Two Ridiculous Summation Formulae arising from the summation formulae of Salahuddin et al

Intazar Husain[1] and Salahuddin[2]

[1] Department of Applied Sciences and Humanities, Jamia Millia Islamia, New Delhi, India

[2] P.D.M College of Engineering, Bahadurgarh,Haryana, India

Abstract - ***In this paper, we have established two summation formulae with the help of contiguous relation , derived formulae of Salahuddin et al and using Mathematica.***

Key Words: *Contiguous relation, Summation formulae,Prudnikov et al.*

Abstract

In this paper, we have established two summation formulae with the help of contiguous relation, derived formulae of Salahuddin et al and using Mathematica.

Key Words : Contiguous relation, Summation formulae,Prudnikov et al.

2010 MSC NO: 33C05, 33C20, 33C60

1. INTRODUCTION AND RESULTS REQUIRED

Special functions and their uses are now spectacular in their scope. Moreover , their rapid growth in pure Mathematics and its applications to the traditional fields of Physics, Engineering and Statistics but in new fields of applications like scientific computing, Optimization, Biology, Environmental Science and Economics ,Management etc. they are emerging. Summation formulae for hypergeometric function has an important role in applied mathematics.

Prudnikov et al[2,p.414] derived the following seven summation formulae

$$ {}_2F_1\left[\begin{matrix} a, & -a\ ; \\ c & ;\end{matrix}\ \frac{1}{2}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^c}\left[\frac{1}{\Gamma(\frac{c+a+1}{2})\,\Gamma(\frac{c-a}{2})} + \frac{1}{\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c-a+1}{2})}\right] \qquad (1) $$

$$ {}_2F_1\left[\begin{matrix} a, & 1-a\ ; \\ c & ;\end{matrix}\ \frac{1}{2}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-1}}\left[\frac{1}{\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c-a+1}{2})}\right] \qquad (2) $$

$$ {}_2F_1\left[\begin{matrix} a, & 2-a\ ; \\ c & ;\end{matrix}\ \frac{1}{2}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(a-1)\,2^{c-2}}\left[\frac{1}{\Gamma(\frac{c+a-2}{2})\,\Gamma(\frac{c-a+1}{2})} - \frac{1}{\Gamma(\frac{c+a-1}{2})\,\Gamma(\frac{c-a}{2})}\right] \qquad (3) $$

$$ {}_2F_1\left[\begin{matrix} a, & 3-a \ ; & \frac{1}{2} \\ c & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(a-1)(a-2)\,2^{c-3}}\left[\frac{(c-2)}{\Gamma(\frac{c+a-2}{2})\,\Gamma(\frac{c-a+1}{2})} - \frac{2}{\Gamma(\frac{c+a-3}{2})\,\Gamma(\frac{c-a}{2})}\right] \quad (4) $$

$$ {}_2F_1\left[\begin{matrix} a, & 4-a \ ; & \frac{1}{2} \\ c & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(1-a)(2-a)(3-a)\,2^{c-4}}\left[\frac{(a-2c+3)}{\Gamma(\frac{c+a-4}{2})\,\Gamma(\frac{c-a+1}{2})} + \frac{(a+2c-7)}{\Gamma(\frac{c+a-3}{2})\,\Gamma(\frac{c-a}{2})}\right] \quad (5) $$

$$ {}_2F_1\left[\begin{matrix} a, & 5-a \ ; & \frac{1}{2} \\ c & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-5}\{\prod_{\gamma=1}^{4}(\gamma-a)\}}\left[\frac{\{2(c-2)(c-4)-(a-1)(a-4)\}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-4}{2})} + \frac{(12-4c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-5}{2})}\right] \quad (6) $$

$$ {}_2F_1\left[\begin{matrix} a, & 6-a \ ; & \frac{1}{2} \\ c & ; & \end{matrix}\right] = $$

$$ = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-6}\{\prod_{\delta=1}^{5}(\delta-a)\}}\left[\frac{(4c^2+2ac-a^2-a-34c+62)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-5}{2})} - \frac{(4c^2-2ac-a^2+13a-22c+20)}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-6}{2})}\right] \quad (7) $$

