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Effects of Variable Viscosity and Thermal Conductivity on MHD Free Convection Flow of Dusty Fluid along a Vertical Stretching Sheet with Heat Generation

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Abstract - *A numerical study has been carried out on a steady two-dimensional laminar MHD heat and mass transfer free convection flow of a viscous incompressible dusty fluid near an isothermal linearly stretching sheet in the presence of a uniform magnetic field with heat generation. The present work examines the effects of temperature dependent viscosity and thermal conductivity on velocity, temperature and species concentration of the fluid. Effects of viscous dissipation and Joule heating are taken into consideration in the energy equation. Non-linear partial differential equations governing the motion are reduced to a system of ordinary differential equations using suitable similarity transformations. Resultant equations are then solved numerically using fourth order Runge-Kutta shooting method. Some important features of velocity, temperature and species concentration of fluid and solid particles are obtained for various physical parameters involved in the problem which are of physical and engineering interest are analyzed, discussed and presented through graphs. Finally, numerical values of skin-friction coefficient, Nusselt number and Sherwood number are presented in tabular form, which respectively give the wall shear stress, rate of heat transfer and rate of mass transfer.*

Key Words: Dusty fluid, variable viscosity and thermal conductivity, viscous dissipation, Joule heating.

1.INTRODUCTION

Investigations of two-dimensional boundary layer flow of free convection heat and mass transfer over a vertical stretching surface are important due to its applications in industries, and many manufacturing processes such as aerodynamic extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, which would be in the form of an electrolyte, and polymer sheet extruded continuously from a die are few practical applications of moving surfaces.

A closed form exponential solution for planar viscous flow of linear stretching sheet has been presented by Crane [1]. Initially, Gebhart [2] observed the problem taking into account the viscous dissipation. Effects of MHD and viscous dissipation on heat transfer analysis were studied by many authors such as Mahmoud [3], Vajravelu and Hadjinicalaou [4], Samad et al. [5] and Anjali Devi [6]. Cortell[7] analyzed MHD flow of a power-law fluid over a stretching sheet. Chen [8] studied mixed convection of power law fluid past a stretching surface with thermal radiation and magnetic field. Cortell [9] observed the influence of viscous dissipation and work done by deformation on MHD flow and heat transfer of viscoelastic fluid over a stretching sheet. Tsai et al. [10] have discussed unsteady flow over a stretching surface with non-uniform heat source.

To study the two-phase flows, in which solid spherical particles are distributed in a fluid are of interest due to wide range of real world applications. The study of heat transfer in the boundary layer induced by stretching surface with a given temperature distribution in a conducting dusty fluid is important in several manufacturing process in industries like extrusion of plastic sheets, glass fibre and paper production, metal spinning and the cooling of metallic plate in a cooling bath. In view of these applications, Saffman [11] carried out pioneering work on the stability of laminar flow of a dusty gas which describes the fluid-particle system and derived the equations of motion of a gas carrying dust particles. Marble[12] studied dynamics of a gas containing small solid particles. Baral[13] observed plane parallel flow of a conducting dusty gas. Chakrabarti [14] investigated the boundary layer flow of a dusty gas. Gupta and Gupta [15] have discussed flow of dusty gas in a channel with arbitrary time varying pressure. Datta and Mishra[16] have discussed the flow of dusty fluid in boundary layer over a semi-infinite flat plate. Vajravelu and Nayfeh[17] analyzed the MHD flow of a dusty fluid over a stretching sheet with the effects of fluid-particle interaction, particle loading and suction on the flow characteristics and compared their analytical solution with numerical ones. Unsteady hydromagnetic boundary layer flow and heat transfer of dusty fluid over a stretching sheet with variable wall temperature (VWT) and variable heat flux (VHF) have been studied by Gireesha et al. [18,19]. In these papers the effects of magnetic field on an unsteady boundary layer flow and heat transfer of a dusty fluid over an unsteady stretching surface in the presence of non- uniform heat source/sink are discussed.

