International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395 -0056
Volume: 03 Issue: 02 | Feb-2016 www.irjet.net p-ISSN: 2395-0072

NOTES ON (T, S)-INTUITIONISTIC FUZZY SUBHEMIRINGS OF A HEMIRING

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Abstract - ***In this paper, we made an attempt to study the algebraic nature of a (T, S)-intuitionistic fuzzy subhemiring of a hemiring.2000 AMS Subject classification: 03F55, 06D72, 08A72.***

Key Words: *T-fuzzy subhemiring, anti S-fuzzy subhemiring, (T, S)-intuitionistic fuzzy subhemiring, product.*

1. INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (*R ; + ; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras (*R ; + ; .*) share the same properties as a ring except that (*R ; +*) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh[23], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov[4,5], as a generalization of the notion of fuzzy set. The notion of anti fuzzy left h-ideals in hemiring was introduced by Akram.M and K.H.Dar [1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan [16], [17], [18]. In this paper, we introduce the some Theorems in (T, S)-intuitionistic fuzzy subhemiring of a hemiring.

2. PRELIMINARIES

2.1 Definition

A (T, S)-norm is a binary operations T: $[0, 1]\times[0, 1] \rightarrow [0, 1]$ and S: $[0, 1]\times[0, 1] \rightarrow [0, 1]$ satisfying the following requirements;

(i) T(0, x)= 0, T(1, x) = x (boundary condition)

(ii) T(x, y) = T(y, x) (commutativity)

(iii) T(x, T(y, z))= T (T(x,y), z)(associativity)

(iv) if $x \leq y$ and $w \leq z$, then $T(x, w) \leq T(y, z)$(monotonicity).

(v) S(0, x) = x, S (1, x) = 1 (boundary condition)

(vi) S(x, y) = S (y, x)(commutativity)

(vii) S (x, S(y, z))= S (S(x, y), z) (associativity)

(viii) if $x \leq y$ and $w \leq z$, then S (x, w) ≤ S (y, z)(monotonicity).

2.2 Definition

Let (R, +, .) be a hemiring. A fuzzy subset A of R is said to be a T-fuzzy subhemiring (fuzzy subhemiring with respect to T-norm) of R if it satisfies the following conditions:

(i) $\mu_A(x+y) \geq T(\mu_A(x), \mu_A(y))$,

(ii) $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in R.

2.3 Definition

Let (R, +, .) be a hemiring. A fuzzy subset A of R is said to be an anti S-fuzzy subhemiring (anti fuzzy subhemiring with respect to S-norm) of R if it satisfies the following conditions:

(i) $\mu_A(x+y) \leq S(\mu_A(x), \mu_A(y))$,

(ii) $\mu_A(xy) \leq S(\mu_A(x), \mu_A(y))$, for all x and y in R.

2.4 Definition

Let (R, +, .) be a hemiring. An intuitionistic fuzzy subset A of R is said to be an (T, S)-intuitionistic fuzzy subhemiring(intuitionistic fuzzy subhemiring with respect to (T, S)-norm) of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \geq T(\mu_A(x), \mu_A(y))$,

(ii) $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$,

(iii) $\nu_A(x + y) \leq S(\nu_A(x), \nu_A(y))$,

(iv) $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y))$, for all x and y in R.

2.5 Definition

Let A and B be intuitionistic fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as $A\times B = \{\langle (x, y), \mu_{A\times B}(x, y), \nu_{A\times B}(x, y) \rangle /$ for all x in G and y in H }, where $\mu_{A\times B}(x, y) = \min\{ \mu_A(x), \mu_B(y) \}$ and $\nu_{A\times B}(x, y) = \max\{ \nu_A(x), \nu_B(y) \}$.

2.6 Definition

Let A be an intuitionistic fuzzy subset in a set S, the strongest intuitionistic fuzzy relation on S, that is an intuitionistic fuzzy relation on A is V given by $\mu_V(x, y) = \min\{ \mu_A(x), \mu_A(y) \}$ and $\nu_V(x, y) = \max\{ \nu_A(x), \nu_A(y) \}$, for all x and y in S.

2.7 Definition

Let (R, +, .) and (R', +, .) be any two hemirings. Let $f : R \to R'$ be any function and A be an (T, S)-intuitionistic fuzzy subhemiring in R, V be an (T, S)-intuitionistic fuzzy subhemiring in f(R)= R', defined by $\mu_V(y) = \sup_{x\in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x\in f^{-1}(y)} \nu_A(x)$, for all x in R and y in R'. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

2.8 Definition

Let A be an (T, S)-intuitionistic fuzzy subhemiring of a hemiring (R, +, ·) and a in R. Then the pseudo (T, S)-intuitionistic fuzzy coset $(aA)^p$ is defined by $((a\mu_A)^p)(x) = p(a)\mu_A(x)$ and $((a\nu_A)^p)(x) = p(a)\nu_A(x)$, for every x in R and for some p in P.

3. PROPERTIES

3.1 Theorem

Intersection of any two (T, S)-intuitionistic fuzzy subhemirings of a hemiring R is a (T, S)-intuitionistic fuzzy subhemiring of a hemiring R.