The contiguous relation is defined as Abramowitz et al[1,p.558]

$$ b\ {}_2F_1\left[\begin{matrix} a, & b+1 \ ; & z \\ c & ; & \end{matrix}\right] = (b-c+1)\ {}_2F_1\left[\begin{matrix} a, & b \ ; & z \\ c & ; & \end{matrix}\right] + (c-1)\ {}_2F_1\left[\begin{matrix} a, & b \ ; & z \\ c-1 & ; & \end{matrix}\right] \quad (8) $$

Salahuddin et al[3,4,5] derived the following thirteen summation formulae

$$ {}_2F_1\left[\begin{matrix} a, & 7-a \ ; & \frac{1}{2} \\ c & ; & \end{matrix}\right] = $$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-7}\{\prod_{\varsigma=1}^{6}(\varsigma - a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-6}{2})}(-3a^2c + 12a^2 + 21ac - 84a + 4c^3 - 48c^2 + 158c - 120) + \frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-7}{2})}(2a^2 - 14a - 8c^2 + 64c - 108)\Big] \quad (9)$$

$${}_2F_1\left[\begin{matrix} a, & 8-a\ ; \\ c & ; \end{matrix}\ \frac{1}{2}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-8}\{\prod_{\xi=1}^{7}(\xi - a)\}}\Big[\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-7}{2})}(-a^3 - 4a^2c + 30a^2 + 4ac^2 - 4ac - 107a + 8c^3 - 124c^2 + 576c - 762) + \frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-8}{2})}(-a^3 + 4a^2c - 6a^2 + 4ac^2 - 68ac + 181a - 8c^3 + 92c^2 - 288c + 210)\Big] \quad (10)$$

$${}_2F_1\left[\begin{matrix} a, & 9-a\ ; \\ c & ; \end{matrix}\ \frac{1}{2}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-9}\{\prod_{\varpi=1}^{8}(\varpi - a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-8}{2})}(a^4 - 18a^3 - 8a^2c^2 + 80a^2c - 85a^2 + 72ac^2 - 720ac + 1494a + 8c^4 - 160c^3 + 1056c^2 - 2560c + 1680) + \frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-9}{2})}(8a^2c - 40a^2 - 72ac + 360a - 16c^3 + 240c^2 - 1072c + 1360)\Big] \quad (11)$$

$${}_2F_1\left[\begin{matrix} a, & 10-a\ ; \\ c & ; \end{matrix}\ \frac{1}{2}\right] =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-10}\{\prod_{\upsilon=1}^{9}(\upsilon - a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-10}{2})}(-a^4 - 4a^3c + 42a^3 + 12a^2c^2 - 72a^2c - 107a^2 + 8ac^3 - 252ac^2 + 1772ac - 3054a - 16c^4 + 312c^3 - 2000c^2 + 4704c - 3024) + \frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-9}{2})}(a^4 - 4a^3c + 2a^3 - 12a^2c^2 + 192a^2c -$$

$$-553a^2+8ac^3-12ac^2-868ac+3406a+16c^4-392c^3+3320c^2-11224c+12264)] \quad (12)$$

$${}_2F_1\begin{bmatrix} a, & 11-a\ ; & \frac{1}{2} \\ c & ; & \end{bmatrix}=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-11}\{\prod_{\varphi=1}^{10}(\varphi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-10}{2})}(5a^4c-30a^4-110a^3c+660a^3-20a^2c^3+360a^2c^2-1305a^2c-$$

$$-810a^2+220ac^3-3960ac^2+21010ac-31020a+16c^5-480c^4+5240c^3-25200c^2+50544c-30240)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-11}{2})}(-2a^4+44a^3+24a^2c^2-288a^2c+530a^2-264ac^2+3168ac-8492a-32c^4+768c^3-6352c^2+$$