Nomenclature

(u,v)	velocity components of the fluid,	τ_T	thermal equilibrium time and is the time required by the dust cloud to adjust its temperature to the fluid,
(u_p,v_p)	velocity components of dust phase,	τ_v	relaxation time of the dust particle i.e., the required by a dust particle to adjust its velocity relative to the fluid,
ρ	density of the fluid,	λ	thermal conductivity of the fluid,
ρ_p	density of the particle phase,	λ_∞	thermal conductivity of the ambient fluid,
μ	coefficient of dynamic viscosity,	C	species concentration of the fluid,
μ_∞	coefficient of dynamic viscosity of ambient fluid,	C_p	concentration of the dust phase within the boundary layer,
υ_∞	kinematic viscosity of the fluid in the free stream,	Dm	coefficient of mass diffusivity of the fluid,
N	number density of the particle phase,	Dm_p	coefficient of mass diffusivity of dust phase,
K	Stokes' resistance(drag co-efficient),	θ_r	viscosity variation parameter,
m	mass of the dust particle,	θ_c	thermal conductivity variation parameter,
g	acceleration due to gravity,	l^*	mass concentration,
β^*	volumetric coefficient of thermal expansion,	τ	relaxation time of particle phase,
T	temperature of the fluid inside the boundary layer,	β	fluid particle interaction parameter,
T_∞	temperature of the fluid at free stream,	Gr	Grashof number,
T_p	temperature of the dust particles inside the boundary layer,	M	magnetic field parameter,
$T_{p\infty}$	temperature of the dust particles in the free-stream,	Re	Reynolds number,
C_{pw} and $C_{p\infty}$	species concentration of dust particles at the surface of the plate and sufficiently far away from the flat surface respectively,	Pr	Prandtl number,
ω	density ratio,	Ec	Eckert number,
σ	electrical conductivity,	Sc	Schmidt number for fluid phase,
c	stretching rate,	Sc_p	Schmidt number for dust phase,
l	characteristic length,	C_f	skin-friction coefficient,
c_p	specific heat of fluid at constant pressure,	Nu	Nusselt number,
c_m	specific heat of dust particles at constant pressure,	Sh	Sherwood number.

Evgeny and Sergei [20] discussed the stability of the laminar boundary layer flow of dusty gas on a flat plate. Further, XIE Ming-liang et al. [21] extended the work of [16] and studied the hydrodynamic stability of a particle-laden flow in growing flat plate boundary layer. Palani and Ganesan [22] studied heat transfer effects on dusty gas flow past a semi-infinite inclined plate. Agranat [23] studied dusty boundary layer flow and heat transfer, with the effect of pressure gradient. Ezzat et al. [24] analyzed the space approach to the hydro-magnetic flow of a dusty fluid through a porous medium. Kannan and Venkataraman [25] examined the free convection in an infinite porous dusty medium induced by pulsating point heat source. Recently, Gireesha et al. [26] studied the boundary layer flow and heat transfer of a dusty fluid flow over a stretching sheet with non-uniform heat source/sink.

In all the above mentioned papers on dusty fluid, the thermophysical transport properties of the ambient fluid were assumed to be constant. However, it is well known from the work of Herwig and Wicken[27], Lai and Kulacki[28], Setayesh and Sahai[29], Attia[30] that these physical properties may change with temperature, especially the fluid viscosity and the thermal conductivity. For lubricating fluids, the properties of the fluid are no longer assumed to be constant. An increase in temperature leads to increase in the transport phenomena by reducing the physical properties across the thermal boundary layer. Therefore to predict the flow and heat transfer characteristics more accurately, it is necessary to take into account the temperature dependent fluid properties.

In view of these applications, the problem studied here the effects of temperature dependent viscosity and thermal conductivity on MHD free convection flow of dusty fluid along a vertical stretching sheet with heat generation. Here the thermophysical transport properties namely, the viscosity and the thermal conductivity are assumed to be function of temperature. In addition to this, we have also included the contribution of viscous dissipation and Joule heating in the energy equation.

2. MATHEMATICAL FORMULATION

Consider a steady two-dimensional heat and mass transfer flow of an electrically conducting viscous incompressible dusty fluid along an isothermal stretching vertical sheet with heat generation/absorption. The stretching sheet coincides with the plane $y = 0$ where the flow is confined to $y > 0$. A uniform magnetic field of strength B_0 is applied along the y-axis. This is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number [31, 32]. The viscous dissipation and Joule heating terms are taken into consideration in the energy equation. Two equal and opposite forces are introduced along the x-axis so that the sheet is stretched keeping the origin fixed as shown in Fig. 1.