Proof: Let A and B be any two (T, S)-intuitionistic fuzzy subhemirings of a hemiring R and x and y in R. Let A = { (x, $\mu_A(x)$, $\nu_A(x)$) / x∈R } and B = { (x, $\mu_B(x)$, $\nu_B(x)$) / x∈R } and also let C = A∩B = { (x, $\mu_C(x)$, $\nu_C(x)$) / x∈R }, where min { $\mu_A(x)$, $\mu_B(x)$ } = $\mu_C(x)$ and max { $\nu_A(x)$, $\nu_B(x)$ } = $\nu_C(x)$. Now, $\mu_C(x+y)$ = min {$\mu_A(x+y)$, $\mu_B(x+y)$}≥ min{ T($\mu_A(x)$, $\mu_A(y)$), T($\mu_B(x)$, $\mu_B(y)$) }≥ T(min{ $\mu_A(x)$, $\mu_B(x)$ }, min { $\mu_A(y)$, $\mu_B(y)$ }) = T($\mu_C(x)$, $\mu_C(y)$). Therefore, $\mu_C(x+y)$ ≥ T($\mu_C(x)$, $\mu_C(y)$), for all x and y in R. And, $\mu_C(xy)$ = min { $\mu_A(xy)$, $\mu_B(xy)$} ≥ min {T ($\mu_A(x)$, $\mu_A(y)$), T($\mu_B(x)$, $\mu_B(y)$) }≥ T(min { $\mu_A(x)$, $\mu_B(x)$ }, min {$\mu_A(y)$, $\mu_B(y)$ }) = T ($\mu_C(x)$, $\mu_C(y)$). Therefore, $\mu_C(xy)$ ≥ T($\mu_C(x)$, $\mu_C(y)$), for all x and y in R.

Now, ν_C(x+y) = max { $\nu_A(x+y)$, $\nu_B(x+y)$ } ≤ max {S($\nu_A(x)$, $\nu_A(y)$), S($\nu_B(x)$, $\nu_B(y)$)} ≤ S(max{ $\nu_A(x)$, $\nu_B(x)$ }, max { $\nu_A(y)$, $\nu_B(y)$ }) = S ($\nu_C(x)$, $\nu_C(y)$). Therefore, $\nu_C(x+y)$ ≤ S($\nu_C(x)$, $\nu_C(y)$), for all x and y in R. And, $\nu_C(xy)$ = max {$\nu_A(xy)$, $\nu_B(xy)$} ≤ max { S($\nu_A(x)$, $\nu_A(y)$), S ($\nu_B(x)$, $\nu_B(y)$)} ≤ S(max {$\nu_A(x)$, $\nu_B(x)$}, max {$\nu_A(y)$, $\nu_B(y)$ }) = S($\nu_C(x)$, $\nu_C(y)$). Therefore, $\nu_C(xy)$ ≤ S ($\nu_C(x)$, $\nu_C(y)$), for all x and y in R. Therefore C is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R.

3.2 Theorem

The intersection of a family of (T, S)-intuitionistic fuzzy subhemirings of hemiring R is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R.

Proof: It is trivial.

2.3 Theorem

If A and B are any two (T, S)-intuitionistic fuzzy subhemirings of the hemirings R_1 and R_2 respectively, then A×B is an (T, S)-intuitionistic fuzzy subhemiring of R_1×R_2.

Proof: Let A and B be two (T, S)-intuitionistic fuzzy subhemirings of the hemirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in R_1×R_2. Now, $\mu_{A\times B}$ [(x_1, y_1)+(x_2, y_2)] = $\mu_{A\times B}$ (x_1+x_2, y_1+y_2) = min {$\mu_A(x_1+x_2)$, $\mu_B(y_1+y_2)$} ≥ min{T($\mu_A(x_1)$, $\mu_A(x_2)$), T($\mu_B(y_1)$, $\mu_B(y_2)$) }≥ T(min{$\mu_A(x_1)$, $\mu_B(y_1)$}, min{$\mu_A(x_2)$, $\mu_B(y_2)$})= T($\mu_{A\times B}$(x_1, y_1), $\mu_{A\times B}$(x_2, y_2)). Therefore, $\mu_{A\times B}$[(x_1, y_1) + (x_2, y_2)] ≥ T($\mu_{A\times B}$ (x_1, y_1), $\mu_{A\times B}$ (x_2, y_2)). Also, $\mu_{A\times B}$[(x_1, y_1) (x_2, y_2)] = $\mu_{A\times B}$ (x_1x_2, y_1y_2) = min { μ_A(x_1x_2), μ_B(y_1y_2) }≥ min {T ($\mu_A(x_1)$, $\mu_A(x_2)$), T ($\mu_B(y_1)$, $\mu_B(y_2)$)}≥ T(min{$\mu_A(x_1)$, $\mu_B(y_1)$}, min {$\mu_A(x_2)$, $\mu_B(y_2)$}) = T($\mu_{A\times B}$ (x_1, y_1), $\mu_{A\times B}$ (x_2, y_2)). Therefore, $\mu_{A\times B}$[(x_1, y_1)(x_2, y_2)] ≥T($\mu_{A\times B}$(x_1, y_1), $\mu_{A\times B}$(x_2, y_2)). Now, $\nu_{A\times B}$[(x_1, y_1) + (x_2, y_2)] = $\nu_{A\times B}$(x_1+x_2, y_1+ y_2) = max {$\nu_A(x_1+x_2)$, ν_B(y_1+ y_2)} ≤ max { S ($\nu_A(x_1)$, $\nu_A(x_2)$), S ($\nu_B(y_1)$, $\nu_B(y_2)$) }≤ S(max{$\nu_A(x_1)$, $\nu_B(y_1)$}, max{$\nu_A(x_2)$, $\nu_B(y_2)$ }) = S($\nu_{A\times B}$ (x_1, y_1), $\nu_{A\times B}$ (x_2, y_2)). Therefore, $\nu_{A\times B}$ [(x_1, y_1)+(x_2, y_2)] ≤ S ($\nu_{A\times B}$ (x_1, y_1), $\nu_{A\times B}$ (x_2, y_2)). Also, $\nu_{A\times B}$ [(x_1, y_1)(x_2, y_2)] = $\nu_{A\times B}$ (x_1x_2, y_1y_2) = max { ν_A(x_1x_2), ν_B(y_1y_2) }≤ max { S($\nu_A(x_1)$, $\nu_A(x_2)$), S($\nu_B(y_1)$, $\nu_B(y_2)$) }≤ S(max {$\nu_A(x_1)$, $\nu_B(y_1)$ }, max{$\nu_A(x_2)$, $\nu_B(y_2)$}) = S($\nu_{A\times B}$(x_1, y_1), $\nu_{A\times B}$(x_2, y_2)).