$$+20928c-22320)\Big] \quad (13)$$

$${}_2F_1\begin{bmatrix} a, & 12-a\ ; & \frac{1}{2} \\ c & ; & \end{bmatrix}=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-12}\{\prod_{\chi=1}^{11}(\chi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-12}{2})}(a^5-6a^4c+9a^4-12a^3c^2+300a^3c-1103a^3+32a^2c^3-$$

$$-408a^2c^2+46a^2c+6351a^2+16ac^4-800ac^3+10364ac^2-46852ac+62182a-32c^5+944c^4-10112c^3+47656c^2-$$

$$-93776c+55440)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-11}{2})}(a^5+6a^4c-69a^4-12a^3c^2+12a^3c+769a^3-32a^2c^3+840a^2c^2-$$

$$-5662a^2c+8301a^2+16ac^4-32ac^3-4612ac^2+42380ac-96002a+32c^5-1136c^4+15104c^3-$$

$$-92536c^2+255392c-245640)\Big] \quad (14)$$

$${}_2F_1\begin{bmatrix} a, & 13-a\ ; & \frac{1}{2} \\ c & ; & \end{bmatrix}=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-13}\{\prod_{\beta=1}^{12}(\beta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-12}{2})}(-a^6+39a^5+18a^4c^2-252a^4c+275a^4-468a^3c^2+6552a^3c$$

$$-18135a^3 - 48a^2c^4 + 1344a^2c^3 - 9834a^2c^2 + 5964a^2c + 74246a^2 + 624ac^4 - 17472ac^3 + 167388ac^2 - 631176ac$$

$$+752856a + 32c^6 - 1344c^5 + 21824c^4 - 172032c^3 + 674384c^2 - 1187424c + 665280) +$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-13}{2})}(-12a^4c + 84a^4 + 312a^3c - 2184a^3 + 64a^2c^3 - 1344a^2c^2$$

$$+6620a^2c - 2436a^2 - 832ac^3 + 17472ac^2 - 112424ac + 216216a - 64c^5 + 2240c^4 - 29312c^3 + 176512c^2 -$$

$$-478752c + 453600)] \qquad (15)$$

$${}_2F_1\begin{bmatrix} a, & 14-a\ ; & \frac{1}{2} \\ c & ; & \end{bmatrix} =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-14}\{\prod_{\gamma=1}^{13}(\gamma-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-14}{2})}(a^6 + 6a^5c - 87a^5 - 24a^4c^2 + 150a^4c + 925a^4 - 32a^3c^3 + 1392a^3c^2$$

$$-12706a^3c + 24615a^3 + 80a^2c^4 - 1728a^2c^3 + 5368a^2c^2 + 58986a^2c - 242486a^2 + 32ac^5 - 2320ac^4 + 47328ac^3$$

$$-391568ac^2 + 1344076ac - 1496568a - 64c^6 + 2656c^5 - 42560c^4 + 330752c^3 - 1278144c^2 + 2222160c - 1235520) +$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-13}{2})}(-a^6 + 6a^5c -$$

$$-3a^5 + 24a^4c^2 - 570a^4c + 2225a^4 - 32a^3c^3 + 48a^3c^2 + 7454a^3c - 39225a^3 - 80a^2c^4 + 3072a^2c^3 - 35608a^2c^2 + 133626a^2c -$$

$$-68104a^2 + 32ac^5 - 80ac^4 - 19872ac^3 + 313808ac^2 - 1676564ac + 2856228a + 64c^6 - 3104c^5 + 59360c^4 - 566848c^3 +$$

$$+2810304c^2 - 6724560c + 5897520)] \qquad (16)$$

$${}_2F_1\begin{bmatrix} a, & 15-a\ ; & \frac{1}{2} \\ c & ; & \end{bmatrix} =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-15}\{\prod_{\varepsilon=1}^{14}(\varepsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-14}{2})}(-7a^6c + 56a^6 + 315a^5c - 2520a^5 + 56a^4c^3 - 1344a^4c^2 + 5103a^4c +$$

$$+16520a^4 - 1680a^3c^3 + 40320a^3c^2 - 271215a^3c + 449400a^3 - 112a^2c^5 + 4480a^2c^4 - 54040a^2c^3 + 150080a^2c^2 + 845824a^2c -$$

