International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395 -0056
Volume: 03 Issue: 03 | Mar-2016 www.irjet.net p-ISSN: 2395-0072

LATTICE REDUCTION AIDED DETECTION TECHNIQUES FOR MIMO SYSTEMS

Susmita Prasad[1], Samarendra Nath Sur[2]

Dept. of Electronics and Communication Engineering, Sikkim Manipal Institute of Technology, Majhitar, Sikkim, India

---***---

Abstract - *A Multiple input multiple output (MIMO) technology is seen to provide the best solution to high data rate and reliable wireless communication. For the purpose of detection, Maximum Likelihood receivers are most optimal but highly complex especially with higher order constellation. There are a number of other detectors, linear and non-linear, which are less complex but suboptimal. In this paper we utilize a novel class receivers based on Lattice Reduction for MIMO Systems which achieve near maximum-likelihood detector performance with lower complexity. [1] Lenstra-Lenstra-Lovasz Algorithm [2] is used for lattice reduction purpose. Performance comparisons are made between LRA receivers and other conventional receivers in both independent and correlated channels by simulations. It will be shown that LRA based receivers outperform the conventional ones, especially in correlated channels.*

Key Words: **MIMO Systems, Zero-Forcing Detection, Minimum Mean Square Error Detection, Wireless Communication, Lattice-Reduction, Maximum-Likelihood Detection, Bit Error Rate**

1. INTRODUCTION

There is a huge demand for high data rate wireless communication services which has caused notable research interests in the multiple input and multiple output (MIMO) technologies. In MIMO, a number of independent data streams are simultaneously send over a communication channel by the use of multiple antennas at the transmitter and receiver sides in a rich scattering environment. Each receiving antenna acquires a superimposition of all of these transmitted streams. The process of separating out each independent data streams is called the MIMO detection. [3]

A brute-force Maximum-Likelihood (ML) detection provides optimal solution to the MIMO symbol detection [4], but its implementation is highly complex especially with either a larger size constellation or large number of antennas. Therefore, the real challenge lies in designing the hardware for the MIMO symbol detectors such that bit-error-rate (BER) performance comparable to the ML detector is achieved while having low hardware complexity and high throughput. Many low-complexity methods like Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) detection exhibits considerably lower complexity which map well to hardware but have greatly reduced BER performance compared to the ML detector. [5] It is clearly desirable to explore detection algorithms that achieve ML or near-ML performance.

Lattice reduction (LR)-aided detectors incorporate lattice reduction algorithms into the algorithms of ZF or MMSE detectors.[6] For L-R aided MIMO detection, the Lenstra-Lenstra-Lovasz algorithm has been used exclusively till date. The LLL reduction is used to improve the performance of the MIMO detection schemes. The algorithm optimizes the generating matrix of the lattice, to obtain a nicer description of the lattice. [7] As of now many research papers have shown the utilization of LLL algorithm for the purpose of lattice reduction. Few of these papers include 'Lattice Reduction Aided Detection for MIMO-OFDM-CDM Communication Systems' by J. Adeane, M.R.D. Rodrigues and I.J.Wessel and 'Lattice-Reduction-Aided Receivers for MIMO-OFDM in Spatial Multiplexing System' by Inaki Berenguer, Jaime Adeaner, Ian J. Wassell and Xiaodong Wang.

Performance comparisons between LRA receiver and other linear receivers will be provided. It will be shown that even with higher order constellation and when the channels are correlated, LRA significantly outperforms other suboptimal detectors in terms of BER.

2. SYSTEM MODEL

Let us consider a $N_T \times N_R$ MIMO Communication system where N_T is the number of transmit antennas and N_R is the number of receive antennas. The data symbol is de-multiplexed into N_T data symbols and then mapped onto rectangular QAM symbols. The modulated data stream is now simultaneously transmitted over N_T antennas over a rich scattering channel.

