Optimum Allocation in Multi-Objective Geometric Programming in Multivariate Double Sampling Design

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Abstract - *The problem of optimum allocation in multivariate double sampling design has been formulated as a Multi-Objective Geometric Programming Problem (MOGPP). The fuzzy programming approach has described for converting the (MOGPP) into Single Objective Geometric Programming Problem (SOGPP) with the use of membership function. The formulated SOGPP has been solved with the help of LINGO Software and the dual solution is obtained. The optimum allocations of sample sizes are obtained with the help of dual solutions and primal-dual relationship* theorem*. A numerical example is given to illustrate the procedure.*

Key Words: Multivariate Double Sampling Design, Multi-Objective, Fuzzy programming, Geometric Programming, Optimum allocations.

1. INTRODUCTION

Multivariate double sampling design is used to estimate the unknown strata weights in stratified sampling. At first stage, a large simple random sample from the population with un-stratification is drawn and sampled units from each stratum are recorded to estimate the unknown strata weights. A stratified random sample is then obtained comprising of simple random subsamples out of the previously selected units of the strata. The double sampling design was first introduced by Neyman (1938) in survey research. Kokan (1963) proposed a nonlinear-programming solution in multivariate surveys but did not discuss its applicability to double sampling. Kokan and Khan (1967) described an analytical solution of an allocation problem in multivariate surveys and also discuss its application to double sampling. Multivariate stratified sample survey deals with more than one (say p) characteristics on each unit of a stratified population. A stratified sample survey is used to convert the heterogeneous population into the homogeneous population. Optimum allocation is an attempt to attain specified level of significance at the minimum cost. The problem of optimum allocation in multivariate stratified sample survey was initially described by Neyman (1934). After that the successful applications of mathematical programming techniques in multivariate stratified sample surveys are due to the following authors as: Ghosh (1958), Murthy (1967), Cochran (1977), Jahan and Ahsan (1995), Khan et al. (1997), Garcia and Cortez (2006), Diaz Garcia et al. (2007), Ansari et al. (2011), Khan et al. (2012), Ali et al (2011), Gupta et al. (2013) and many others.

Geometric programming is an important methodology for solving algebraic nonlinear optimization problems. One of the significant property of geometric programming is that a problem with highly nonlinear constraints can be stated equivalently as one with only linear constraints. This is because there is a strong duality theorem for geometric programming problems. If the primal problem is in posynomial form, then a global minimizing solution to that problem can be obtained by solving the dual maximization. The dual constraints are linear and linearly constrained programs are generally easier to solve than ones with nonlinear constraints. It's attractive structural properties, as well as its elegant theoretical basis, have led to a number of interesting applications and the development of numerous useful results. Many authors have used geometric programming in different situations and various fields. Some of them are: Duffin et al. (1967), Ahmad and Charles (1987), Maqbool et al. (2011), MOGPP was discussed by Ojha and Biswal (2010), Ojha and Das (2010), Islam, S. (2010), Bazikar and Saraj (2014), Dey and Roy (2014), Shafiullah et al. (2015), Shafiullah and Agarwal (2016) and many others.

The real-life decision-making problems of sample surveys, social, economic, environmental and technical areas are of multiple-objectives. Multi-objective programming is a powerful mathematical procedure and applicable in decision making to a wide range of problems in the government organizations, managements, econometrics, non-profitable organizations and private sector etc. A system with indistinguishable information can neither be formulated nor be solved effectively by traditional mathematics-based optimization techniques nor probability-based stochastic optimization approaches. However, fuzzy set theory and fuzzy programming techniques provide a useful and efficient tool for modeling and optimizing such systems. Zadeh (1965, 1978) first introduced the concept of fuzzy set theory. After that, Bellman and Zadeh (1970) used the fuzzy set theory to the decision-making problem. Gupta et al. (2014) and Zimmermann (1978) have described the fuzzy set theory, fuzzy programming, and linear programming respectively. Fuzzy multi-objective programming is discussed by many authors such as Sakawa and Yano (1989, 1994), Islam and Roy (2006) and many others.
In this paper, the problem of optimum allocation in multivariate double sampling design has been formulated as a Multi-Objective Geometric Programming Problem (MOGPP). The fuzzy programming approach has described

for converting the (MOGPP) into Single Objective Geometric Programming Problem (SOGPP) with the use of membership function. The formulated SOGPP has been solved with the help of LINGO Software and the dual solution is obtained. The optimum allocations of sample sizes are obtained with the help of dual solutions and primal-dual relationship theorem. A numerical example is projected to illustrate the procedure.