Therefore, $\nu_{A\times B}$[(x_1, y_1)(x_2, y_2)] ≤ S($\nu_{A\times B}$(x_1, y_1), $\nu_{A\times B}$(x_2, y_2)). Hence A×B is an (T, S)-intuitionistic fuzzy subhemiring of hemiring of R_1×R_2.

3.4 Theorem

If A is a (T, S)-intuitionistic fuzzy subhemiring of a hemiring (R, +, ·), then $\mu_A(x) \leq \mu_A(0)$ and $\nu_A(x) \geq \nu_A(0)$, for x in R, the zero element 0 in R.

Proof: For x in R and 0 is the zero element of R. Now, $\mu_A(x) = \mu_A(x+0) \geq T(\mu_A(x), \mu_A(0))$, for all x in R. So, $\mu_A(x) \leq \mu_A(0)$ is only possible. And $\nu_A(x) = \nu_A(x+0) \leq S(\nu_A(x), \nu_A(0))$ for all x in R. So, $\nu_A(x) \geq \nu_A(0)$ is only possible.

3.5 Theorem

Let A and B be (T, S)-intuitionistic fuzzy subhemiring of the hemirings R_1 and R_2 respectively. Suppose that 0 and $0_|$ are the zero element of R_1 and R_2 respectively. If A×B is an (T, S)-intuitionistic fuzzy subhemiring of $R_1 \times R_2$, then at least one of the following two statements must hold. (i) $\mu_B(0_|) \geq \mu_A(x)$ and $\nu_B(0_|) \leq \nu_A(x)$, for all x in R_1, (ii) $\mu_A(0) \geq \mu_B(y)$ and $\nu_A(0) \leq \nu_B(y)$, for all y in R_2.

Proof: Let A×B be an (T, S)-intuitionistic fuzzy subhemiring of $R_1 \times R_2$. By contraposition, suppose that none of the statements (i) and (ii) holds. Then we can find a in R_1 and b in R_2 such that $\mu_A(a) > \mu_B(0_|)$, $\nu_A(a) < \nu_B(0_|)$ and $\mu_B(b) > \mu_A(0)$, $\nu_B(b) < \nu_A(0)$. We have, $\mu_{A\times B}(a, b) = \min\{\mu_A(a), \mu_B(b)\} > \min\{\mu_B(0_|), \mu_A(0)\} = \min\{\mu_A(0), \mu_B(0_|)\} = \mu_{A\times B}(0, 0_|)$. And, $\nu_{A\times B}(a, b) = \max\{\nu_A(a), \nu_B(b)\} < \max\{\nu_B(0_|), \nu_A(0)\} = \max\{\nu_A(0), \nu_B(0_|)\} = \nu_{A\times B}(0, 0_|)$. Thus A×B is not an (T, S)-intuitionistic fuzzy subhemiring of $R_1 \times R_2$. Hence either $\mu_B(0_|) \geq \mu_A(x)$ and $\nu_B(0_|) \leq \nu_A(x)$, for all x in R_1 or $\mu_A(0) \geq \mu_B(y)$ and $\nu_A(0) \leq \nu_B(y)$, for all y in R_2.

3.6 Theorem

Let A and B be two intuitionistic fuzzy subsets of the hemirings R_1 and R_2 respectively and A×B is an (T, S)-intuitionistic fuzzy subhemiring of $R_1 \times R_2$. Then the following are true:

(i) if $\mu_A(x) \leq \mu_B(0_|)$ and $\nu_A(x) \geq \nu_B(0_|)$, then A is an (T, S)-intuitionistic fuzzy subhemiring of R_1.

(ii) if $\mu_B(x) \leq \mu_A(0)$ and $\nu_B(x) \geq \nu_A(0)$, then B is an (T, S)-intuitionistic fuzzy subhemiring of R_2.

(iii) either A is an (T, S)-intuitionistic fuzzy subhemiring of R_1 or B is an (T, S)-intuitionistic fuzzy subhemiring of R_2.

Proof: Let A×B be an (T, S)-intuitionistic fuzzy subhemiring of $R_1 \times R_2$ and x and y in $R_1$and $0_|$ in R_2. Then $(x, 0_|)$ and $(y, 0_|)$ are in $R_1 \times R_2$. Now, using the property that $\mu_A(x) \leq \mu_B(0_|)$ and $\nu_A(x) \geq \nu_B(0_|)$, for all x in R_1. We get, $\mu_A(x+y) = \min\{\mu_A(x+y), \mu_B(0_| + 0_|)\} = \mu_{A\times B}((x+y), (0_| + 0_|)) = \mu_{A\times B}[(x, 0_|) + (y, 0_|)] \geq T(\mu_{A\times B}(x, 0_|), \mu_{A\times B}(y, 0_|)) = T(\min\{\mu_A(x), \mu_B(0_|)\}, \min\{\mu_A(y), \mu_B(0_|)\}) = T(\mu_A(x), \mu_A(y))$. Therefore, $\mu_A(x+y) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in R_1. Also, $\mu_A(xy) = \min\{\mu_A(xy), \mu_B(0_| 0_|)\} = \mu_{A\times B}((xy), (0_| 0_|)) = \mu_{A\times B}[(x, 0_|)(y, 0_|)] \geq T(\mu_{A\times B}(x, 0_|), \mu_{A\times B}(y, 0_|)) = T(\min\{\mu_A(x), \mu_B(0_|)\}, \min\{\mu_A(y), \mu_B(0_|)\}) = T(\mu_A(x), \mu_A(y))$. Therefore, $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in R_1. And, $\nu_A(x+y) = \max\{\nu_A(x+y), \nu_B(0_| + 0_|)\} = \nu_{A\times B}((x+y), (0_| + 0_|)) = \nu_{A\times B}[(x, 0_|) + (y, 0_|)] \leq S(\nu_{A\times B}(x, 0_|), \nu_{A\times B}(y, 0_|)) = S(\max\{\nu_A(x), \nu_B(0_|)\}, \max\{\nu_A(y), \nu_B(0_|)\}) = S(\nu_A(x), \nu_A(y))$. Therefore, $\nu_A(x+y) \leq S(\nu_A(x), \nu_A(y))$, for all x and y in R_1. Also, $\nu_A(xy) = \max\{\nu_A(xy), \nu_B(0_| 0_|)\} = \nu_{A\times B}((xy), (0_| 0_|)) = \nu_{A\times B}[(x,$