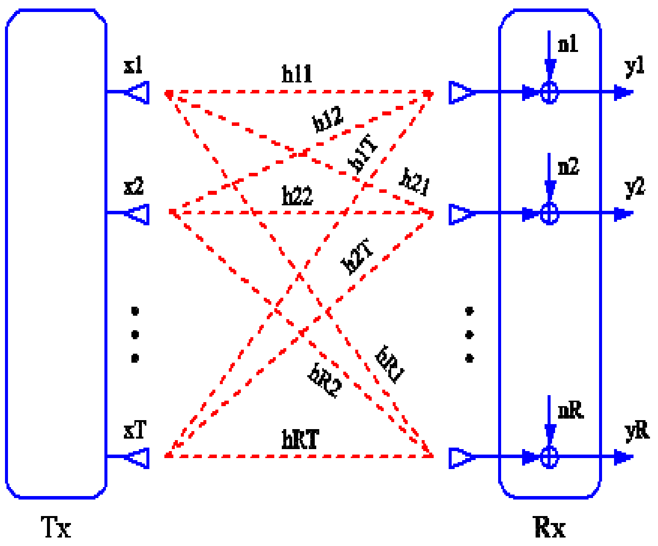

Figure 1: MIMO System [8]

It is possible to relate the received data vector to the transmitted data vector as

$\bar{y} = \mathcal{H}\bar{x} + \bar{n}$ ……… (1)

This is the baseband model where the received data vector

$\bar{y} = [y_1, y_2 \ldots\ldots y_{N_R}]^T$, transmitted data vector $\bar{x} = [x_1, x_2 \ldots\ldots x_{N_T}]^T$,noise vector $\bar{n} = [n_1, n_2 \ldots\ldots n_{N_R}]^T$ and $\mathcal{H}$ is the $N_T \times N_R$ matrix of complex flat fading channel coefficients between transmit and receive antennas. n is modelled as a zero mean white Gaussian random vector with covariance matrix σ_n^2 I_{N_R}.[9]

Since $\bar{x}, \bar{y}, \bar{n}$ *and* $\mathcal{H}$ are complex valued we can equivalently write

$$\begin{bmatrix} \mathcal{R}_y \\ \mathcal{I}_y \end{bmatrix} = \begin{bmatrix} \mathcal{R}_{\mathcal{H}} & -\mathcal{I}_{\mathcal{H}} \\ \mathcal{I}_{\mathcal{H}} & \mathcal{R}_{\mathcal{H}} \end{bmatrix} \begin{bmatrix} \mathcal{R}_x \\ \mathcal{I}_x \end{bmatrix} + \begin{bmatrix} \mathcal{R}_n \\ \mathcal{I}_n \end{bmatrix} \text{.........(2)} .$$

This gives us the real model of the form:

$$y_{rl} = \mathcal{H}_{rl} x_{rl} + n_{rl} \text{.........(3)}$$

2.1 LRA DETECTION

In LRA detection, the channel matrix $\mathcal{H}_{rl}$ is considered as the generator matrix of some lattice. The columns of the channel matrix are generator basis of this lattice.

Let $\mathcal{H}_{rl}$ be defined as

$$\mathcal{H}_{rl} = \begin{bmatrix} h_{11} & \cdots & h_{1N_R} \\ \vdots & \ddots & \vdots \\ h_{N_T 1} & \cdots & h_{N_T N_R} \end{bmatrix} \text{.........(4)}$$

Here the disadvantage of $\mathcal{H}_{rl}$ is that the receiver signal easily falls out of its decision region by even a small amount of noise if the basis vectors of $\mathcal{H}_{rl}$ is highly correlated i.e. the angle between the vectors is very narrow. Therefore there is a need of Lattice Reduction whose supreme goal is to transform the generator matrix $\mathcal{H}_{rl}$ of the lattice to another generator matrix $\mathcal{H}_{rl}{}'$ of that same lattice by finding out the change of basis P.

$\mathcal{H}_{rl}{}' = \mathcal{H}_{rl} P$ ....(5)

The new generator matrix $\mathcal{H}_{rl}{}'$ can now be designed to be near orthogonal such that it improves the reliability of many low-complexity suboptimal detectors. There are many existing LR algorithms that help in achieving this.