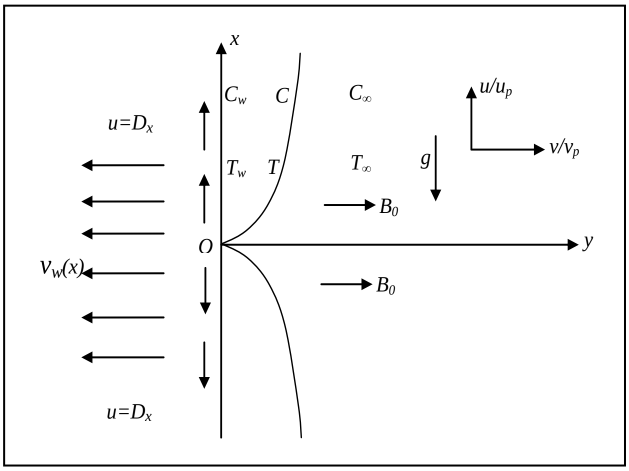

Fig.1: Flow configuration and coordinate system

In the present study dust particles are assumed to be spherical and uniform in size and shape and are uniformly distributed throughout the fluid. Also it is assumed that fluid has constant physical properties except viscosity and thermal conductivity. Using the above assumptions and based on Saffman[11] model of the steady motion of an incompressible fluid and following Vajravelu and Nayfeh[17] and Eldable et al.[33], governing equations of the flow are:

For the fluid phase:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\frac{KN}{\rho}(u_p-u)+g\beta^*(T-T_\infty)-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda\frac{\partial^2 T}{\partial y^2}\right)+\frac{Nc_p}{\tau_T}(T_p-T)$$
$$+\frac{N}{\tau_v}(u_p-u)^2+Q_0(T-T_\infty)+\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{\vec{J}^2}{\sigma} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(Dm\frac{\partial C}{\partial y}\right) \tag{4}$$

For the dust phase:

$$\frac{\partial}{\partial x}(\rho_p u_p)+\frac{\partial}{\partial y}(\rho_p v_p)=0 \tag{5}$$

$$u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}=\frac{K}{m}(u-u_p) \tag{6}$$

$$u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}=\frac{K}{m}(v-v_p) \tag{7}$$

$$u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}=-\frac{c_p}{c_m\tau_T}(T_p-T) \tag{8}$$

$$u_p\frac{\partial C_p}{\partial x}+v_p\frac{\partial C_p}{\partial y}=\frac{\partial}{\partial y}\left(Dm_p\frac{\partial C_p}{\partial y}\right) \tag{9}$$

The appropriate boundary conditions are given by:

$$\left.\begin{aligned}
&\text{At } y=0:\\
&u=U_w=cx,\ v=0,\ T=T_w=T_\infty+A\left(\frac{x}{l}\right)^2,\\
&C=C_w=C_\infty+B\left(\frac{x}{l}\right)^2, C_p=C_{pw}=C_{p\infty}+E\left(\frac{x}{l}\right)^2\\
&\text{As } y\to\infty:\\
&u\to 0,\ u_p\to 0,\ v_p\to v,\ \rho_p\to\omega\rho,\\
&T\to T_\infty,\ T_p\to T_\infty,\ C\to C_\infty,\ C_p\to C_{p\infty}
\end{aligned}\right\} \tag{10}$$

where $c>0$ is the stretching rate, ω is the density ratio, A, B and E are positive constants and $l=\sqrt{\frac{\upsilon_\infty}{c}}$ is a characteristic length, C_{pw} and $C_{p\infty}$ are the species concentration of the dust particles at the surface of the plate and sufficiently far away from the flat surface respectively.

To convert the governing equations into a set of similarity equations, we introduce the following transformations as,

$$\left.\begin{aligned}
&u = cxf'(\eta),\ v = -\sqrt{\upsilon_\infty c}\, f(\eta),\ \eta = \sqrt{\frac{c}{\upsilon_\infty}}\, y, \\
&u_p = cxF(\eta),\ v_p = \sqrt{\upsilon_\infty c}\, G(\eta),\ \rho_r = H(\eta), \\
&\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}, \\
&\varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},\ \varphi_p(\eta) = \frac{C_p - C_{p\infty}}{C_{pw} - C_{p\infty}} \\
&\text{where}\quad T - T_\infty = A\left(\frac{x}{l}\right)^2 \theta(\eta), \\
&C - C_\infty = B\left(\frac{x}{l}\right)^2 \varphi(\eta), \\
&C_p - C_{p\infty} = E\left(\frac{x}{l}\right)^2 \varphi_p(\eta).
\end{aligned}\right\} \quad (11)$$

where $\rho_r = \dfrac{\rho_p}{\rho}$ is the relative density and the prime $(')$ denotes derivative with respect to η.