$0_l)(y, 0_l)] \leq S(\nu_{A\times B}(x, 0_l), \nu_{A\times B}(y, 0_l)) = S(\max\{\nu_A(x), \nu_B(0_l)\}, \max\{\nu_A(y), \nu_B(0_l)\}) = S(\nu_A(x), \nu_A(y))$. Therefore, $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y))$, for all x and y in R_1. Hence A is an (T, S)-intuitionistic fuzzy subhemiring of R_1. Thus (i) is proved. Now, using the property that $\mu_B(x) \leq \mu_A(0)$ and $\nu_B(x) \geq \nu_A(0)$, for all x in R_2, let x and y in R_2 and 0 in R_1. Then (0, x) and (0, y) are in $R_1\times R_2$.We get, $\mu_B(x+y) = \min\{\mu_B(x+y), \mu_A(0+0)\} = \min\{\mu_A(0+0), \mu_B(x+y)\} = \mu_{A\times B}((0+0), (x+y)) = \mu_{A\times B}[(0, x)+(0, y)] \geq T(\mu_{A\times B}(0, x), \mu_{A\times B}(0, y)) = T(\min\{\mu_A(0), \mu_B(x)\}, \min\{\mu_A(0), \mu_B(y)) = T(\mu_B(x), \mu_B(y))$. Therefore, $\mu_B(x+y) \geq S(\mu_B(x), \mu_B(y))$, for all x and y in R_2. Also, $\mu_B(xy) = \min\{\mu_B(xy), \mu_A(00)\} = \min\{\mu_A(00), \mu_B(xy)\} = \mu_{A\times B}((00), (xy)) = \mu_{A\times B}[(0, x)(0, y)] \geq T(\mu_{A\times B}(0, x), \mu_{A\times B}(0, y)) = T(\min\{\mu_A(0), \mu_B(x)\}, \min\{\mu_A(0), \mu_B(y)\}) = T(\mu_B(x), \mu_B(y))$. Therefore, $\mu_B(xy) \geq T(\mu_B(x), \mu_B(y))$, for all x and y in R_2. And, $\nu_B(x+y) = \max\{\nu_B(x+y), \nu_A(0+0)\} = \max\{\nu_A(0+0), \nu_B(x+y)\} = \nu_{A\times B}((0+0), (x+y)) = \nu_{A\times B}[(0, x)+(0, y)] \leq S(\nu_{A\times B}(0, x), \nu_{A\times B}(0, y)) = S(\max\{\nu_A(0), \nu_B(x)\}, \max\{\nu_A(0), \nu_B(y)\}) = S(\nu_B(x), \nu_B(y))$. Therefore, $\nu_B(x+y) \leq S(\nu_B(x), \nu_B(y))$, for all x and y in R_2. Also, $\nu_B(xy) = \max\{\nu_B(xy), \nu_A(00)\} = \max\{\nu_A(00), \nu_B(xy)\} = \nu_{A\times B}((00), (xy)) = \nu_{A\times B}[(0, x)(0, y)] \leq S(\nu_{A\times B}(0, x), \nu_{A\times B}(0, y)) = S(\max\{\nu_A(0), \nu_B(x)\}, \max\{\nu_A(0), \nu_B(y)\}) = S(\nu_B(x), \nu_B(y))$. Therefore, $\nu_B(xy) \leq S(\nu_B(x), \nu_B(y))$, for all x and y in R_2. Hence B is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R_2. Thus (ii) is proved. (iii) is clear.

3.7 Theorem

Let A be an intuitionistic fuzzy subset of a hemiring R and V be the strongest intuitionistic fuzzy relation of R. Then A is an (T, S)-intuitionistic fuzzy subhemiring of R if and only if V is an (T, S)-intuitionistic fuzzy subhemiring of R×R.