A lattice reduction algorithm is an algorithm that can be used to improve the performance of MIMO detection schemes, provided that the channel state information (i.e. the matrix $\mathcal{H}_{rl}$) is known at the receiver.[10] This algorithm finds another basis $\mathcal{H}_{rl}{}'$ which enjoys better properties than $\mathcal{H}_{rl}$, for example with respect to inversion, and hence makes it easier to detect the transmitted symbol in this lattice when noise is present.

2.2 ZF AND MMSE ALGORITHMS

At the receiver, $\bar{y} = \mathcal{H}\bar{x} + \bar{n}$. In order to detect the message, we perform inverse operation, $\mathcal{H}^{-1}\bar{y} = \bar{x} + \mathcal{H}^{-1}\bar{n}$......(6). Thus the simple receiving consists of the inverting of channel matrix $\mathcal{H}$ along with an extra term $\mathcal{H}^{-1}\bar{n}$. But its inverse modelling is difficult as the inverse exists only when the matrix is square. Therefore, we define a generalized inverse considering $N_R > N_T$. Two of these approaches are discussed below :

In case of zero forcing detection technique, we choose the minimum error vector from among all the possible transmit vectors $\bar{x}$, i.e. $\bar{x}$ is to be selected in such a way that error $||\bar{y} - \mathcal{H}\bar{x}||^2$ is minimized. [11]

On vector differentiation we get,

$\hat{x} = (\mathcal{H}^T\mathcal{H})^{-1}\mathcal{H}^T\bar{y}$......(7)

This is the approximate solution which minimizes the error called the Least Square Error Solution.

For complex channel matrix,

$\hat{x} = (\mathcal{H}^H\mathcal{H})^{-1}\mathcal{H}^H\bar{y}$......(8)

where $\mathcal{H}^{+} = (\mathcal{H}^H\mathcal{H})^{-1}\mathcal{H}^H$ ...... (9) is the pseudo inverse of $\mathcal{H}$.

Zero Forcing suffers from noise amplification especially when the number of transmitters and receivers are same. This can be shown with the example of Single Input and Single Output System.

Let us say,

$y = hx + n$

Inverse modelling will result in

$h^{-1}y = x + h^{-1}n$

$\frac{y}{h} = x + \frac{n}{h}$.........(10)

If the value of h is small ($h \simeq 0$), the noise blows up causing instability.

The minimum mean square error detection takes noise into account and thereby leading to improved performance and elimination of noise enhancement. The estimate of the transmitted vector at the MMSE receiver is given by

$\hat{x} = P_d\mathcal{H}^H \, (P_d\mathcal{H}\mathcal{H}^H + \sigma_n^2 \, \mathrm{I})^{-1}\bar{y}$ ....(11)

where P_d is the transmitted data power and σ_n^2 is the noise power at the receiver.

Equivalently, $\hat{x}$ can be expressed as

$\hat{x} = P_d \, (P_d\mathcal{H}^H\mathcal{H} + \sigma_n^2 \, \mathrm{I})^{-1}\mathcal{H}^H\bar{y}$ ....(12)

At very high SNR,

$P_d > \sigma_n^2$ $\quad \hat{x} \simeq (\mathcal{H}^H\mathcal{H})^{-1}\mathcal{H}^H\bar{y}$ [12] This is nothing but the transmit vector estimate of ZF receiver. So we can say that ZF is the limiting form of MMSE for SNR approaches to infinity. [13]

2.3 LLL ALGORITHMS

Lenstra-Lenstra-Lovasz Algorithm is a popular Lattice Reduction aided detection algorithm named after its founder whose running time is polynomial in the dimension of the lattice. [14]

Implementation of LLL lattice reduction[15]

