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Static Stress Analysis of Functionally Graded Circular Cylindrical Shell Subjected to Internal Pressure

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Abstract – *In this paper, a functionally graded circular cylindrical shell is studied under the internal pressure. For various arrangements of stiffeners the FGM circular cylindrical shell is studied for its deflection and the von-Mises stresses are analyzed. . Analytical modeling is based on first order shear deformation theory (FOST) [1, 2] and a finite element computational tool ABAQUS [3] is used to model the FGM cylindrical shell. Material properties are estimated by power law index.*

Key Words: **FGM, ABAQUS, Analytical solution, von-Mises stresses, Circular Cylindrical shells**

1. INTRODUCTION

Functionally graded materials(FGMs) are a class of composite materials that have been taken into consideration in the last two decades because of their special performance compared with conventional materials. FGMs are inhomogeneous materials made from different phases of material constituents, usually ceramic and metal, and their material properties change gradually along a certain direction, usually in the thickness one. The term FGM was originally presented in the 1984 by a group of scientists in Japan when they conducted research into materials that are resistant to extremely high temperatures for aircrafts and aerospace applications. Due to particular characteristics of functionally materials, these can resist high temperatures in various environments.

Shell structures are widely used in industrial applications such as deep drawing, roll forming and pressure vessels. With the increase in demand, FGMs have been widely used in general structures. Hence many functionally graded (FG) structures have been extensively studied, such as FG plates, FG cylindrical shells.

Many researchers are being working on Functionally Graded material. FGM Stiffened shells are used in several engineering applications to improve efficiency of the structures in terms of strength/weight. A.S. Oktem, J.L. Mantari, C. Guedes Soares [4] carried out the static analysis of FGM plates and doubly curved shells using a higher order deformation theory. Erasmo Viola, Francesco Tornabene, Nicholas Fantuzzi [5] studied the static analysis of doubly curved laminated shells and panels using 2D higher order shear deformation theory. Dao Huy Bich, Dao Van Dung, Vu Hoai Nam, Nguyen Thi Phuong [6] carried out the nonlinear static and dynamic buckling analysis under axial compression ,GDE are derived and is solved by Galerkin procedure to obtain static critical buckling load and the nonlinear dynamic responses are found by fourth order Runge-Kutta method. J.L. Mantari, E.V. Granados [7] studied the static analysis of FG sandwich plates using new first order shear deformation theory. S.M.Shiyekar, Prashant Lavate [8] carried out bending analysis of FG plates using FEM computational tools ABAQUS. Ankit Gupta, Mohammad Talha [9] presented an overview of recent development in modeling and analysis of FGM and structures. M. Memar Ardestani, B. Soltani, Sh. Shams [10] carried out the analysis of stiffened FGM plates using first order shear deformation theory. Erasmo Viola, Luigi Rossetti, Nicholas Fantuzzi [11] analyzed the FGM Conical shells and panels subjected to meridian, circumferential and normal uniform loadings using 2D unconstrained third order shear deformation theories. Dao Huy Bich, Dao Van Dung, Vu Hoai Nam [12] studied the nonlinear dynamic analysis of an eccentrically stiffened FGM cylindrical panel based on classical shell theory with geometric nonlinearity in Von Karman- Donnell sense. Erasmo Viola, Luigi Rossetti, Nicholas Fantuzzi [13] presented the numerical investigation of FGM cylindrical shells subjected to mechanical loadings by using 2D unconstrained third order shear deformation theory.Ebrahim Asadi, Wenchao Wang, Mohamad S. Qatu [14] studied the exact static and free vibration solutions for isotropic and symmetric and anti-symmetric cross-ply cylindrical shells for different length to thickness and length to radius ratios.