$$-3383296a^2 + 1680ac^5 - 67200ac^4 + 999600ac^3 - 6787200ac^2 + 20482140ac - 21070560a + 64c^7 - 3584c^6 + 80864c^5 -$$

$$-940800c^4 + 5987520c^3 - 20296192c^2 + 32464368c - 17297280) + \frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-15}{2})}(2a^6 - 90a^5 - 48a^4c^2 + 768a^4c -$$

$$-1474a^4 + 1440a^3c^2 - 23040a^3c + 77970a^3 + 160a^2c^4 - 5120a^2c^3 + 46640a^2c^2 - 90880a^2c - 226192a^2 - 2400ac^4 +$$

$$+76800ac^3 - 861600ac^2 + 3955200ac - 6138120a - 128c^6 + 6144c^5 - 116160c^4 + 1095680c^3 - 5363584c^2 +$$

$$+12679168c - 11009376)] \qquad (17)$$

$${}_2F_1\begin{bmatrix} a, & 16-a & ; & \frac{1}{2} \\ c & & ; & \end{bmatrix} =$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-16}\{\prod_{\zeta=1}^{15}(\zeta - a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-16}{2})}(-a^7 + 8a^6c - 12a^6 + 24a^5c^2 - 792a^5c + 3710a^5 - 80a^4c^3 + 1080a^4c^2 +$$

$$+6280a^4c - 66600a^4 - 80a^3c^4 + 5280a^3c^3 - 85480a^3c^2 + 435480a^3c - 458929a^3 + 192a^2c^5 - 6240a^2c^4 + 45200a^2c^3 +$$

$$+271560a^2c^2 - 3746640a^2c + 8942052a^2 + 64ac^6 - 6336ac^5 + 186000ac^4 - 2408160ac^3 + 15005072ac^2 - 42553152ac +$$

$$+41722740a - 128c^7 + 7104c^6 - 158720c^5 + 1827360c^4 - 11505152c^3 + 38596416c^2 - 61194240c + 32432400) +$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-15}{2})}(-a^7 - 8a^6c + 124a^6 + 24a^5c^2 - 24a^5c - 2818a^5 + 80a^4c^3 - 3000a^4c^2 + 26360a^4c - 40760a^4 - 80a^3c^4 +$$

$$+160a^3c^3 + 45080a^3c^2 - 534760a^3c + 1499471a^3 - 192a^2c^5 + 10080a^2c^4 - 175760a^2c^3 + 1189560a^2c^2 - 2226480a^2c -$$

$$-2760884a^2 + 64ac^6 - 192ac^5 - 75120ac^4 + 1782560ac^3 - 16394608ac^2 + 65703616ac - 93008652a + 128c^7 - 8128c^6 +$$

$$+210944c^5 - 2878240c^4 + 22080512c^3 - 94015552c^2 + 202146816c - 165145680)\Big] \qquad (18)$$

$$ {}_2F_1\left[\begin{matrix} a, & 17-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = $$

$$ = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-17}\{\prod_{\vartheta=1}^{16}(\vartheta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-16}{2})}(a^8 - 68a^7 - 32a^6c^2 + 576a^6c - 638a^6 + 1632a^5c^2 - 29376a^5c + 101320a^5 + $$

$$ +160a^4c^4 - 5760a^4c^3 + 44640a^4c^2 + 129600a^4c - 1341071a^4 - 5440a^3c^4 + 195840a^3c^3 - 2303840a^3c^2 + $$

$$ +9743040a^3c - 9832052a^3 - 256a^2c^6 + 13824a^2c^5 - 246560a^2c^4 + 1411200a^2c^3 + 4297408a^2c^2 - 64103040a^2c + $$

$$ +143207628a^2 + 4352ac^6 - 235008ac^5 + 4977600ac^4 - 52289280ac^3 + 282566656ac^2 - 727036416ac + 670152240a + $$

$$ +128c^8 - 9216c^7 + 275456c^6 - 4423680c^5 + 41249792c^4 - 224907264c^3 + 683065344c^2 - 1014128640c + 518918400 + $$