Proof: Suppose that A is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R. We have, $\mu_V(x+y) = \mu_V[(x_1, x_2) + (y_1, y_2)] = \mu_V(x_1+y_1, x_2+y_2) = \min\{\mu_A(x_1+y_1), \mu_A(x_2+y_2)\} \geq \min\{T(\mu_A(x_1), \mu_A(y_1)), T(\mu_A(x_2), \mu_A(y_2))\} \geq T(\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\mu_V(x), \mu_V(y))$. Therefore, $\mu_V(x+y) \geq T(\mu_V(x), \mu_V(y))$, for all x and y in R×R. And, $\mu_V(xy) = \mu_V[(x_1, x_2)(y_1, y_2)] = \mu_V(x_1y_1, x_2y_2) = \min\{\mu_A(x_1y_1), \mu_A(x_2y_2)\} \geq \min\{T(\mu_A(x_1), \mu_A(y_1)), T(\mu_A(x_2), \mu_A(y_2))\} \geq T(\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\mu_V(x), \mu_V(y))$. Therefore, $\mu_V(xy) \geq T(\mu_V(x), \mu_V(y))$, for all x and y in R×R. We have, $\nu_V(x+y) = \nu_V[(x_1, x_2) + (y_1, y_2)] = \nu_V(x_1+y_1, x_2+y_2) = \max\{\nu_A(x_1+y_1), \nu_A(x_2+y_2)\} \leq \max\{S(\nu_A(x_1), \nu_A(y_1)), S(\nu_A(x_2), \nu_A(y_2))\} \leq S(\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\}) = S(\nu_V(x_1, x_2), \nu_V(y_1, y_2)) = S(\nu_V(x), \nu_V(y))$. Therefore, $\nu_V(x+y) \leq S(\nu_V(x), \nu_V(y))$, for all x and y in R×R. And, $\nu_V(xy) = \nu_V[(x_1, x_2)(y_1, y_2)] = \nu_V(x_1y_1, x_2y_2) = \max\{\nu_A(x_1y_1), \nu_A(x_2y_2)\} \leq \max\{S(\nu_A(x_1), \nu_A(y_1)), S(\nu_A(x_2), \nu_A(y_2))\} \leq S(\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\}) = S(\nu_V(x_1, x_2), \nu_V(y_1, y_2)) = S(\nu_V(x), \nu_V(y))$. Therefore, $\nu_V(xy) \leq S(\nu_V(x), \nu_V(y))$, for all x and y in R×R. This proves that V is an (T, S)-intuitionistic fuzzy subhemiring of R×R. Conversely assume that V is an (T, S)-intuitionistic fuzzy subhemiring of R×R, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R, we have $\min\{\mu_A(x_1 + y_1), \mu_A(x_2 + y_2)\} = \mu_V(x_1 + y_1, x_2 + y_2) = \mu_V[(x_1, x_2) + (y_1, y_2)] = \mu_V(x+y) \geq T(\mu_V(x), \mu_V(y)) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\})$. If $x_2 = 0$, $y_2 = 0$, we get, $\mu_A(x_1+ y_1) \geq T(\mu_A(x_1), \mu_A(y_1))$, for all x_1 and y_1 in R. And, $\min\{\mu_A(x_1y_1), \mu_A(x_2y_2)\} = \mu_V(x_1y_1, x_2y_2) = \mu_V[(x_1, x_2)(y_1, y_2)] = \mu_V(xy) \geq T(\mu_V(x), \mu_V(y)) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\})$. If $x_2 = 0$, $y_2=0$, we get, $\mu_A(x_1y_1) \geq T(\mu_A(x_1), \mu_A(y_1))$, for all x_1 and y_1 in R.

We have, $\max\{\nu_A(x_1+ y_1), \nu_A(x_2+ y_2)\}= \nu_V(x_1+ y_1, x_2+ y_2) = \nu_V[(x_1, x_2)+ (y_1, y_2)] = \nu_V(x+y) \leq S(\nu_V(x), \nu_V(y)) = S(\nu_V(x_1, x_2), \nu_V(y_1, y_2)) = S(\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\})$. If $x_2 = 0$, $y_2=0$, we get, $\nu_A(x_1+y_1) \leq S(\nu_A(x_1), \nu_A(y_1))$, for all x_1 and y_1 in R.

And, $\max\{\nu_A(x_1y_1), \nu_A(x_2y_2)\} = \nu_V(x_1y_1, x_2y_2) = \nu_V[(x_1, x_2)(y_1, y_2)] = \nu_V(xy) \leq S(\nu_V(x), \nu_V(y)) = S(\nu_V(x_1, x_2), \nu_V(y_1, y_2)) = S(\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\})$. If $x_2 = 0$, $y_2 = 0$, we get $\nu_A(x_1y_1) \leq S(\nu_A(x_1), \nu_A(y_1))$, for all x_1 and y_1 in R.

Therefore A is an (T, S)-intuitionistic fuzzy subhemiring of R.

3.8 Theorem

If A is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring (R, +, .), then $H = \{ x / x \in R: \mu_A(x) = 1, \nu_A(x) = 0\}$ is either empty or is a subhemiring of R.

Proof: It is trivial.

3.9 Theorem

If A be an (T, S)-intuitionistic fuzzy subhemiring of a hemiring (R, +, .), then (i) if $\mu_A(x+y)=0$, then either $\mu_A(x)= 0$ or $\mu_A(y) = 0$, for all x and y in R.

(ii) if $\mu_A(xy) = 0$, then either $\mu_A(x) = 0$ or $\mu_A(y) = 0$, for all x and y in R.

(iii) if $\nu_A(x+y)=1$, then either $\nu_A(x)= 1$ or $\nu_A(y) = 1$, for all x and y in R.

(iv) if $\nu_A(xy) = 1$, then either $\nu_A(x)= 1$ or $\nu_A(y) = 1$, for all x and y in R.

Proof: It is trivial.

3.10 Theorem

If A is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring (R, +, .), then $H = \{ \langle x, \mu_A(x) \rangle : 0 < \mu_A(x) \leq 1$ and $\nu_A(x) = 0\}$ is either empty or a T-fuzzy subhemiring of R.

Proof: It is trivial.

3.11 Theorem

If A is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring (R, +, .) then $H = \{ \langle x, \mu_A(x) \rangle : 0 < \mu_A(x) \leq 1\}$ is either empty or a T-fuzzy subhemiring of R.

Proof: It is trivial.

3.12 Theorem

If A is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring (R, +, .), then $H = \{ \langle x, \nu_A(x) \rangle : 0 < \nu_A(x) \leq 1\}$ is either empty or an anti S-fuzzy subhemiring of R.

Proof: It is trivial.