2. FORMULATION OF FSDT

1.1 Introduction

The FSDT developed by Mindlin [15] accounts for the shear deformation effect by the way of a linear variation of the in-plane displacements through the thickness. It is noted that the theory developed by Reissner [16] also accounts for the shear deformation effect. However, the Reissner theory is

not similar with the Mindlin theory like erroneous perception of many researchers through the use of misleading descriptions such as “Reissner-Mindlin plates” and “FSDT of Reissner”.The major difference between two theories was established by Wang et al. by derivating the bending relationships between Mindlin and Reissner quantities for a general plate problems. Since the Reissner theory was based on the assumption of a linear bending stress distribution, its formulation will inevitably lead to the displacement variation being not necessarily linear across the plate thickness. Thus, it is incorrect to refer to the Reissner theory as the FSDT which implies a linear variation of the displacements through the thickness. Another difference between two theories is that the normal stress which was included in the Reissner theory was omitted in the Mindlin.
The FSDT was used to model FG shells. Reddy and Chin [17] studied the dynamic response of FG cylinders and plates subjected to two different types of thermal loadings using the FSDT and the finite element method.

1.2 Definition of displacement field

$$u = u_0 + z\theta_x \qquad (1a)$$
$$v = v_0 + z\theta_y$$
$$w = w_0$$

In the above relations, the terms u, v and w are the displacements of a general point (x, y, z) in the domain in *x*, *y* and *z* directions respectively. The parameters u_0, v_0 are the inplane displacements and w_0 is the transverse displacement of a point (*x, y*) on the element middle plane. The functions θ_x *and* θ_y are the rotations of the normal to the element middle plane about *y*- and *x*-axis respectively.

1.3 Strain- displacement relations

With the definition of strains from the linear theory of elasticity, assuming $h/R_x, h/R_y \ll 1$, the general strain-displacement relations in the curvilinear co-ordinate system are given as follows:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{w}{R_x}$$
$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{w}{R_y}$$
$$\varepsilon_z = \frac{\partial w}{\partial z}$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$
$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - \frac{u}{R_x}$$
$$\gamma_{yz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial y} - \frac{v}{R_y}$$

Substituting the expressions for displacements at any point within the space given by Eqs. (1a) for the displacement models considered herein, the linear strains in terms of middle surface displacements, for each displacement model can be obtained as follows:

$$\varepsilon_x = \varepsilon_{x0} + z\chi_x$$
$$\varepsilon_y = \varepsilon_{y0} + z\chi_y$$
$$\varepsilon_z = 0$$
$$\gamma_{xy} = \varepsilon_{xy0} + z\chi_{xy}$$
$$\gamma_{xz} = \emptyset_x + z\chi_{xz}$$
$$\gamma_{yz} = \emptyset_y + z\chi_{yz}$$

Where, $(\chi_{xz}, \chi_{yz}) = (-\frac{\theta_x}{R_x}, -\frac{\theta_y}{R_y})$

1.4 Stress-strain relations and stress resultants

Assuming the principal material axes (1,2,3) and the shell axes (x,y,z) in the curvilinear co-ordinate system ,the three dimensional stress-strain relations for an cylindrical shell with reference to the principal material axes for the theory to be developed based on the displacement are defined as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \\ \tau_{13} \\ \tau_{23} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} \qquad (2a)$$

Where,

$$C_{11} = \frac{E_1}{(1-\nu_{12}\nu_{21})}, C_{12} = \frac{\nu_{21}E_1}{(1-\nu_{12}\nu_{21})},$$
$$C_{22} = \frac{E_2}{(1-\nu_{12}\nu_{21})}, \quad \frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2},$$
$$C_{33} = G_{12}, \quad C_{44} = G_{13}, \quad C_{55} = G_{23}$$

These equations in compacted form may be written as

$$\boldsymbol{\sigma} = \boldsymbol{C\varepsilon}$$

As mentioned earlier, the relations given by Eq. (2a) are the stress-strain constitutive relations for the cylindrical shell referred to shell’s principal material axes (1,2,3). The principal material axes of shell may not coincide with the reference axes of the shell (x,y,z).It is therefore necessary to transform the constitutive relations from the shell’s material axes (1,2,3) to reference axes (x,y,z).This is conveniently accomplished through the transformations .The final relations are as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & Q_{45} \\ 0 & 0 & 0 & Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (2b)$$

These equations in compacted form may be written as

$$\boldsymbol{\sigma} = \boldsymbol{Q}\,\boldsymbol{\varepsilon}$$

in which the coefficients of the Q matrix, called as reduced elastic constants are defined in Appendix A.