The viscosity of the fluid is assumed to be an inverse linear function of temperature, and it can be expressed as following Lai and Kulacki [14]:

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \delta\left(T - T_\infty\right)\right] \quad (12)$$

or, $\dfrac{1}{\mu} = \alpha\left(T - T_r\right)$,

where $\alpha = \dfrac{\delta}{\mu_\infty}$ and $T_r = T_\infty - \dfrac{1}{\delta}$

Also thermal conductivity of the fluid is varying with temperature. Following Choudhury and Hazarika [20], we assumed thermal conductivity of the fluid as:

$$\frac{1}{\lambda} = \frac{1}{\lambda_\infty}\left[1 + \xi\left(T - T_\infty\right)\right] \quad (13)$$

or, $\dfrac{1}{\lambda} = \zeta\left(T - T_c\right)$,

where $\zeta = \dfrac{\xi}{\lambda_\infty}$ and $T_c = T_\infty - \dfrac{1}{\xi}$.

Here, α, δ, ξ, ζ, T_r and T_c are constants and their values depend on the reference state and thermal properties of the fluid i.e., υ (kinematic viscosity) and λ (thermal conductivity).

Two dimensionless reference temperatures θ_r and θ_c can be defined as

$$\left.\begin{aligned}
&\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\delta(T_w - T_\infty)} \quad \text{and} \\
&\theta_c = \frac{T_c - T_\infty}{T_w - T_\infty} = -\frac{1}{\xi(T_w - T_\infty)}
\end{aligned}\right\} \quad (14)$$

and called the viscosity variation parameter and the thermal conductivity variation parameter respectively.

Substituting Eqs. (11)-(14) in the Eqs. (2)-(10), we get

$$\frac{\theta_r}{\theta - \theta_r} f''' - \frac{\theta_r}{(\theta - \theta_r)^2}\theta' f'' - ff'' + f'^2 - l^*\beta H(F - f') + Mf' - Gr\theta = 0 \quad (15)$$

$$GF' + F^2 + \beta(F - f') = 0 \quad (16)$$

$$GG' + \beta(f + G) = 0 \quad (17)$$

$$G^2 H' - \beta H(f + G) + GFH = 0 \quad (18)$$

$$\frac{\theta_c}{\theta - \theta_c}\theta'' - \frac{\theta_c}{(\theta - \theta_c)^2}\theta'^2 + \Pr(2f'\theta - f\theta') - \frac{N}{\rho c \tau_T}\Pr(\theta_p - \theta) - \frac{N}{\rho c \tau_v}\Pr Ec(F - f')^2 - \Pr Q\theta - \frac{\theta_r}{\theta - \theta_r}\Pr Ec f''^2 - \Pr EcMf'^2 = 0 \quad (19)$$

$$G\theta_p' + 2F\theta_p + \frac{c_p}{cc_m \tau_T}(\theta_p - \theta) = 0 \quad (20)$$

$$\frac{\theta_r}{\theta - \theta_r}\varphi'' - \frac{\theta_r}{(\theta - \theta_r)^2}\theta'\varphi' + Sc(2f'\varphi - f\varphi') = 0 \quad (21)$$

$$\frac{\theta_r}{\theta - \theta_r}\varphi_p'' - \frac{\theta_r}{(\theta - \theta_r)^2}\theta'\varphi_p' + Sc_p(2F\varphi_p + G\varphi_p') = 0 \quad (22)$$

where the dimensionless parameters are defined as follows:

$l^* = mN/\rho$ mass concentration,
$\tau = m/K$ relaxation time of the particle phase,
$\beta = 1/c\tau$ fluid particle interaction parameter,
$\rho_r = \rho_p/\rho$ relative density,

$Gr = \dfrac{g\beta^*(T_w - T_\infty)}{c^2 x}$ Grashof number,

$Sc = \dfrac{\upsilon}{Dm}$ Schmidt number for fluid phase,

$Sc_p = \dfrac{\upsilon}{Dm_\infty}$ Schmidt number for dust phase,

$M = \dfrac{\sigma B_0^2}{\rho c}$ magnetic field parameter,

$\Pr = \dfrac{\mu_\infty c_p}{\lambda_\infty}$ Prandtl number,

$Q = \dfrac{Q_0}{c\rho c_p}$ heat generating parameter,

$Ec = \dfrac{U_w^2}{c_p(T_w - T_\infty)}$ Eckert number.