3.13 Theorem

If A is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring (R,+, .), then □A is an (T, S)-intuitionistic fuzzy subhemiring of R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R. Consider A = { ⟨ x, $\mu_A(x)$, $\nu_A(x)$ ⟩ }, for all x in R, we take $\Box$A=B ={ ⟨x, $\mu_B(x)$, $\nu_B(x)$ ⟩ }, where $\mu_B(x) = \mu_A(x)$, $\nu_B(x) = 1-\mu_A(x)$. Clearly, $\mu_B(x+y) \geq T(\mu_B(x), \mu_B(y))$, for all x and y in R and $\mu_B(xy) \geq T(\mu_B(x), \mu_B(y))$, for all x and y in R. Since A is an (T, S)-intuitionistic fuzzy subhemiring of R, we have $\mu_A(x+y) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in R, which implies that $1-\nu_B(x+y) \geq T((1-\nu_B(x)), (1-\nu_B(y)))$, which implies that $\nu_B(x+y) \leq 1-T((1-\nu_B(x)), (1-\nu_B(y))) \leq S(\nu_B(x), \nu_B(y))$. Therefore, $\nu_B(x+y) \leq S(\nu_B(x), \nu_B(y))$, for all x and y in R. And $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in R, which implies that $1-\nu_B(xy) \geq T((1-\nu_B(x)), (1-\nu_B(y)))$

which implies that $\nu_B(xy) \leq 1-T((1-\nu_B(x)), (1-\nu_B(y))) \leq S(\nu_B(x), \nu_B(y))$. Therefore, $\nu_B(xy) \leq S(\nu_B(x), \nu_B(y))$, for all x and y in R. Hence B = $\Box$A is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R.

3.14 Theorem

If A is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring (R, +, .), then ◊A is an (T, S)-intuitionistic fuzzy subhemiring of R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R.That is A = { ⟨ x, $\mu_A(x)$, $\nu_A(x)$ ⟩ }, for all x in R. Let ◊A =B= { ⟨ x, $\mu_B(x)$, $\nu_B(x)$ ⟩ }, where $\mu_B(x) = 1-\nu_A(x)$, $\nu_B(x) = \nu_A(x)$. Clearly, $\nu_B(x+y) \leq S(\nu_B(x), \nu_B(y))$, for all x and y in R and $\nu_B(xy) \leq S(\nu_B(x), \nu_B(y))$, for all x and y in R. Since A is an (T, S)-intuitionistic fuzzy subhemiring of R, we have $\nu_A(x+y) \leq S(\nu_A(x), \nu_A(y))$, for all x and y in R, which implies that $1-\mu_B(x+y) \leq S((1-\mu_B(x)), (1-\mu_B(y)))$ which implies that $\mu_B(x+y) \geq 1-S((1-\mu_B(x)), (1-\mu_B(y))) \geq T(\mu_B(x), \mu_B(y))$. Therefore, $\mu_B(x+y) \geq T(\mu_B(x), \mu_B(y))$, for all x and y in R. And $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y))$, for all x and y in R, which implies that $1-\mu_B(xy) \leq S((1-\mu_B(x)), (1-\mu_B(y)))$, which implies that $\mu_B(xy) \geq 1-S((1-\mu_B(x)), (1-\mu_B(y))) \geq T(\mu_B(x), \mu_B(y))$. Therefore, $\mu_B(xy) \geq T(\mu_B(x), \mu_B(y))$, for all x and y in R. Hence B = ◊A is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R.

3.15 Theorem

Let (R, +, .) be a hemiring and A be a non empty subset of R. Then A is a subhemiring of R if and only if B = $< \chi_A, \overline{\chi_A} >$ is an (T, S)-intuitionistic fuzzy subhemiring of R, where χ_A is the characteristic function.

Proof: It is trivial.

3.16 Theorem

Let A be an (T, S)-intuitionistic fuzzy subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then A∘f is an (T, S)-intuitionistic fuzzy subhemiring of R.

Proof: Let x and y in R and A be an (T, S)-intuitionistic fuzzy subhemiring of a hemiring H. Then we have, $(\mu_A\circ f)(x+y) = \mu_A(f(x+y)) = \mu_A(f(x)+f(y)) \geq T(\mu_A(f(x)), \mu_A(f(y))) = T((\mu_A\circ f)(x), (\mu_A\circ f)(y))$, which implies that $(\mu_A\circ f)(x+y) \geq T((\mu_A\circ f)(x), (\mu_A\circ f)(y))$. And, $(\mu_A\circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(x)f(y)) \geq T(\mu_A(f(x)), \mu_A(f(y))) = T((\mu_A\circ f)(x), (\mu_A\circ f)(y))$, which implies that $(\mu_A\circ f)(xy) \geq T((\mu_A\circ f)(x), (\mu_A\circ f)(y))$. Then we have, $(\nu_A\circ f)(x+y) = \nu_A(f(x+y)) = \nu_A(f(x)+f(y)) \leq S(\nu_A(f(x)), \nu_A(f(y))) = S((\nu_A\circ f)(x), (\nu_A\circ f)(y))$, which implies that $(\nu_A\circ f)(x+y) \leq S((\nu_A\circ f)(x), (\nu_A\circ f)(y))$. And $(\nu_A\circ f)(xy) = \nu_A(f(xy)) =$

$\nu_A(f(x)f(y)) \le S(\nu_A(f(x)), \nu_A(f(y))) = S((\nu_A \circ f)(x), (\nu_A \circ f)(y))$, which implies that $(\nu_A \circ f)(xy) \le S((\nu_A \circ f)(x), (\nu_A \circ f)(y))$. Therefore $(A \circ f)$ is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R.

3.17 Theorem

Let A be an (T, S)-intuitionistic fuzzy subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. Then $A \circ f$ is an (T, S)-intuitionistic fuzzy subhemiring of R.