The components of the strain vector $\bar{\varepsilon}$ and the corresponding components of the stress- resultant vector $\bar{\sigma}$ are defined as follows:

$$\begin{bmatrix} N_x & M_x \\ N_y & M_y \\ N_{xy} & M_{xy} \end{bmatrix} = \sum_{L=1}^{NL} \int_{Z_L}^{Z_{L+1}} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} (1, z^2, z, z^3)dz$$

$$= \sum_{L=1}^{NL} \int_{Z_L}^{Z_{L+1}} \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} \\ Q_{12} & Q_{22} & Q_{23} & Q_{24} \\ Q_{13} & Q_{23} & Q_{33} & Q_{34} \\ Q_{14} & Q_{24} & Q_{34} & Q_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} (1, z^2, z, z^3)dz$$

$$\begin{bmatrix} Q_x & S_x \\ Q_y & S_y \end{bmatrix} = \sum_{L=1}^{NL} \int_{Z_L}^{Z_L+1} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} (1, z^2, z, z^3)dz$$

$$\bar{\sigma} = (N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y, S_x, S_y)$$

$$\bar{\varepsilon} = (\varepsilon_{x0}, \varepsilon_{y0}, \varepsilon_{xy0}, \chi_x, \chi_y, \chi_{xy}, \emptyset_x, \emptyset_y, \chi_{xz}, \chi_{yz})$$

1.5 Equilibrium equations

For equilibrium equations, the total potential energy must be stationary and using the definitions of stress-resultants and mid-surface strains stated in above sections principal of virtual work yields

$$\delta\Pi = \delta(U - W) = 0$$

Where, U is the strain energy and W represents the work done by external forces. These are evaluated as follows:

$$\delta U = \int_x \int_y \int_z (\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \sigma_z \delta\varepsilon_z + \tau_{xy}\delta\gamma_{xy} + \tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz})dx\,dy\,dz. \quad (3a)$$

Integration through thickness and by substituting in terms of strains and introducing stress resultants, the above relations transform in the following form

$$\delta\Pi = \int_x \int_y (N_x\delta\varepsilon_{x0} + N_y\delta\varepsilon_{y0} + N_{xy}\delta\varepsilon_{xy0} + M_x\delta\chi_x + M_y\delta\chi_y + M_{xy}\delta\chi_{xy} + Q_x\delta\emptyset_x + Q_y\delta\emptyset_y + S_x\delta\chi_{xz} + S_y\delta\chi_{yz} - \delta w_0 q)\,dxdy = 0 \quad (3b)$$

In the above equation is the distributed transverse load. The governing equations of equilibrium can be derived from eq. (3b) by integrating the displacement gradients in mid surface strains by parts and setting the coefficients of derivatives of mid-surface displacements to zero separately. Thus one obtains the following equilibrium equations.

$$\frac{\partial N_x}{\partial x} + \frac{\partial(N_{xy} + c_0 M_{xy})}{\partial y} + \frac{Q_x}{R_x} = 0$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial(N_{xy} - c_0 M_{xy})}{\partial x} + \frac{Q_y}{R_y} = 0$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \frac{N_x}{R_x} - \frac{N_y}{R_y} + q = 0 \quad (3c)$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x + \frac{S_x}{R_x} = 0$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y + \frac{S_y}{R_y} = 0$$

In addition following line integrals are also obtained

$$\int_x (N_y\delta v_0 + N_{xy}\delta u_0 + M_y\delta\theta_y + M_{xy}\delta\theta_x + C_0 M_{xy}\delta u_0 + Q_y\delta w_0 + S_y\delta\theta_z)dx + \int_y (N_x\delta u_0 + N_{xy}\delta v_0 + M_x\delta\theta_x + M_{xy}\delta\theta_y - C_0 M_{xy}\delta v_0 + Q_x\delta w_0 + S_x\delta\theta_z) \quad (3d)$$

1.6 Closed form solutions

Sinusoidal variation of transverse load is considered as under:

$$q = \sum_{m,n}^{\infty} q_{mn} \sin\alpha x \sin\beta y$$

$\alpha = \frac{m\pi}{a}, \beta = \frac{m\pi}{b}$ in which a and b are the dimensions of shell middle surface along the x and y axes respectively.