$$ + \frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-17}{2})}(16a^6c - 144a^6 - 816a^5c + 7344a^5 - 160a^4c^3 + 4320a^4c^2 - 22480a^4c - 30960a^4 + 5440a^3c^3 - $$

$$ -146880a^3c^2 + 1157360a^3c - 2484720a^3 + 384a^2c^5 - 17280a^2c^4 + 247840a^2c^3 - 1092960a^2c^2 - 1901760a^2c + $$

$$ +15669504a^2 - 6528ac^5 + 293760ac^4 - 4999360ac^3 + 39804480ac^2 - 146267456ac + 194890176a - 256c^7 + 16128c^6 - $$

$$ -414976c^5 + 5610240c^4 - 42628864c^3 + 179788032c^2 - 383195904c + 310867200)\Big] \quad (19) $$

$$ {}_2F_1\left[\begin{matrix} a, & 18-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = $$

$$ = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-18}\{\prod_{\eta=1}^{17}(\eta-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-18}{2})}(-a^8 - 8a^7c + 148a^7 + 40a^6c^2 - 256a^6c - 3362a^6 + 80a^5c^3 - 4440a^5c^2 + $$

$$ +49664a^5c - 103400a^5 - 240a^4c^4 + 5520a^4c^3 + 18760a^4c^2 - 849520a^4c + 3240271a^4 - 192a^3c^5 + 17760a^3c^4 - 440560a^3c^3 + $$

$$ +4091160a^3c^2 - 12923320a^3c + 3622852a^3 + 448a^2c^6 - 20352a^2c^5 + 253360a^2c^4 + 576240a^2c^3 - 31091248a^2c^2 + $$

$$ +192701168a^2c - 344444908a^2 + 128ac^7 - 16576ac^6 + 660032ac^5 - 12228640ac^4 + 118499872ac^3 - 604789504ac^2 + $$

$$+1488844864ac-1324543920a-256c^8+18304c^7-542976c^6+8650240c^5 -79993344c^4+432549376c^3-1303568384c^2+$$

$$+1923025920c-980179200)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-17}{2})}(a^8-8a^7c+4a^7-40a^6c^2 +1264a^6c-6214a^6+80a^5c^3-120a^5c^2-$$

$$-32416a^5c+213904a^5+240a^4c^4-12720a^4c^3+186440a^4c^2-743120a^4c -456391a^4-192a^3c^5+480a^3c^4+216080a^3c^3-$$

$$-4278120a^3c^2+27569480a^3c-52277444a^3-448a^2c^6+30720a^2c^5 -745840a^2c^4+7817520a^2c^3-30345632a^2c^2-$$

$$-19224224a^2c+253516684a^2+128ac^7-448ac^6-259264ac^5+8556320ac^4 -118218848ac^3+813195488ac^2-$$

$$-2692403360ac+3335839536a+256c^8-20608c^7+696192c^6-12817024c^5 +139638144c^4-913535872c^3+3463541888c^2-$$

$$-6848013696c+5284782720)] \tag{20}$$

$${}_2F_1\left[\begin{matrix} a, & 19-a\ ; & \frac{1}{2} \\ c & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-19}\{\prod_{\lambda=1}^{18}(\lambda-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-18}{2})}(9a^8c-90a^8-684a^7c+6840a^7 -120a^6c^3+3600a^6c^2-14046a^6c-99540a^6+$$

$$+6840a^5c^3-205200a^5c^2+1664856a^5c-2968560a^5+432a^4c^5-21600a^4c^4 +277080a^4c^3+327600a^4c^2-20793831a^4c+$$

$$+70898310a^4-16416a^3c^5+820800a^3c^4-14644440a^3c^3+111013200a^3c^2 -315518940a^3c+131909400a^3-576a^2c^7+$$

$$+40320a^2c^6-992880a^2c^5+9324000a^2c^4+4429536a^2c^3-636886080a^2c^2 +3695816316a^2c-6211091160a^2+10944ac^7-$$