Input: Lattice Basis

$h_1 = \mathcal{H}[:,1], \dots\dots\dots, h_N = \mathcal{H}[:,N] \in C^N$,

$\frac{1}{4} < \delta < 1$

$k = 2$

While $k \leq N$ do

$\quad$ *for* $n = k-1, \dots\dots,1$ do

$\qquad h_k = h_k - [\mu_{k,n}]h_n$;

$\quad$ *end for*

$\quad$ % Compute $\hat{h}_k$

$\quad \hat{h}_1 = h_1$

$\quad \hat{h}_n = h_n - \sum_{m=1}^{n-1}\mu_{n,m}\hat{h}_m$ *for* $n = 2, \dots N$

$\quad$ *If* $\delta \parallel \hat{h}_{k-1} \parallel^2 > \parallel \hat{h}_k + \mu_{k,k-1}\hat{h}_{k-1} \parallel^2$

$\quad$ *then* $h_{k-1} \leftrightarrow h_k$ % Exchange

$\quad k = \max(k-1, 2)$

$\quad$ *else* $k = k+1$;

end while

$h'_u = h_u$ *for* $u = 1,2,3 \dots N$ % final result

Output: Reduced lattice Basis $\mathcal{H}' = [h'_1, h'_2, \dots\dots . h'_N]$ and P defined as $\mathcal{H}' = \mathcal{H}P$

3. SIMULATIONS AND RESULTS

In this paper all the results are simulated using MATLAB software. Basically this paper represents the comparative study of different receiver system under correlated channel condition.

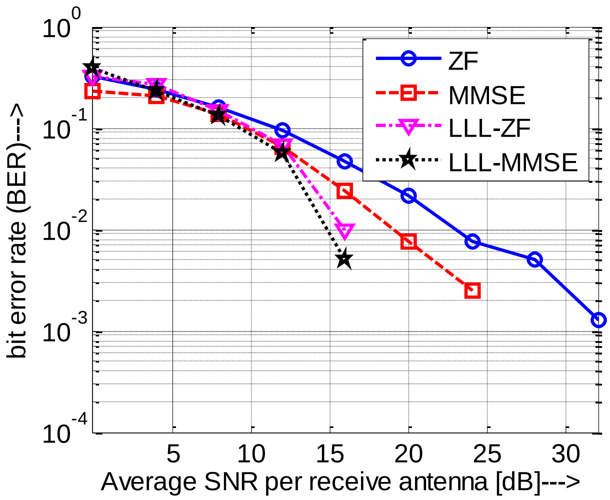

Figure 1: BER Performance of a 4 × 4 MIMO System with uncorrelated channels and QPSK modulation.

This figure represents the study of BER performance of LLL receivers and conventional receivers with the system being QPSK modulated and having uncorrelated channels.

We observe that as the SNR increases, the BER of the receivers decrease.

It is also observed that the LRA receivers have better BER performance than the conventional receivers.

Table 1: Comparison of BER performance of linear and LRA aided receivers

Receiver	To obtain BER 0.01 SNR required in dB
ZF	22.94
MMSE	18.98
LLL-ZF	16
LLL-MMSE	14.84

LLL-MMSE outperform ZF and MMSE receivers by 8.1 dB and 4.14 dB respectively at 0.01 BER. LLL-ZF outperform ZF and MMSE receivers by 6.94 dB and 2.98 dB respectively at 0.01 BER.