The exact form of spatial variation of mid-surface displacements is given by

$$u_0 = \sum_{m,n}^{\infty} u_{0mn} \cos\alpha x \sin\beta y$$

$$v_0 = \sum_{m,n}^{\infty} v_{0mn} \sin \alpha x \cos \beta y$$
$$w_0 = \sum_{m,n}^{\infty} w_{0mn} \sin \alpha x \sin \beta y$$
$$(4a)$$
$$\theta_x = \sum_{m,n}^{\infty} \theta_{xmn} \cos \alpha x \sin \beta y$$
$$\theta_y = \sum_{m,n}^{\infty} \theta_{ymn} \sin \alpha x \cos \beta y$$

Substitution of Eq.(3d) and series of Eq. (4a) into Eq.(3c) yields a set of linear algebraic equations in terms of the unknown amplitudes $u_{0mn}, v_{0mn}, w_{0mn}, \theta_{xmn}, \theta_{ymn}$ .These equations can be expressed in matrix form as

$$\boldsymbol{C\Delta} = \boldsymbol{F}$$

Here C, Δ and F are given in Appendix B.

3. NUMERICALS AND RESULTS

In this paper, a $FGM_{(a,b,c)}$ circular cylindrical shell is studied when it is subjected to internal pressure of 20KN/m^2 .where a,b,c are vibrational parameters through which the volume of ceramic changes throughout the thickness of the shell. Here we have analyzed the simply supported $FGM_{1,1,4}$ circular cylindrical shell having dimensions R=500mm,L=1000mm.

The shell is studied for different arrangements of stiffeners:

ARRANGEMENT A: Longitudinal (external) stiffeners

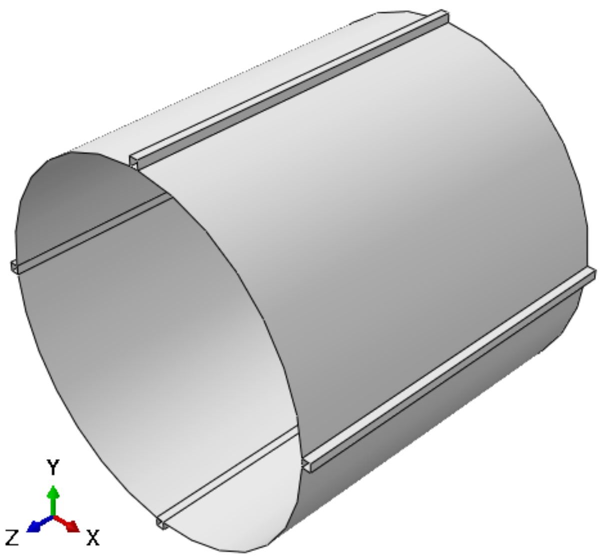

Fig 1

ARRANGEMENT B: Ring (external) stiffeners

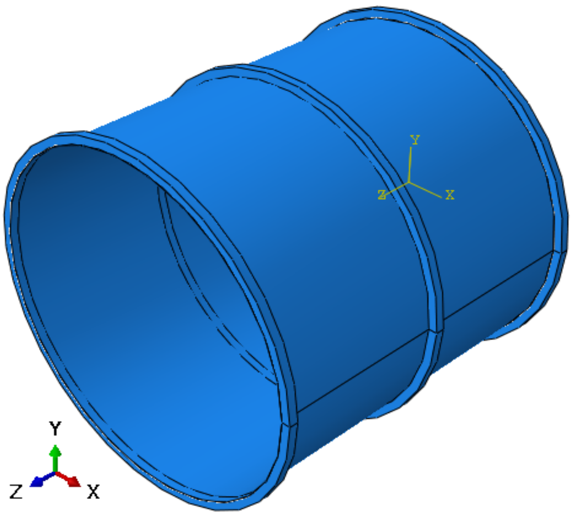

Fig 2

ARRANGEMENT C: Ring (internal) stiffeners

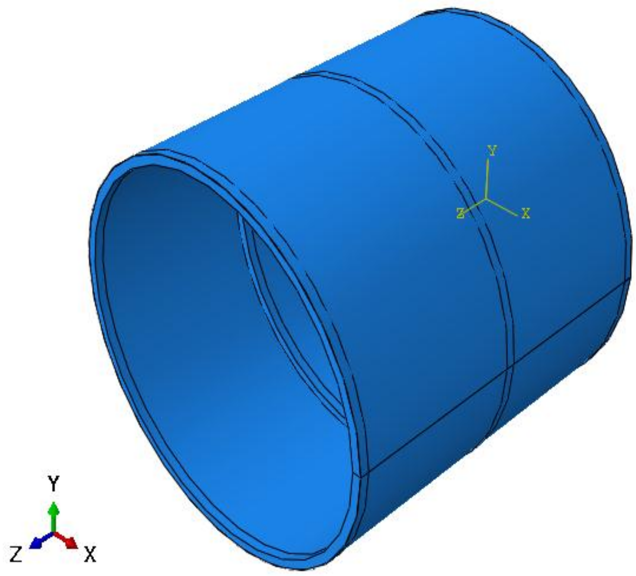

Fig 3

For different power law index ,the FG circular cylindrical shell is analyzed and is tabulated in table1.

Table1. Showing the results of $FGM_{1,1,4}$ circular cylindrical shell subjected to internal pressure.

Power law index(p)	Type of stiffeners	Max. displacement	Max. S-Misses stresses	Max. normal stresses(S_{11})	Max. normal stresses(S_{22})	Max. shear stresses(S_{12})
P=0	Longitudinal (external)	0.25055	5.434e02	5.573e02	2.567e02	1.288e02
	Ring(external)	0.34285	5.219e02	5.070e02	4.725e02	1.944e02
	Ring(internal)	0.19015	5.164e02	4.906e02	3.925e02	2.106e02
P=0.2	Longitudinal (external)	0.39585	5.608e02	5.750e02	2.674e02	1.328e02
	Ring(external)	0.32645	5.386e02	5.232e02	4.872e02	2.007e02
	Ring(internal)	0.47855	5.329e02	5.062e02	4.052e02	2.174e02
P=2	Longitudinal (external)	0.5815	6.808e02	6.976e02	3.424e02	1.612e02
	Ring(external)	0.548	6.550e02	6.364e02	5.896e02	2.448e02
	Ring(internal)	0.2175	6.482e02	6.153e02	4.941e02	2.646e02
P=5	Longitudinal (external)	1.9045	7.580e02	7.765e02	3.899e02	1.795e02
	Ring(external)	0.3972	7.281e02	7.075e02	6.546e02	2.724e02
	Ring(internal)	0.631	7.205e02	6.838e02	5.498e02	2.942e02

The deformed shapes of $FGM_{1,1,4}$ are shown in fig 4,fig 5 and fig 6 for ARRANGEMENT A ,ARRANGEMENT B and ARRANGEMENT C respectively.

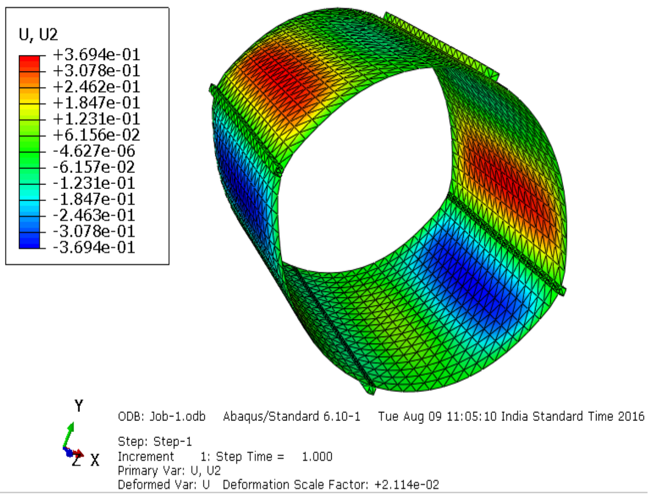

Fig 4: Deformed shape of $FGM_{1,1,4}$ circular cylindrical shell with ARRANGEMENT A type stiffeners.