The boundary conditions defined in Eqn.(10) will becomes:

$$\left.\begin{array}{l} f = 0,\ f' = 1,\ \theta = 1,\ \varphi = 1,\ \varphi_p = 1 \ \text{ at } \ \eta = 0 \\ f' = 0,\ F=0,\ G=-f,\ H=\omega,\ \theta = 0,\ \theta_p = 0,\ \varphi = 0, \\ \varphi_p = 0 \ \text{ as } \ \eta \to \infty \end{array}\right\} \quad (23)$$

2.1 Skin-friction, Nusselt number and Sherwood number

Skin-friction coefficient (C_f), Nusselt number(Nu) and Sherwood number(Sh) are the parameters of physical and engineering interest for the present problem, which physically indicate the wall shear stress, rate of heat transfer and rate of mass transfer respectively.
Skin-friction coefficient is defined as

$$C_f = \frac{2\tau_w}{\rho_\infty u_0^2}, \text{ where } \tau_w = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0} \text{ is the shearing stress.}$$

After using the non-dimensional variables we get the skin-friction co-efficient as

$$C_f = -\frac{2\theta_r}{1-\theta_r} \mathrm{Re}^{-1/2} f''(0).$$

The Nusselt number which is defined as

$$Nu = \frac{-xq_w}{\lambda_\infty(T_w - T_\infty)}, \quad \text{where } q_w = -\lambda \left.\frac{\partial T}{\partial y}\right)_{y=0} \text{ is}$$

the heat transfer from the sheet.
Using the non-dimensional variables we get

$$Nu = \frac{\theta_c}{1-\theta_c} \mathrm{Re}^{1/2}\, \theta'(0).$$

The Sherwood number which is defined as

$$Sh = \frac{xm_w}{Dm_\infty(C_w - C_\infty)},$$

where $m_w = -Dm \left.\dfrac{\partial C}{\partial y}\right)_{y=0}$ is the mass flux at the surface and Dm_∞ is the diffusion constant at free stream.
Using the non-dimensional variables we get

$$Sh = -\mathrm{Re}^{-1/2}\, Sc^{-1} \frac{\theta_r}{1-\theta_r} \varphi'(0).$$

3. RESULTS AND DISCUSSION

Equations (15)-(22) with the boundary conditions (23) were solved numerically using fourth order Runge-Kutta method using shooting technique. Numerical values are computed by developing suitable codes in MATLAB for the method. Numerical computations of these solutions have been carried out to study the effects of various physical parameters such as viscosity variation parameter(θ_r), thermal conductivity variation parameter(θ_c), magnetic parameter(M), Prandtl number(Pr) and number density parameter of dust particle(N) on velocity, temperature and species concentration profiles of both the fluid and dust particles. A representative set of numerical results are shown graphically for velocity, temperature and species concentration profiles from Fig.2-Fig.11. In the following discussion some parameters are given the following fixed values: *Re=100, Gr=0.5, M=1, Pr=0.71, Q=0.75,* $Sc=Sc_p=0.22$, $l^*=0.2$, *β=0.5, N=0.5, ρ=1, c=0.6,* $\tau_T=0.5$, $\tau_v=1$, *ω=0.1, Ec=0.05,* $c_p=c_m=0.2$, $\theta_r=5$ and $\theta_c=3$, unless otherwise stated.

The effect of viscosity variation parameter θ_r on velocity profiles of fluid phase $f'(\eta)$ and dust phase $F(\eta)$ verses η is depicted in Fig. 2. It is observed that the velocity profiles decrease with an increasing values of fluid viscosity parameter θ_r for both the phases. This is because of the fact that with an increase in the value of fluid viscosity variation parameter decreases the velocity boundary layer thickness.

Fig. 3 illustrates the effect of thermal conductivity variation parameter on velocity profiles. It is seen that velocity increases with the increasing values of θ_c. It is due to the reason that temperature decreases with the increasing