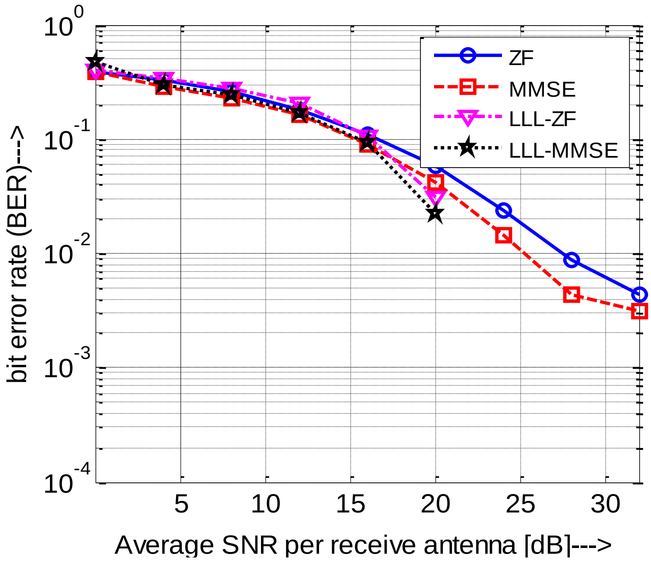

Figure 2: BER Performance of a 4 × 4 MIMO System with uncorrelated channel and 16QAM Modulation.

This figure represents the study of BER performance of LLL receivers and conventional receivers with the channels being uncorrelated and system being 16QAM modulated.

It is observed that as the SNR increases, the BER of the receivers decrease. We observe that the LLL receivers have better BER performance than the conventional receivers.

Table 2: Comparison of BER performance of linear and LRA aided receivers

Receiver	To obtain BER 0.03 the SNR required in dB
ZF	22.87
MMSE	21.15
LLL-ZF	19.97
LLL-MMSE	19.1

LLL-MMSE outperform ZF and MMSE receivers by 3.77 dB and 2.05 dB respectively at 0.03 BER. LLL-ZF outperform ZF and MMSE receivers by 2.9 dB and 1.18 dB respectively at 0.03 BER. It is clear from figures 1 and 2 that the Lattice Reduction Aided Receivers outperform the conventional receivers.

Table 3: Comparison of BER curves of receivers for QPSK modulated and 16QAM modulated systems

Receivers	To obtain BER 0.03	
	SNR required by a QPSK modulated system (in dB)	SNR required by a 16QAM modulated system (in dB)
ZF	18.16	22.87
MMSE	15.02	21.15
LLL-ZF	13.67	19.97
LLL-MMSE	13.02	19.1

This table represents comparison between figure 1 and figure 2 i.e. performance of the receivers when the size of the constellation is increased.

We observe that as the constellation is increased from QPSK to 16QAM, SNR required for LLL-MMSE receiver to achieve a BER of 0.03 increases by 6.08dB. Also, similarly in case of ZF receiver 4.71dB increment in SNR is required for getting 0.03 BER.

It can be concluded that as the size of the constellation is increased the BER performance of the receivers degrades.

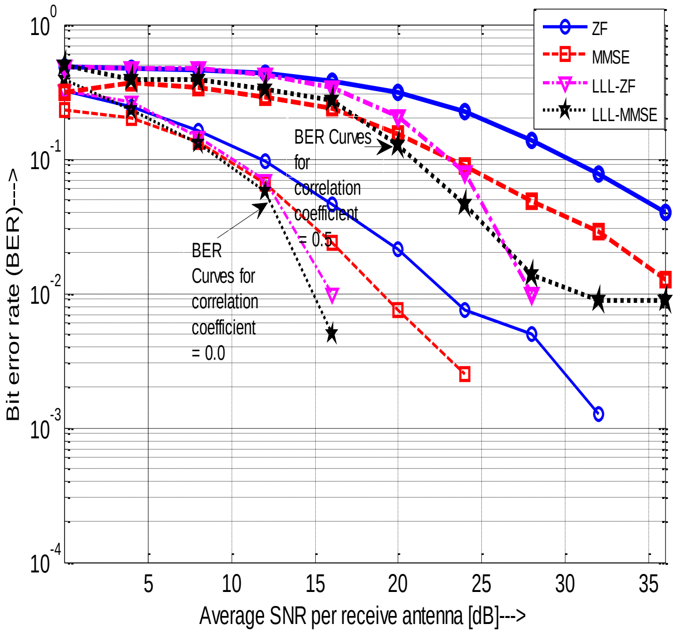

Figure 3. BER Performance of a 4 × 4 MIMO QPSK System with correlation coefficients of the channel as 0.0 and 0.5.

