RAYLEIGH'S QUOTIENT AND LUMPED PARAMETER METHOD FOR FINDING TORSION AND BENDING

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Abstract - *In the Rayleigh's Quotient and the Lumped Parameter method, we study about this and known that how it is use full in the finding the bending and torsion frequency of the Micro – Bridge and Micro – Cantilever. Not only in this section we study but until we find the bending and torsion of the Rayleigh's Quotient and the Lumped Parameter method. Calculating the modal or resonant response of flexible structure can be performed by the means of analytical and numerical method. Numerical method is solved by the partial differential equation with complex boundary condition and geometry shape. Analytical method dedicated to evaluating the resonant response of elastic member emprise distributed parameter methods and lumped parameter method. Rayleigh's quotient occupies a unique position in vibration. It is not only fundamental to vibration theory, but it also have a practical value whereas The lumped – parameter method which transform the real, distributed. Parameter properties – elastic (stiffness) and inertial (mass of moment of inertia) into equivalent lumped parameter ones Ke (equivalent stiffness) me (equivalent mass), or Je (equivalent mechanical moment of inertia) which are computed separately.*

Key Words:

1. **Model Analytical Methods**
2. **Rayleigh's Quotient**
3. **Rayleigh's Quotient Method**
   - **3.1 Bending**
   - **3.2 Torsion**
4. **Lumped Parameter Methods**
   - **4.1 Bending**
   - **4.2 Equivalent Stiffness**
   - **4.3 Equivalent Mass**
   - **4.4 Torsion**

1.INTRODUCTION

Micro cantilever and the Micro Bridge is the simplest mechanical device that operate as standalone system in a variety of electromechanical system (MEMS) application, such as nano scale reading/writing in topology detection/creation, optical detection, material properties characterization, resonant sensing, mass detection, or micro/Nano electronics circuitry component such as switches or filters.

The method of obtaining the bending and the torsion resonant of Micro cantilever and the Micro Bridge (fixed-free, flexible members) by the **Rayleigh's Quotient approximate Method.**

1.1 Rayleigh's Quotient

Rayleigh's quotient occupies a unique position in vibration. It is not only fundamental to vibration theory, but it also have a practical value, as it can be used as a means of estimating the fundamental frequency of a system or as a tool in speeding up convergence to the solution of the Eigen value problems in matrix iteration.

1.2 Lumped Parameter Methods

The lumped – parameter method which transform the real, distributed. Parameter properties – elastic (stiffness) and inertial (mass of moment of inertia) into equivalent lumped parameter ones Ke (equivalent stiffness) me (equivalent mass), or Je (equivalent mechanical moment of inertia) which are computed separately.

2. MODAL ANALYTICAL METHOD

Calculating the modal or resonant response of flexible structure can be performed by the means of analytical and numerical method.

Numerical method is solved by the partial differential equation with complex boundary condition and geometry shape. Analytical method dedicated to evaluating the resonant response of elastic member emprise distributed parameter methods and lumped parameter method.

3. RAYLEIGH'S QUOTIENT

Rayleigh's quotient occupies a unique position in vibration. It is not only fundamental to vibration theory, but it also have a practical value, as it can be used as a means of estimating the fundamental frequency of a system or as a tool in speeding up convergence to the solution of the Eigen value problems in matrix iteration.

$$\lambda[m]\{u\} = [k]\{u\}$$

Where,
[m] – inertia matrix
[k] – stiffness matrix
Let us consider the Eigen value problem associated with the shaft clamped with the boundary condition x = 0, to x = L. I(x) is the mass of the polar moment of inertia per unit of length. GJ(x) is the torsion stiffness at point x.
The angular displacement of the shaft is given by the written equation:-