Proof: Let x and y in R and A be an (T, S)-intuitionistic fuzzy subhemiring of a hemiring H. Then we have, $(\mu_A \circ f)(x+y) = \mu_A(f(x+y)) = \mu_A(f(y)+f(x)) \ge T(\mu_A(f(x)), \mu_A(f(y))) = T((\mu_A \circ f)(x), (\mu_A \circ f)(y))$, which implies that $(\mu_A \circ f)(x+y) \ge T(\mu_A \circ f)(x), (\mu_A \circ f)(y))$. And, $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(y)f(x)) \ge T(\mu_A(f(x)), \mu_A(f(y))) = T((\mu_A \circ f)(x), (\mu_A \circ f)(y))$, which implies that $(\mu_A \circ f)(xy) \ge T((\mu_A \circ f)(x), (\mu_A \circ f)(y))$. Then we have, $(\nu_A \circ f)(x+y) = \nu_A(f(x+y)) = \nu_A(f(y)+f(x)) \le S(\nu_A(f(x)), \nu_A(f(y))) = S((\nu_A \circ f)(x), (\nu_A \circ f)(y))$, which implies that $(\nu_A \circ f)(x+y) \le S((\nu_A \circ f)(x), (\nu_A \circ f)(y))$.

And, $(\nu_A \circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(y)f(x)) \le S(\nu_A(f(x)), \nu_A(f(y))) = S((\nu_A \circ f)(x), (\nu_A \circ f)(y))$, which implies that $(\nu_A \circ f)(xy) \le S((\nu_A \circ f)(x), (\nu_A \circ f)(y))$. Therefore $A \circ f$ is an (T, S)-intuitionistic fuzzy subhemiring of the hemiring R.

3.18 Theorem

Let A be an (T, S)-intuitionistic fuzzy subhemiring of a hemiring (R, +, .), then the pseudo (T, S)-intuitionistic fuzzy coset $(aA)^p$ is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R, for every a in R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R.

For every x and y in R, we have, $((a\mu_A)^p)(x+y) = p(a)\mu_A(x+y) \ge p(a)\,T(\mu_A(x), \mu_A(y)) = T(p(a)\mu_A(x), p(a)\mu_A(y)) = T(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Therefore, $((a\mu_A)^p)(x+y) \ge T(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Now, $((a\mu_A)^p)(xy) = p(a)\mu_A(xy) \ge p(a)\,T(\mu_A(x), \mu_A(y)) = T(p(a)\mu_A(x), p(a)\mu_A(y)) = T(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Therefore, $((a\mu_A)^p)(xy) \ge T(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. For every x and y in R, we have, $((a\nu_A)^p)(x+y) = p(a)\nu_A(x+y) \le p(a)\,S(\nu_A(x), \nu_A(y)) = S(p(a)\nu_A(x), p(a)\nu_A(y)) = S(((a\nu_A)^p)(x), ((a\nu_A)^p)(y))$. Therefore, $((a\nu_A)^p)(x+y) \le S(((a\nu_A)^p)(x), ((a\nu_A)^p)(y))$. Now, $((a\nu_A)^p)(xy) = p(a)\,\nu_A(xy) \le p(a)\,S(\nu_A(x), \nu_A(y)) = S(p(a)\,\nu_A(x), p(a)\,\nu_A(y)) = S(((a\nu_A)^p)(x), ((a\nu_A)^p)(y))$. Therefore, $((a\nu_A)^p)(xy) \le S(((a\nu_A)^p)(x), ((a\nu_A)^p)(y))$. Hence $(aA)^p$ is an (T, S)-intuitionistic fuzzy subhemiring of a hemiring R.

3.19 Theorem

Let (R, +, .) and (R^{l}, +, .) be any two hemirings. The homomorphic image of an (T, S)-intuitionistic fuzzy subhemiring of R is an (T, S)-intuitionistic fuzzy subhemiring of R^{l}.

Proof: Let (R, +, .) and (R^{l}, +, .) be any two hemirings. Let $f : R \to R^{l}$ be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)\,f(y)$, for all x and y in R. Let V = f(A), where A is an (T, S)-intuitionistic fuzzy subhemiring of R. We have to prove that V is an (T, S)-

intuitionistic fuzzy subhemiring of R^{l}. Now, for f(x), f(y) in R^{l}, μ_v(f(x) + f(y)) = μ_v(f(x+y)) ≥ μ_A(x + y) ≥ T (μ_A(x), μ_A(y)) which implies that μ_v(f(x) + f(y)) ≥ T (μ_v(f(x)), μ_v(f(y))). Again, μ_v(f(x)f(y)) = μ_v(f(xy)) ≥ μ_A(xy) ≥ T (μ_A(x), μ_A(y)),which implies that μ_v(f(x)f(y)) ≥ T (μ_v(f(x)), μ_v(f(y))).

Now,for f(x),f(y) in R^{l}, ν_v(f(x)+f(y)) = ν_v(f(x+y)) ≤ ν_A(x+ y) ≤ S (ν_A(x), ν_A(y)), ν_v(f(x) + f(y)) ≤ S (ν_v(f(x)), ν_v(f(y))).

Again, ν_v(f(x)f(y)) = ν_v(f(xy)) ≤ ν_A(xy) ≤ S (ν_A(x), ν_A(y)), which implies that ν_v(f(x)f(y)) ≤ S (ν_v(f(x)), ν_v(f(y))). Hence V is an (T, S)-intuitionistic fuzzy subhemiring of R^{l}.

3.20 Theorem

Let (R, +, .) and (R^{l}, +, .) be any two hemirings. The homomorphic preimage of an (T, S)-intuitionistic fuzzy subhemiring of R^{l} is a (T, S)-intuitionistic fuzzy subhemiring of R.