This figure shows the comparison of BER performances of the receivers when the correlation coefficient of the channel is increased (here from 0.0 to 0.5) and when the system is QPSK modulated.

We observe that as the correlation coefficient of the channel increases, the BER performance of the receivers degrades.

Table 4. Comparison of BER curves of receivers with correlation coefficients 0.0 and 0.5

Receivers	To obtain BER 0.04	
	SNR required by a system having correlation coefficient of 0.0	SNR required by a system having correlation coefficient of 0.5
ZF	16.67	35.8
MMSE	13.9	29.31
LLL-ZF	13.1	25.3
LLL-MMSE	12.55	24.46

As the correlation coefficient of the channel is increased from 0.0 to 0.5 the SNR required by a LLL-MMSE receiver to obtain 0.04 BER increases by 11.91 dB. Also, for a ZF receiver this requirement is 19.13dB.

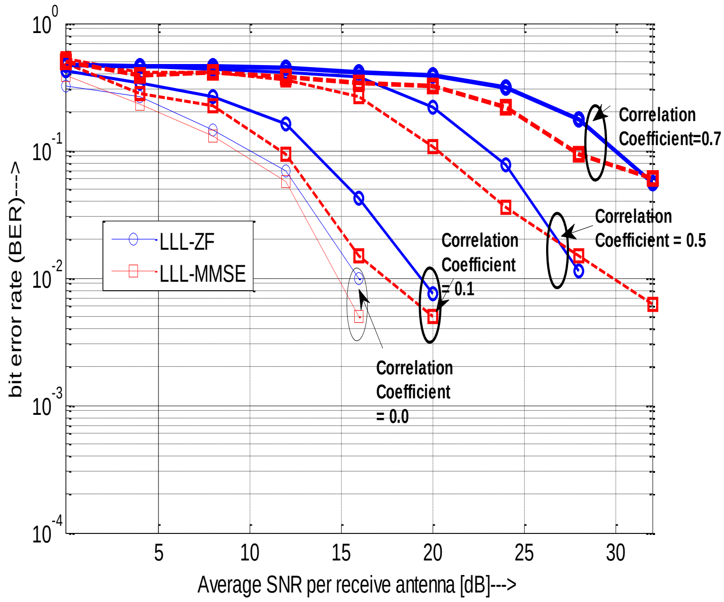

Figure 4: BER Performance of a 4 × 4 MIMO QPSK System with correlation coefficients of the channel as 0.0, 0.1, 0.5 and 0.7

This figure shows comparison of BER performances of LLL-MMSE and LLL-ZF Receivers with increasing

correlation coefficients – 0, 0.1, 0.5 and 0.7 and QPSK modulation.
We observe that more correlated the channel, lesser is the BER performance of the receiver.

Table 5. Comparison of BER curves of LLL-ZF and LLL-MMSE receiver for increasing values of correlation coefficients

Correlation coefficient of the channel	To obtain BER 0.01	
	SNR required (in dB) for LLL-ZF receiver	SNR required (in dB) for LLL-MMSE receiver
0.0	10.03	9.198
0.1	13.4	11.69
0.5	22.87	20.2
0.7	30.01	27.7

At BER 0.01 and channel correlation coefficient 0.7, LLL-MMSE outperforms LLL-ZF receiver by 2.31 dB.
Also, as the correlation of the channel is increased from 0.1 to 0.7, the SNR requirement by the LLL-MMSE receiver increases by 16.01 dB to obtain BER 0.01.

4. CONCLUSIONS

In this paper, we have investigated several detection schemes for MIMO Communication Systems. We have used Lenstra-Lenstra-Lovasz algorithm for lattice reduction. We can conclude from the simulation results that LRA receivers outperforms the traditional linear receivers. Performance of the LRA receivers is found close to that of ML receivers. This shows that the LLL algorithm has capability of improving BER performance of conventional receivers.