$$-766080ac^6+21827808ac^5-325310400ac^4+2707726176ac^3-12394025280ac^2 +28254838896ac-23908836960a+256c^9-$$

$$-23040c^8+880512c^7-18627840c^6+238347264c^5-1891123200c^4 +9158978048c^3-25507261440c^2+35661692160c-$$

$$-17643225600)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-19}{2})}(-2a^8+152a^7+80a^6c^2-1600a^6c +3148a^6-4560a^5c^2+91200a^5c-371488a^5-$$

$$-480a^4c^4+19200a^4c^3-185680a^4c^2-126400a^4c+4559182a^4+18240a^3c^4-729600a^3c^3+9799440a^3c^2-50068800a^3c+$$

$$+73373288a^3+896a^2c^6-53760a^2c^5+1107680a^2c^4-8467200a^2c^3-743936a^2c^2+274718720a^2c-822056088a^2-$$

$$-17024ac^6+1021440ac^5-24338240ac^4+292569600ac^3-1853708096ac^2+5798641920ac-6885423072a-$$

$$-512c^8+40960c^7-1374464c^6+25123840c^5-271685888c^4+1764075520c^3-6639757056c^2+13042437120c-$$

$$-10013310720)] \tag{21}$$

2. MAIN SUMMATION FORMULAE

$${}_2F_1\left[\begin{matrix}a, & 20-a & ; & \frac{1}{2}\\ c & & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-20}\{\prod_{\Upsilon=1}^{19}(\Upsilon-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-20}{2})}(33522128640+47215599696a+14182895460a^2+$$

$$+345040520a^3-140133105a^4+962073a^5+330750a^6-9330a^7+15a^8+a^9-67958134272c-$$

$$-57343402272ac-9605975576a^2c+295428296a^3c+58846422a^4c-2100880a^5c-32820a^6c+$$

$$+1640a^7c-10a^8c+48842214912c^2+25998562336ac^2+2187966784a^2c^2-168954152a^3c^2-$$

$$-6101120a^4c^2+380720a^5c^2-2240a^6c^2-40a^7c^2-17641896960c^3-5917427456ac^3-182014144a^2c^3+$$

$$+27821280a^3c^3-9440a^4c^3-19680a^5c^3+160a^6c^3+3666323456c^4+750095264ac^4-1895280a^2c^4-$$

$$-1926160a^3c^4+23280a^4c^4+240a^5c^4-465172736c^5-54369728ac^5+1155616a^2c^5+55104a^3c^5-$$

$$-672a^4c^5+36595328c^6+2174144ac^6-61824a^2c^6-448a^3c^6-1740800c^7-41984ac^7+1024a^2c^7+$$

$$+45824c^8+256ac^8-512c^9)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-19}{2})}(-190253266560-131460917904a-15315714660a^2+$$

$$+1718684120a^3+100625805a^4-10839927a^5+135450a^6+7470a^7-195a^8+a^9+258458522112c+$$

$$+117489033888ac+5199265016a^2c-1259577944a^3c+961578a^4c+3256720a^5c-84780a^6c+40a^7c+$$

$$+10a^8c-139931759232c^2-40815588704ac^2+198370336a^2c^2+283436248a^3c^2-7330880a^4c^2-$$

$$-224080a^5c^2+7840a^6c^2-40a^7c^2+40472263680c^3+7213462784ac^3-274206656a^2c^3-26053920a^3c^3+$$

$$+1017440a^4c^3-480a^5c^3-160a^6c^3-6993636736c^4-700147936ac^4+42392880a^2c^4+896240a^3c^4-$$

$$-47280a^4c^4+240a^5c^4+757008896c^5+36475712ac^5-2849056a^2c^5+1344a^3c^5+672a^4c^5-$$

$$-51764608c^6-836416ac^6+88704a^2c^6-448a^3c^6+2170880c^7-1024ac^7-1024a^2c^7-50944c^8$$

