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 v}{\partial \lambda^2}\right) \qquad (6)$$

Where, υ is the kinematic viscosity, and p the pressure

Introducing the stream function $\psi(x,y)$, we know that

$$u=\frac{\partial \psi}{\partial y} \quad \& \quad v=-\frac{\partial \psi}{\partial x} \qquad (7)$$

Introducing the dimensionless variable in equation (7) we get

$$u=\frac{1}{h}\frac{\partial \psi}{\partial \lambda} \quad \& \quad v=-\frac{\partial \psi}{\partial x} \qquad (8)$$

The equation of continuity can be satisfied by a stream function of the form

$$\psi(x,\ \lambda)=(h\,u_0-v_w\ x)\,f(\lambda) \qquad (9)$$

From equation (5) & (6) eliminating P, then using equation (8) and (9)

We get $\frac{\partial}{\partial \lambda}\left[\frac{v_w}{h}\left(f'^2-f\,f''\right)+\frac{v}{h^2}f'''\right]=0 \qquad (10)$

Integrating the above equation (10)

We get $\frac{v_w}{h}\left(f'^2-f\,f''\right)+\frac{v}{h^2}f'''=A \qquad (11)$

Or

$$Re\left(f'^2-f\,f''\right)+f'''=A \qquad (12)$$

Where above equation is Frankel Skan Family equation: $Re=v_w\,h/v$ and Re is Reynolds Number

The above differential equation is Non-linear differential equation and solving by using perturbation method. The boundary condition of $f(\lambda)$ are

$$f(1)=1,\ f(-1)=-1,\ and$$

$$f'(1)=f'(-1)=0 \qquad (13)$$

Hence the solution of the equation of motion and continuity is given by a nonlinear third order differential equation (11), subjection to the boundary conditions (12)

3. Method of Solution

The non-linear ordinary differential equation (12), we find the analytical solution of the differential equation subjection to the condition (13) must in general be integrated numerically. However for special case when Re and h are small, the approximate analytical results can be obtained by use of a regular perturbation approach. In this situation f may be expanded in the form

Let

$$f(\lambda)=f_0(\lambda)+\mathrm{Re}f_1(\lambda)+\mathrm{Re}^2f_2(\lambda)+\mathrm{Re}^3f_3(\lambda)+\mathrm{Re}^4f_4(\lambda)+\cdots=\sum_{n=0}^{\infty}Re^n f_n(\lambda) \qquad (14)$$

&

$$A = K_0 + A_1 Re^1 + A_2 Re^2 + A_3 Re^3 + A_4 Re^4 \dots = \sum_{n=0}^{\infty} A_n R^n \quad (15)$$

Where the boundary conditions are

$$f_0(-1) = -1, \quad f_0'(-1) = 0, \quad f_0(1) = 1, \quad f_0'(1) = 0 \quad (16)$$

And $f_n(0) = f_n'(0) = 0, \quad f_n(1) = f_n'(1) = 0$ when $n > 0$ (17)

Putting the value of $f(\lambda)$ & A in above equation (12), we get

$$Re\Big[\big(f_0'(\lambda) + Re^1 f_1'(\lambda) + Re^2 f_2'(\lambda) + Re^3 f_3'(\lambda) + Re^4 f_4'(\lambda) + \dots\big)^2 - \big(f_0(\lambda) + Re^1 f_1(\lambda) + Re^2 f_2(\lambda) + Re^3 f_3(\lambda) + Re^4 f_4(\lambda) + \dots\big).\big(f_0''(\lambda) + Re^1 f_1''(\lambda) + Re^2 f_2''(\lambda) + Re^3 f_3''(\lambda) + Re^4 f_4''(\lambda) + \dots\big)\Big] + f_0'''(\lambda) + Re^1 f_1'''(\lambda) + Re^2 f_2'''(\lambda) + Re^3 f_3'''(\lambda) + Re^4 f_4'''(\lambda) + \dots = A_0 + A_1 Re^1 + A_2 Re^2 + A_3 Re^3 + A_4 Re^4 + \cdots \quad (18)$$

Taking the coefficient of $Re^0, Re^1, Re^2, Re^3, \dots \dots$

We get $f_0''' = A_0$ (19)

$$f_1''' = A_1 - f_0'^2 + f_0 f_0'' \quad (20)$$

$$f_2''' = A_2 - 2f_0' f_1' + f_1 f_0'' + f_0 f_1'' \quad (21)$$

$$f_3''' = A_3 - f_1'^2 - 2f_0' f_1' + f_2 f_0'' + f_1 f_1'' + f_0 f_2'' \quad (22)$$

$$f_4''' = A_4 - 2f_0' f_3' - 2f_1' f_2' + f_0'' f_3 + f_0 f_3'' + f_1'' f_2 + f_1 f_2'' \quad (23)$$

Integrating above equation (19),

We get $f_0 = \frac{K_0 \lambda^3}{6} + \frac{C_1 \lambda^2}{2} + C_2 \lambda + C_3$ (24)

Using the boundary condition (16), we get

$$f_0(\lambda) = \frac{\lambda}{2}(3 - \lambda^2) = 1.3\,\lambda - 0.5\,\lambda^3 \quad (25)$$