REFERENCES

[1] Inaki Berenguer, Jaime Adeaner, Ian J. Wassell and Xiaodong Wang, "Lattice-Reduction-Aided Receivers for MIMO-OFDM in Spatial Multiplexing System", Personal, Indoor and Mobile Radio Communications, 2004. PIMRC 2004. 15th IEEE International Symposium on, On pages: 1517 - 1521 Volume 2, 5-8 Sept. 2004

[2] Chiao-En Chen, "A New Lattice Reduction Algorithm for LR-aided MIMO Linear Detection", in IEEE Transactions on Wireless Communications, On pages 2417 – 2422 Volume:10 , Issue: 8, 23 June 2011

[3] Brain J. Gestner, "Lattice Reduction for MIMO detection : From theoretical analysis to hardware realization ", PhD dissertation, School of Electrical and Computer Engineering, Georgia Institute of technology, May 2011

[4] C. Windpassinger and R. F. H. Fischer, "Low-Complexity Near-Maximum-Likelihood Detection and Precoding for MIMO Systems using Lattice Reduction", in Proc. IEEE Information Theory Workshop(ITW), Paris, France, March 2003

[5]J. Adeane, M.R.D. Rodrigues and I.J.Wessel, "Lattice Reduction Aided Detection for MIMO-OFDM-CDM Communication Systems", IET Communications, Pages 526 – 531 Volume 1 Issue 3, June 2007

[6] Dirk Wubben, Ronald Bohnke, Volker Kuhn and Karl-Dirk Kammeyer, "MMSE-BASED LATTICE-REDUCTION FOR NEAR-ML DETECTION FOR MIMO SYSTEMS", Communications, 2004 IEEE International Conference on, On pages: 798 - 802 Vol.2, 20-24 June 2004

[7]Christoph Windpassinger, Lutz H. -j. Lampe, Robert F. H. Fischer, "From Lattice Reduction-Aided Detection towards Maximum-Likelihood Detection", in MIMO system, in Proceedings of WOC, 2003

[8]http://www.wcsg.ieiit.cnr.it/research/activity/MI-C-MIMO.html

[9]Ying Hung Gan, and Wai Ho Mow, "Complex Lattice Reduction Algorithm for Low-Complexity MIMO Detection", IEEE Transactions on Signal Processing (Volume:57 , Issue: 7), On pages: 2701 – 2710, 24 February 2009

[10]*Advanced 3G and 4G Wireless Communication,* NPTEL National Program on Technology Enhanced Learning, Prof. Aditya K. Jagannatham*:* MIMO MMSE Receiver and Introduction to SVD 2013 [Online].
Available : https://youtu.be/aQqgMcSviko [Accessed: 22- Feb- 2015]

[11] *Advanced 3G and 4G Wireless Communication,* NPTEL National Program on Technology Enhanced Learning, Prof. Aditya K. Jagannatham*:* MIMO System Model and Zero-Forcing Receiver 2013 [Online].
Available : https://youtu.be/1_-9tbFNsPk [Accessed: 21- Feb- 2015]

[12]*Advanced 3G and 4G Wireless Communication,* NPTEL National Program on Technology Enhanced Learning, Prof. Aditya K. Jagannatham*:* CDMA Near-Far Problem and Introduction to MIMO 2013 [Online].
Available : https://youtu.be/p8uQz_3U1CA. [Accessed: 20- Feb- 2015]

[13]Adeane, J.; Rodrigues, M.R.D.; Berenguer, I.; Wassell, I.J. "Improved detection methods for MIMO-OFDM-CDM

communication systems", *Vehicular Technology Conference, 2004. VTC2004-Fall. 2004 IEEE 60th,* On page(s): 1604 - 1608 Vol. 3 Volume: 3, 26-29 Sept. 2004

[14] H. Yao and G.W. Wornell. ‘‘Lattice-reduction-aided detectors for MIMO communication systems.’’ In Proc. IEEE Globecom, pages 424-428, Taipei, Taiwan, Nov. 2002.

[15] Lovasz, L.: ‘An algorithmic theory of numbers, graphs, and convexity’ (SIAM, Philadelphia, PA, 1986).