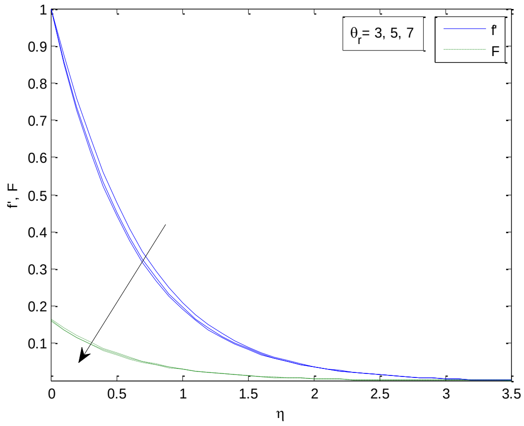

Fig. 2: Velocity profile for different θr

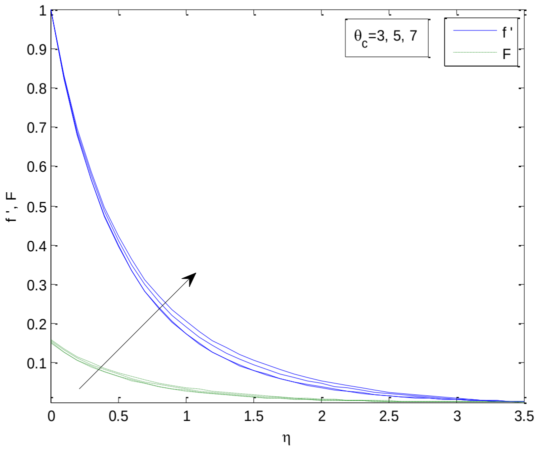

Fig. 3: Velocity profile for different θc

values of θ_c and as a result viscosity decreases and so velocity increases.

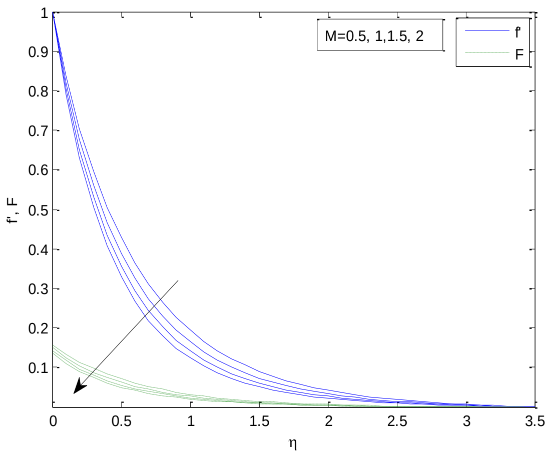

Fig. 4: Velocity profile for different M

Fig. 4 shows the variation of velocity profile for both the phases with various values of *M*. From this plot it is seen that the effect of increasing values of *M* is to decrease the velocity distribution of both the fluid and dust phases. This is due to the fact that the presence of a magnetic field normal to the flow in an electrically conducting fluid produces a Lorentz force, which acts against the flow.

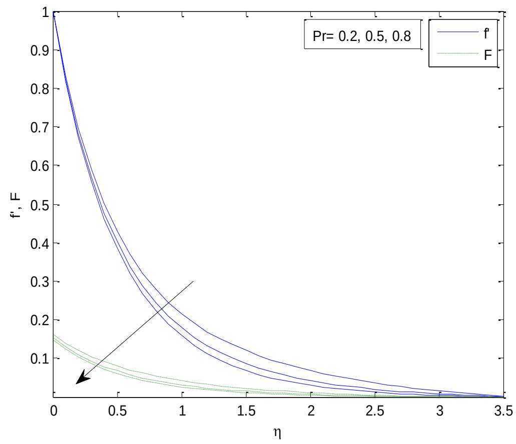

Fig. 5: Velocity profile for different Pr

Variation of velocity profile for different values of Prandtl number(*Pr*) and number density of the particle phase(*N*) are depicted in Fig.5 and Fig. 6, respectively. We observed that the velocity profile decreases with increasing values of both the parameters.

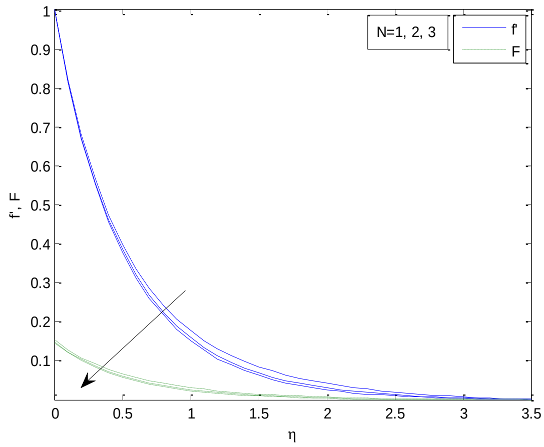

Fig. 6: Velocity profile for different *N*

The fluid and dust particles temperature profiles are presented in Fig. 7-Fig. 10. Fig. 7 represents that an increase in the viscosity variation parameter(θ_r) increases the temperature distribution. This is due to the reason that an increase in the value of θ_r , the thermal boundary layer thickness increases, which results an increase in the temperature.