Using similar process by putting the value of f_1, f_2, f_3 & f_4 in above equation (22), (21), (22) & (23) with boundary condition (17)

We get $f_1(\lambda) = -0.0143\,\lambda^2 + 0.01786\,\lambda^3 - 0.00357143\,\lambda^7$ (26)

$f_2(\lambda) = 0.00039392\,\lambda^2 - 0.00191144\,\lambda^3 + 0.001788\,\lambda^4 - 0.0002383\,\lambda^6 + 0.000255\,\lambda^7 - 0.000298\,\lambda^9 + 0.00001082\,\lambda^{11}$

(27)

$f_3(\lambda) = -0.0022742\,\lambda^2 + 0.0015501\,\lambda^3 + 0.00362425\,\lambda^4 - 0.002983\,\lambda^5 - 0.000452\,\lambda^6 + 0.0008155\,\lambda^7 - 0.0003971\,\lambda^8 + 0.0003721\,\lambda^9 + 0.00008753\,\lambda^{10} - 0.0004417\,\lambda^{11} + 0.0000688\,\lambda^{13} - 0.0000002626\,\lambda^{15}$

(28)

$f_4(\lambda) = 0.00164225\,\lambda^2 - 0.00251\,\lambda^3 + 0.0002844\,\lambda^4 + 0.000233\,\lambda^5 + 0.000506\,\lambda^6 - 0.00018971\,\lambda^7 + 0.000000348\,\lambda^8 + 0.00005641\,\lambda^9 - 0.0000324\,\lambda^{10} + 0.0000283\,\lambda^{11} + 0.0000102\,\lambda^{12} - 0.0000373\,\lambda^{13} - 0.0000018544\,\lambda^{14} + 0.00000899\,\lambda^{15} - 0.00000076\,\lambda^{17} + 0.000000002871\,\lambda^{19}$

(29)

Hence the solution of above non-linear differential equation is given below by putting the value of $f_0,\ f_1,\ f_2$ & f_3.

$f(\lambda) = f_0(\lambda) + \mathrm{Re}f_1(\lambda) + \mathrm{Re}^2 f_2(\lambda) + \mathrm{Re}^3 f_3(\lambda) + \mathrm{Re}^4 f_4(\lambda)\ +$ (30)