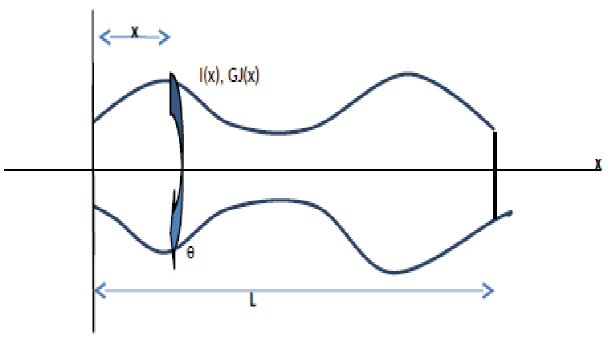
$$\theta(x,t) = \Theta(x)f(t)$$

f(t) is harmonic with frequency ω

Hence, the Eigen value problem can be written in the form of the differential equation:-

$$-\frac{d}{dx}\left[GJ(x)\frac{d\Theta(x)}{dx}\right] = \lambda I(x)\,\Theta(x)$$

$$\Lambda = \omega^2$$



Where Θ(x) is subjected to the boundary condition Θ (0) = 0.

$$GJ(x)\frac{d\Theta(x)}{dx} = 0$$

This above form when we put the boundary condition Θ (x) = L

Multiply the equation (1) by Θ(x) and considering boundary condition of (2), we get-

$$\Lambda = \omega^2 = R(\Theta) = \int_0^L \frac{\Theta(x)\left(\frac{d}{dx}\right)\left\{GJ(x)\left[\frac{d\Theta(x)}{dx}\right]\right\}dx}{\int_0^L I(x)\,\Theta^2(x)\,dx}$$

$$R(\Theta) = \int_0^L \frac{GJ(x)\left[\frac{d\Theta(x)}{dx}\right]^2 dx}{\int_0^L I(x)\Theta^2(x)dx}$$

Where **R(Θ)** is known as the **Rayleigh's Quotient** of the system

4. RAYLEIGH'S QUOTIENT METHODS

Rayleigh's quotient methods is a distributed parameter procedure enabling calculation of various resonant frequencies of freely vibrating elastic structure.

$$T_{max} = U_{max}$$

The next assumption in the harmonic motion of a vibrating component is the product between the spatial function and time dependent function.

$$U(x,t)=U(x)\sin(\acute{\omega}t)$$

Where the deformation con be produced through bending axial or torsion free vibrations.

4.1 Bending

In out of the plane bending of single component MEMS, such as the micro-cantilever and the micro-binding-

$$\omega b^2 = \int_l \frac{EIy(x)\left[\frac{\partial^2 uz(x)}{\partial x^2}\right]^2 dx}{\int_l \rho A(x) Vz(x)^2 dx}$$

The deflection Vz(x) is related to the maximum deflection Uz by means of a bending distribution function as

$$Uz(x)=Uzfz(x)$$

Hence the bending resonant frequency is

$$\omega b^2 = \int_l \frac{EIy(x)\left[\frac{d^2 fb(x)}{\partial x^2}\right]^2 dx}{\int_l \rho A(x) Uz(x)^2 dx}$$

4.2 Torsion

Rayleigh's quotient method con also be applied to torsion problem involving micro-cantilever and micro-bridges-

$$\omega t^2 = 12 \int_l \frac{GIt(x)\left[\frac{d\theta x(x)}{dx}\right]^2 dx}{\int \rho w(x)\, t(x)\, [w(x)^2 + t(x)^2]\, \theta x(Lx)]^2 dx}$$

Whereas the torsion angle at an arbitrary abscissa, θx(x) and the maximum torsion angle θx.

$$\theta x(x) = \theta x Ft(X)$$

$$\omega f^2 = 12 \int_L \frac{GIt(x)\left[\frac{dFt(x)}{dx}\right]^2 dx}{\int_L \rho\omega(x)\, t(x)\, [\omega(x)^2 + t(x)^2]\theta x(X)^2 dx}$$

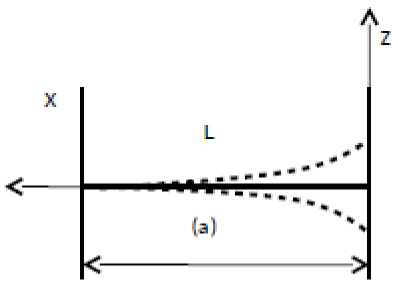
5. LUMPED PARAMETER METHODS

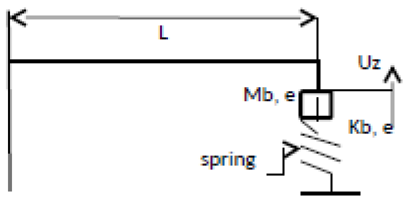
The lumped – parameter method which transform the real, distributed. Parameter properties – elastic (stiffness) and inertial (mass of moment of inertia) into equivalent lumped parameter ones Ke (equivalent stiffness) me (equivalent mass), or Je (equivalent mechanical moment of inertia) which are computed separately. The resonant frequency of interest is expressed as :