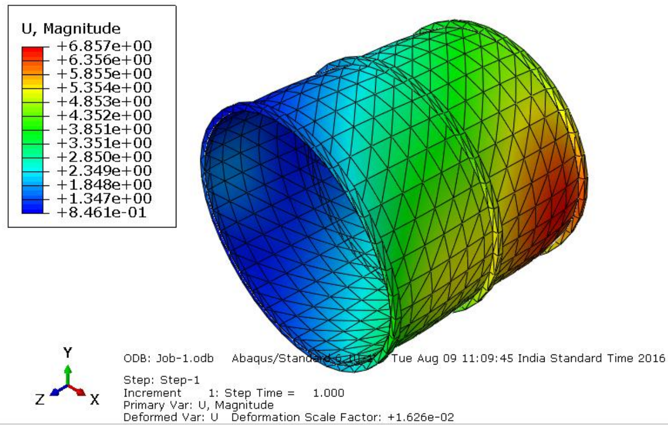

Fig 5: Deformed shape of $FGM_{1,1,4}$ circular cylindrical shell with ARRANGEMENT B type stiffeners.

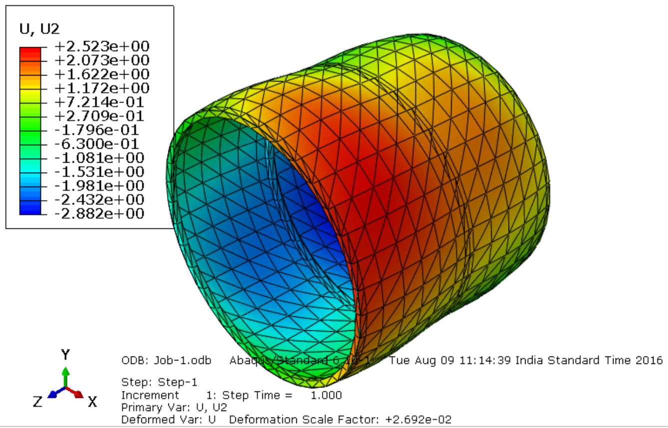

Fig 6: Deformed shape of $FGM_{1,1,4}$ circular cylindrical shell with ARRANGEMENT C type stiffeners.