$$+256ac^8+512c^9)] \quad (22)$$

$${}_2F_1\left[\begin{matrix}a, & 21-a\ ; & \frac{1}{2}\\ c & ; & \end{matrix}\right]=$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-21}\{\prod_{\Psi=1}^{20}(\Psi-a)\}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-20}{2})}(670442572800+946321185600a+284169369024a^2+$$

$$+4885689900a^3-3333875180a^4+41694345a^5+10037727a^6-381150a^7+1230a^8+105a^9-a^{10}-$$

$$-1394694005760c-119837928672 0ac-203053089360a^2c+8433107760a^3c+1530533620a^4c-70408800a^5c-$$

$$-1146200a^6c+92400a^7c-1100a^8c+1048586614272c^2+578478838560ac^2+49539606520a^2c^2-$$

$$-4805882760a^3c^2-177714670a^4c^2+15397200a^5c^2-141500a^6c^2-4200a^7c^2+50a^8c^2-404078540800c^3-$$

$$-143591669760ac^3-4354528640a^2c^3+902932800a^3c^3-2094400a^4c^3-1108800a^5c^3+17600a^6c^3+$$

$$+91700259840c^4+20464187520ac^4-122473120a^2c^4-77439600a^3c^4+1402800a^4c^4+25200a^5c^4-400a^6c^4-$$

$$-13092907520c^5-1739633280ac^5+50240960a^2c^5+3104640a^3c^5-73920a^4c^5+1209103616c^6+$$

$$+87071040ac^6-3652320a^2c^6-47040a^3c^6+1120a^4c^6-72089600c^7-2365440ac^7+112640a^2c^7+2677760c^8+$$

$$+26880ac^8-1280a^2c^8-56320c^9+512c^{10})+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-21}{2})}(362387520000+268742591040a+$$

$$+41471452880a^2-1867829040a^3-305673060a^4+14303520a^5+225720a^6-18480a^7+220a^8-$$

$$-494250063360c-247867413696ac-19713479280a^2c+1984361232a^3c+69962284a^4c-6179040a^5c+$$

$$+56920a^6c + 1680a^7c - 20a^8c + 268936121344c^2 + 89644203264ac^2 + 2511762176a^2c^2 - 547968960a^3c^2 +$$

$$+1404480a^4c^2 + 665280a^5c^2 - 10560a^6c^2 - 78226625536c^3 - 16719935232ac^3 + 112602112a^2c^3 +$$

$$+62139840a^3c^3 - 1126720a^4c^3 - 20160a^5c^3 + 320a^6c^3 + 13598953984c^4 + 1756191360ac^4 - 51029440a^2c^4 -$$

$$-3104640a^3c^4 + 73920a^4c^4 - 1480941056c^5 - 104786304ac^5 + 4397120a^2c^5 + 56448a^3c^5 - 1344a^4c^5 +$$

$$+101871616c^6 + 3311616ac^6 - 157696a^2c^6 - 4296704c^7 - 43008ac^7 + 2048a^2c^7 + 101376c^8 - 1 \quad (23)$$

3. DERIVATION OF THE MAIN FORMULAE

Involving the contiguous relation (8) and the formula of Salahuddin et al(21), one can established the result(22) and on the same way result(23) can be established.

References

1. Abramowitz, Milton., A and Stegun, Irene ; *Handbook of Mathematical Functions with Formulas , Graphs , and Mathematical Tables*. National Bureau of Standards, 1970.
2. Prudnikov, A.P., Brychkov, Yu. A. and Marichev, O.I.; *Integral and Series Vol 3: More Special Functions*,Nauka, Moscow,2003.
3. Salahuddin, Khola, R. K.;New hypergeometric summation formulae arising from the summation formulae of Prudnikov, *South Asian Journal of Mathematics*,4(2014),192-196.
4. Salahuddin, Khola, R. K.;Certain new Hypergeometric Summation formulae arising from the summation formulae of Salahuddin et al, *Sohag Journal of Mathematics*,3(2016),1-7.
5. Salahuddin, Pandit,U. K. and Chaudhary, M. P.;Two Incredible Summation Formula Involving Computational Technique, *Global Journal of Science Frontier Research: F*,15(2015),29-35.