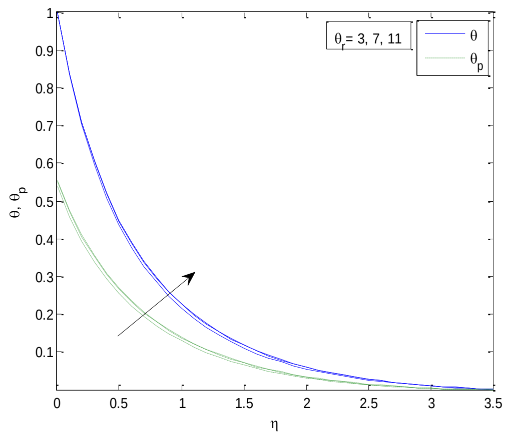

Fig. 7: Temperature profile for different Θr

From the plots in Fig. 8, one can observed that for increasing values of thermal conductivity variation parameter(θ_c) enhances the temperature distribution, which is true for both the fluid phase as well as dust phase. Fig. 9 reveals the effect of magnetic parameter M on temperature profile. It is noticed that temperature profile increases for increasing values of magnetic parameter. This is because of the Lorentz force.

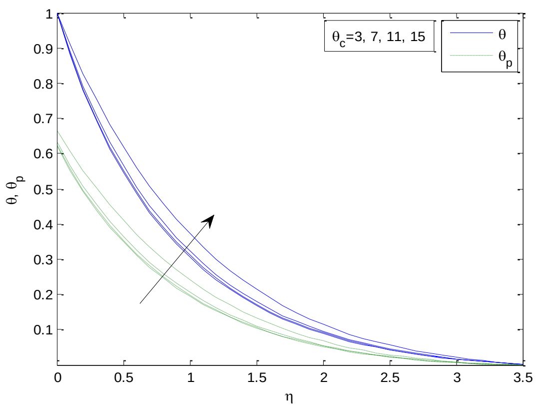

Fig. 8: Temperature profile for different Θc

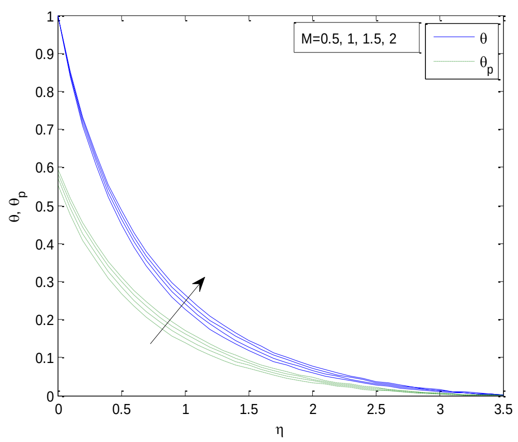

Fig. 9: Temperature profile for different M

Fig. 10 displays the effect Prandtl number(Pr) on temperature profile. The increase in the value of Pr decreases the temperature of fluid and dust phase. Physically, by this we mean that for higher Prandtl number fluid has a relatively high thermal conductivity which reduces temperature.

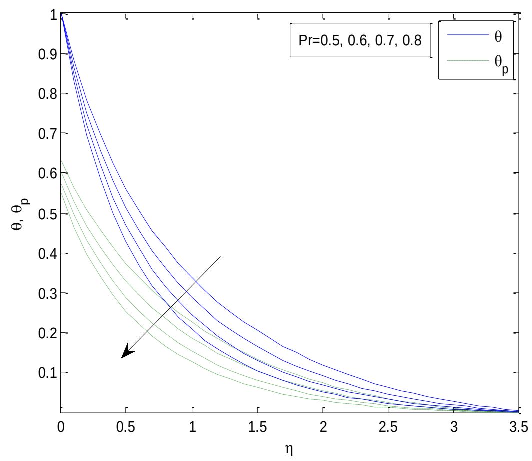

Fig. 10: Temperature profile for different Pr

The fluid and dust particles species concentration profiles presented in Fig. 11 and Fig. 12 for various values of θ_r and θ_c , respectively. From these plots it is observed that species concentration profiles decreases for increasing values of θ_r , while it increases with an increasing values of θ_c .