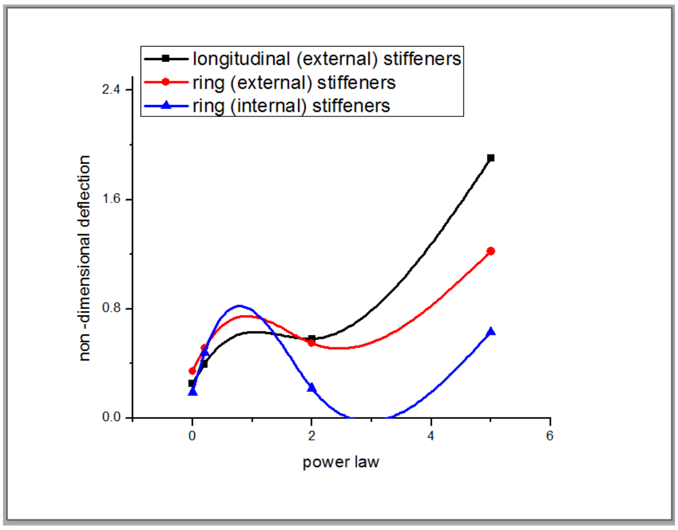

Fig 7: Non-dimensional deflections vs. power law index

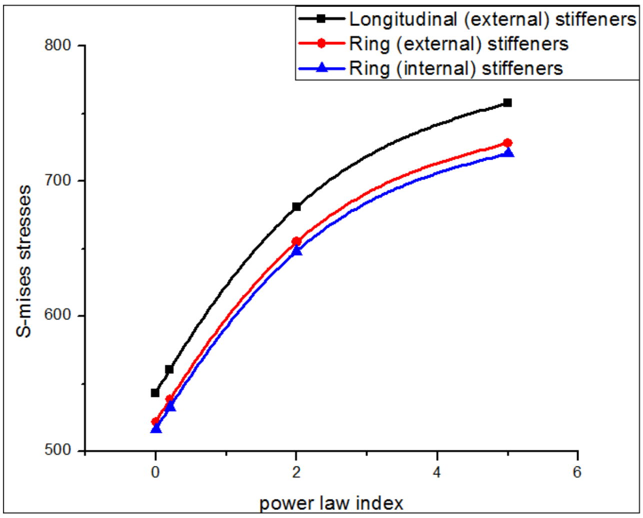

Fig 8: von-Mises max stresses vs. power law index

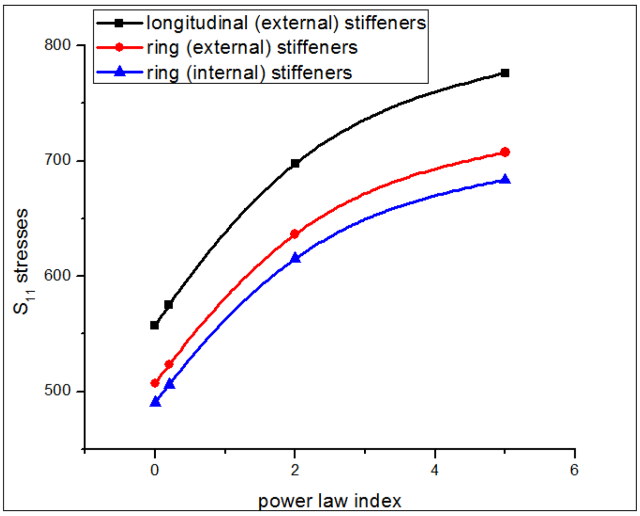

Fig 9: von-Mises normal S_{11} stresses vs. power law index

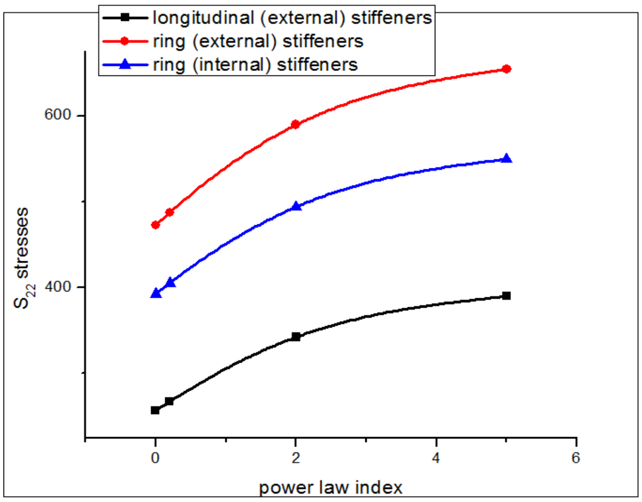

Fig 10: von-Mises normal S_{22} stresses vs. power law index

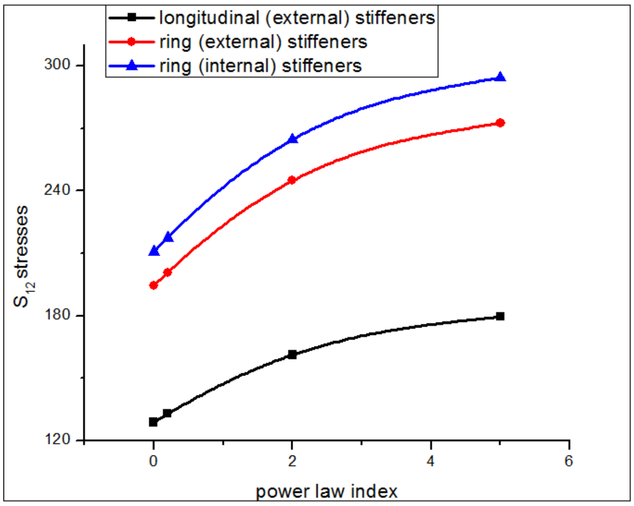

Fig 11: von-Mises shear S_{12} stresses vs. power law index

4. CONCLUSIONS

Functionally graded circular cylindrical shell when subjected to internal pressure, the ring (internal) stiffeners arrangement shows lowest S-Misses stresses while the longitudinal (external) stiffeners shows lowest shear stresses. The ring (internal) stiffener shows lesser deflection for power law index p= 0,2,5 but increases gradually for power law index p=1.The non-dimensional deflection increases as the power law increases.

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