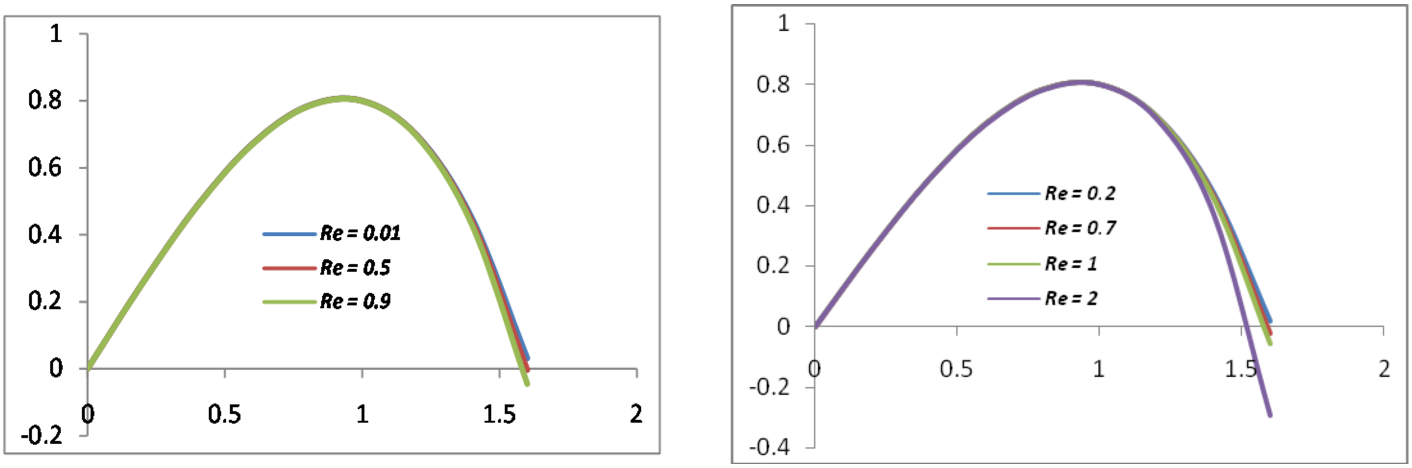
Or

$f(\lambda) = 1.3\,\lambda - 0.5\,\lambda^3 + \mathrm{Re}[-0.0143\,\lambda^2 + 0.01786\,\lambda^3 - 0.00357143\,\lambda^7] + \mathrm{Re}^2[0.00039392\,\lambda^2 - 0.00191144\,\lambda^3 + 0.001788\,\lambda^4 - 0.0002383\,\lambda^6 + 0.000255\,\lambda^7 - 0.000298\,\lambda^9 + 0.00001082\,\lambda^{11}] + \mathrm{Re}^3[-0.0022742\,\lambda^2 + 0.0015501\,\lambda^3 + 0.00362425\,\lambda^4 - 0.002983\,\lambda^5 - 0.000452\,\lambda^6 + 0.0008155\,\lambda^7 - 0.0003971\,\lambda^8 + 0.0003721\,\lambda^9 + 0.00008753\,\lambda^{10} - 0.0004417\,\lambda^{11} + 0.0000688\,\lambda^{13} - 0.0000002626\,\lambda^{15}] + \mathrm{Re}^4[0.00164225\,\lambda^2 - 0.00251\,\lambda^3 + 0.0002844\,\lambda^4 + 0.000233\,\lambda^5 + 0.000506\,\lambda^6 - 0.00018971\,\lambda^7 + 0.000000348\,\lambda^8 + 0.00005641\,\lambda^9 - 0.0000324\,\lambda^{10} + 0.0000283\,\lambda^{11} + 0.0000102\,\lambda^{12} - 0.0000373\,\lambda^{13} - 0.0000018544\,\lambda^{14} + 0.00000899\,\lambda^{15} - 0.00000076\,\lambda^{17} + 0.000000002871\,\lambda^{19}]$

$+\ \ldots$ (31)

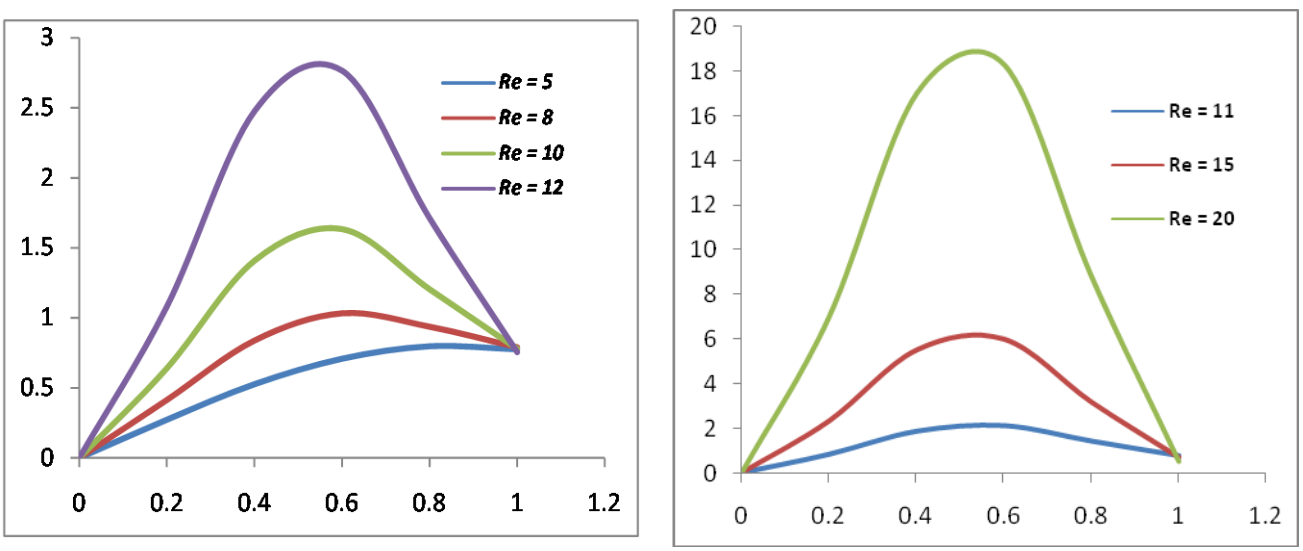
4. Result and discussion

Objective of the present study was to apply totally the numerical technique and in this paper we solve the non-linear differential equation (12) subject to (16) & (17) must in be integrated by analytic procedure to use of a perturbation approach which is consider as power series in the form of (14).



(a) (b)

Fig. 2. Graph between dimensionless variable λ and fluid velocity $f(\lambda)$ in the different Suction Reynolds Number.



(a) (b)

Fig. 3. Graph between dimensionless variable λ and fluid velocity $f(\lambda)$ in the different Large Reynolds Number.

However for we see that increase of Reynolds Number velocity of fluid increases slowly and then decreases sharply, at different suction Reynolds number velocity of fluid is show in small change which is shown in the fig. 2 (a) and (b) where large change of Reynolds Number we see that the velocity of fluid is major change which is given in the fig. 3 (a) and (b). In this figure we see that increase of Reynolds Number the velocity of fluid is sharply increases and then sharply decreases. This variation is shown in the graph given below.

5. Conclusion

In this research paper an analytical procedure is used for solving the differential equations and the main objective is to analysis of laminar flow of viscous fluid, the effect of Suction Reynolds Number and Large Reynolds Number porous bounding walls. The enhancements of Reynolds Number, velocity of fluid increase sharply and then decreases sharply for large Reynolds Number where as the velocity of fluid is small changes for suction Reynolds number. The application of viscous flow of fluid is in engineering and biological problem such as accelerators electrostatics precipitation, petroleum industry, geothermal energy extraction and plasma studies.

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