Proof: Let V = f(A), where V is an (T, S)-intuitionistic fuzzy subhemiring of R^{l}. We have to prove that A is an (T, S)-intuitionistic fuzzy subhemiring of R. Let x and y in R. Then, μ_A(x+y) = μ_v(f(x+y)) = μ_v(f(x)+f(y)) ≥ T(μ_v(f(x)), μ_v(f(y))) = T(μ_A(x), μ_A(y)), since μ_v(f(x)) = μ_A(x), which implies that μ_A(x+y) ≥ T(μ_A(x), μ_A(y)). Again, μ_A(xy) = μ_v(f(xy)) = μ_v(f(x)f(y)) ≥ T (μ_v(f(x)), μ_v(f(y))) = T (μ_A(x), μ_A(y)), since μ_v(f(x)) = μ_A(x) which implies that μ_A(xy) ≥ T(μ_A(x), μ_A(y)). Let x and y in R. Then, ν_A(x+y) = ν_v(f(x+y)) = ν_v(f(x)+f(y)) ≤ S(ν_v(f(x)), ν_v(f(y))) = S(ν_A(x), ν_A(y)), since ν_v(f(x)) = ν_A(x) which implies that ν_A(x+ y) ≤ S(ν_A(x), ν_A(y)). Again, ν_A(xy) = ν_v(f(xy)) = ν_v(f(x)f(y)) ≤ S(ν_v(f(x)), ν_v(f(y))) = S(ν_A(x), ν_A(y)), since ν_v(f(x)) = ν_A(x) which implies that ν_A(xy) ≤ S (ν_A(x), ν_A(y)). Hence A is an (T, S)-intuitionistic fuzzy subhemiring of R.

3.21 Theorem

Let (R, +, .) and (R^{l}, +, .) be any two hemirings. The anti-homomorphic image of an (T, S)-intuitionistic fuzzy subhemiring of R is an (T, S)-intuitionistic fuzzy subhemiring of R^{l}.

Proof: Let (R, +, .) and (R^{l}, +, .) be any two hemirings. Let f : R → R^{l} be an anti-homomorphism. Then, f (x+y) = f (y) + f (x) and f(xy) = f(y) f(x), for all x and y in R. Let V = f(A), where A is an (T, S)-intuitionistic fuzzy subhemiring of R. We have to prove that V is an (T, S)-intuitionistic fuzzy subhemiring of R^{l}. Now, for f(x), f(y) in R^{l}, μ_v(f(x)+f(y)) = μ_v(f(y+x)) ≥ μ_A(y+x) ≥ T(μ_A(y), μ_A(x)) = T(μ_A(x), μ_A(y)), which implies that μ_v(f(x) + f(y)) ≥ T(μ_v(f(x)), μ_v(f(y))). Again, μ_v(f(x)f(y)) = μ_v(f(yx)) ≥ μ_A(yx) ≥ T(μ_A(y), μ_A(x)) = T(μ_A(x), μ_A(y)), which implies that μ_v(f(x)f(y)) ≥ T(μ_v(f(x)), μ_v(f(y))). Now for f(x), f(y) in R^{l}, ν_v(f(x)+f(y)) = ν_v(f(y+x)) ≤ ν_A(y+ x) ≤ S(ν_A(y), ν_A(x)) = S(ν_A(x), ν_A(y)), which implies that ν_v(f(x)+f(y)) ≤ S(ν_v(f(x)), ν_v(f(y))).

Again, ν_v(f(x)f(y)) = ν_v(f(yx)) ≤ ν_A(yx) ≤ S(ν_A(y), ν_A(x)) = S(ν_A(x), ν_A(y)), which implies that ν_v(f(x)f(y)) ≤ S (ν_v(f(x)), ν_v(f(y))). Hence V is an (T, S)-intuitionistic fuzzy subhemiring of R^{l}.

3.22 Theorem

Let (R, +, .) and (R^{l}, +, .) be any two hemirings. The anti-homomorphic preimage of an (T, S)-intuitionistic fuzzy subhemiring of R^{l} is an (T, S)-intuitionistic fuzzy subhemiring of R.

Proof: Let V = f(A), where V is an (T, S)-intuitionistic fuzzy subhemiring of R^{l}. We have to prove that A is an (T, S)-intuitionistic fuzzy subhemiring of R. Let x and y in R. Then, $\mu_A(x+y) = \mu_v(f(x+y)) = \mu_v(f(y)+f(x)) \geq T(\mu_v(f(y)), \mu_v(f(x))) = T(\mu_v(f(x)), \mu_v(f(y))) = T(\mu_A(x), \mu_A(y))$, which implies that $\mu_A(x+y) \geq T(\mu_A(x), \mu_A(y))$. Again, $\mu_A(xy) = \mu_v(f(xy)) = \mu_v(f(y)f(x)) \geq T(\mu_v(f(y)), \mu_v(f(x))) = T(\mu_v(f(x)), \mu_v(f(y))) = T(\mu_A(x), \mu_A(y))$, since $\mu_v(f(x)) = \mu_A(x)$ which implies that $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$. Then, $\nu_A(x+y) = \nu_v(f(x+y)) = \nu_v(f(y)+f(x)) \leq S(\nu_v(f(y)), \nu_v(f(x))) = S(\nu_v(f(x)), \nu_v(f(y))) = S(\nu_A(x), \nu_A(y))$ which implies that $\nu_A(x+y) \leq S(\nu_A(x), \nu_A(y))$. Again, $\nu_A(xy) = \nu_v(f(xy)) = \nu_v(f(y)f(x)) \leq S(\nu_v(f(y)), \nu_v(f(x))) = S(\nu_v(f(x)), \nu_v(f(y))) = S(\nu_A(x), \nu_A(y))$, which implies that $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y))$. Hence A is an (T, S)-intuitionistic fuzzy subhemiring of R.

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