$$\omega^2 = \frac{Ke}{Me}$$

5.1 Bending

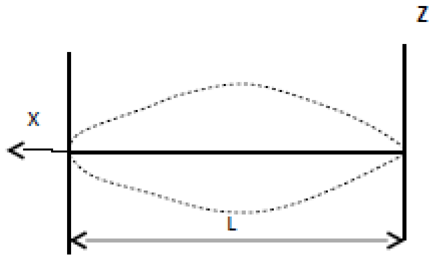
The lumped parameter, approach substitute parameter flexible component by an equivalent, lumped – parameter are;



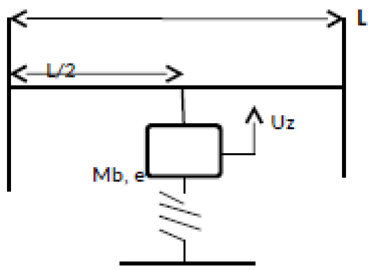


(b)

Figure 1. Cantilever vibrating out of plane (a) real distributed parameter system. (b) Equivalent lumped parameter, mass spring system.



(a)



(b)

Fig: Bridge vibrating out of plane (a) real distributed parameter system, (b) equivalent lumped parameter, mass spring system.

5.2 Equivalent Stiffness

The stiffness of liner springs can be found in several ways, by applying a force Fz at the point of interestand by relating it to static deflection Uz at the same position. It is known that for liner system. Force is proportional to deflection and the proportionality constant is the stiffness.

` Fz ∞ Uz

Fz = Kb,eUz

The equivalent lumped parameter stiffness is:

$$Kb,e = \int_L EIy(x)\left[\frac{d^2 Fb(x)}{d(x)^2}\right]^2 dx$$

5.3 Equivalent Mass

The equivalent mass Mb,e can be found by applying Rayleigh's principal according to which the velocity distribution over a vibration beam is identical to the one of deflection. The kinetic energy of the equivalent mass is simply: -

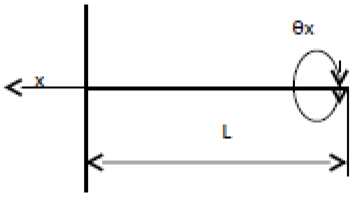
$$Te = \frac{1}{2} Mb,e \left[\frac{dUz(t)}{d(t)}\right]^2$$

The equivalent mass is obtained as :-

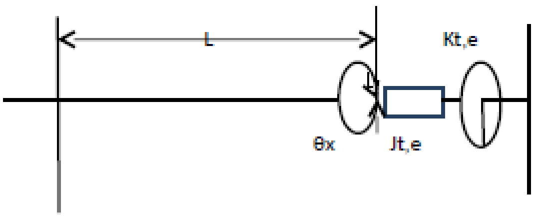
$$Mb,e = \int \rho A(x) Fb(x)^2 dx$$

5.4 Torsion

A similar approach can be followed in studying the torsion of micro-cantilever and micro-bridge by using the lumped parameter approach, and substitute the distributed parameter of a torsion vibrating member by corresponding lumped parameter ones.



(a)



(b)

Fig: free fixed bar vibration torsional (a) real distribution parameter system, (b) equivalent lumped parameter mass spring system.

REFERENCES

1. Dr. Nilanajn Malik, Assistant Professor, Department of Mechanical Engineering Indian Institute of Technology BHU Varanasi.
2. Dr. P. S Dubey, Director, Ashoka Institute of Technology and Management, Varanasi.
3. Prof. A. K Agrawal, Head of Department, Department of Mechanical Engineering Indian Institute of Technology BHU Varanasi.
4. Mr. Vibhas Patel, Head of Department, Department of Mechanical Engineering, Ashoka Institute of Technology and Management, Varanasi.
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