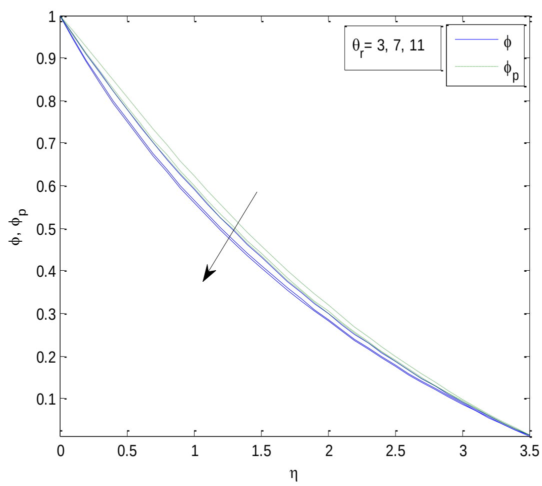

Fig. 11: Concentration profile for different Θr

The calculated values of skin-friction coefficient(C_f), Nusselt number(Nu) and Sherwood number(Sh) for various values of flow governing parameters are presented in Table 1. From the table it is observed that skin-friction coefficient increases with the increasing values of viscosity variation parameter,

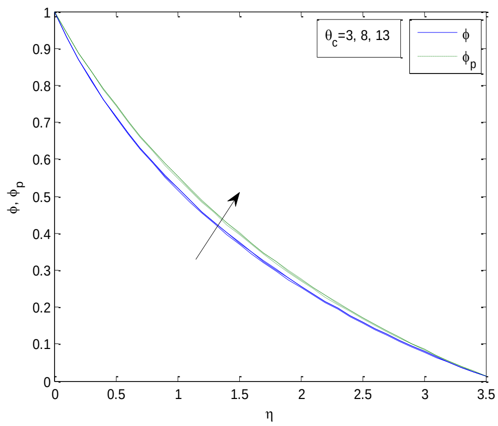

Fig. 12: Concentration profile for different Θc

thermal conductivity variation parameter and magnetic parameter. Viscosity variation parameter leads to a decrease in the values of Nusselt number and Sherwood number. It is also noticed that the value of Nusselt number decreases with increase in the value of magnetic parameters, but it increases for the rising value of Eckert number. We observed that Sherwood number decreases with increase in the value of thermal conductivity variation parameter, magnetic parameter, Eckert number and Schmidt number.

Table1: Effects of θ_r, θ_c, M, Ec and Sc on local skin-friction coefficient (C_f), local Nusselt number (Nu) and local Sherwood number(Sh)

θ_r	θ_c	***M***	***Ec***	***Sc***	***C_f***	***Nu***	***Sh***
5					3.312662	1.901111	3.247656
7	3	1	0.05	0.22	3.264794	1.890416	3.20359
9					3.239164	1.88458	3.179744
	5				2.925444	1.171352	2.97049
5	7	1	0.05	0.22	2.938145	1.244441	2.969897
	9				2.944844	1.284757	2.969607
		0.5			3.312662	1.901111	3.247656
5	3	1.0	0.05	0.22	3.724615	1.858642	3.190413
		1.5			4.094781	1.82174	3.142709
			0.03		3.724182	1.851815	3.190774
5	3	1	0.05	0.22	3.724616	1.858642	3.190413
			0.07		3.725051	1.865482	3.190053
				0.2	3.724615	1.858642	3.345971
5	3	1	0.05	0.3	3.724615	1.858642	2.777084
				0.4	3.724615	1.858642	2.493639

3. CONCLUSIONS

From the present investigation the following conclusions were made:

- When the effects of variable viscosity and thermal conductivity are taken into account, the flow characteristics are significantly changed compared to the constant property case.
- The velocity profile decreases with the increasing values of viscosity variation parameter and Prandtl number while the opposite effect is observed in the case of thermal conductivity variation parameter.
- The effect of magnetic parameter leads to decrease in the velocity profile. On the other hand, it enhances the temperature for both the fluid and dust phases.
- An increasing viscosity variation parameter causes increase in the temperature. But species concentration reduces with it for both the fluid and dust phases.
- Species concentration field enhances with the increase of thermal conductivity variation parameter.
- Skin-friction coefficient increases due to magnetic field.
- The wall shear stress increases with the increase of the viscosity variation parameter and thermal conductivity variation parameter.
- The rate of heat transfer to the fluid decreases with the increase in θ_r and M whereas it increases for increasing value of θ_c and Ec.
- An increase in the value of viscosity variation parameter leads to decrease in the values of Sherwood number.
- The coefficient of skin-friction and Sherwood number decrease with the increase in θ_c and M.
- Rate of mass transfer decreases with the increase in the thermal conductivity variation parameter.

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