2. FORMULATION OF THE PROBLEM

It is assumed that double sampling involves p factors of interest. It is also assumed that it is not possible to keep a separate account of the time or cost to measure or guess each variate at every sampling point and then the total cost of double sampling is

$$C = x_1 c_1 + x_2 c_2$$

where C is the total cost of the survey, c_1 is the cost to obtain one direct estimate (measurement of p factors) and c_2 is the cost to obtain one indirect estimate (guessing the values of p factors), x_1 and x_2 are the sample sizes of direct and indirect sampling respectively. Usually c_1 is large in relation to c_2 and fixed the cost of sampling do not enter into optimization problems. The sample variance function of the direct and indirect samples is given as:

$$\frac{V_{x_{1j}}}{x_1} + \frac{V_{x_{2j}}}{x_2} \le v_j^0$$

where v_j^0 is the specified variance of the j^{th} variate, v_{x_1} and v_{x_2} are the sample variances of j^{th} variate in direct and indirect samples. It may be supposed that

$$f_{0j}(x) = \sum_{i=1}^{2} \frac{V_{ij}}{x_i}, \quad i = 1,2,\cdots,p.$$

The mathematical formulation of the above problem is given as:

$$\left.\begin{array}{l} \text{Minimize } f_{0j}(x) = \sum_{i=1}^{2} \frac{V_{ij}}{x_i} \\ \text{Subject to } c_i x_i \le C \text{ and } x_i \ge 0,\ i = 1,2;\ j = 1,2,\cdots,p \end{array}\right\} \quad (1)$$

3. GEOMETRIC PROGRAMMING APPROACH IN TWO-STAGE SAMPLING DESIGN

Geometric programming always transforms the primal problem of minimizing a posynomial subject to posynomial constraints to a dual problem of maximizing a function of the weights on each constraint. The mathematical form of problem (1) can be expressed in the following way:

$$\left.\begin{array}{l} \text{Minimize } f_{0j}(x) = \sum_{i=1}^{2} \frac{V_{ij}}{x_i} \\ \text{Subject to } c_i x_i \le C \text{ and } x_i \ge 0,\ i = 1,2;\ j = 1,2,\cdots,p \end{array}\right\} \quad (2)$$

In the above equations we have noticed that the objective function 2(i) is non-linear and the constraints 2(ii) are linear and the reduced which in the standard GP (Primal) problem can be stated as:

$$\left.\begin{array}{l} \text{Minimize } f_{0j}(x) \\ \text{Subject to } f_q(x) \le 1 \text{ and } x_i \ge 0,\ i = 1,2;\ j = 1,2,\cdots,p. \end{array}\right\} \quad (3)$$

$$\left.\begin{array}{l} \text{Where } f_q(x) = \sum_{i=1}^{2} c_i x_i \le C \text{ and} \\ f_q(x) = \sum_{i \in j[q]} d_i \left\{ \prod_{j=1}^{n} x_i^{\lambda_{ij}} \right\}, d_i \ge 0, x_i > 0, q = 0,1,\cdots,p. \end{array}\right\} \quad (4)$$

The number of posynomial terms in the function can be denoted by p, the number of variables is denoted by n and the exponents λ_{ij} are real constants. The objective function $C(x)$ for our allocation problem that is given in 2(i) and 2(ii) has $d_i = V_{x_{ij}}$ and $d_i = c_i$, $i = 1,2$.

The dual form of Geometric Programming problem which is stated in (3) can be given as:

$$\left.\begin{array}{lll} Max \left[\prod_{q=0}^{p} \prod_{i \in [q]} \left(\frac{d_i}{w_i} \right)^{w_i} \right] \prod_{q=1}^{p} \left(\sum_{i \in [q]} w_i \right)^{\sum_{i \in [q]} w_i} & & (i) \\ Subject \ to \ \sum_{i \in [0]} w_i = 1 & & (ii) \\ \sum_{q=0}^{p} \sum_{i \in [q]} p_{ij} w_i = 0 & & (iii) \\ and \quad w_i \ge 0,\ q = 0, \dots, p & & \end{array}\right\} \quad (5)$$

Step 1: For the Optimum value of the objective function, the objective function always takes the form

$$C_0(x^*) = \left(\frac{Coeffi.\ of\ first\ term}{w_{01}} \right)^{w_{01}} \times \left(\frac{Coeffi.\ of\ Second\ term}{w_{02}} \right)^{w_{02}} \times \dots\dots \times \left(\frac{Coeffi.\ of\ last\ term}{w_k} \right)^{w_k}$$

$$\left(\sum w's \text{ in the first constraints} \right)^{\sum w's \text{ in the first constraints}} \left(\sum w's \text{ in the last constraints} \right)^{\sum w's \text{ in the last constraints}}$$

Step 2: The equations that can be used for geometric program for the weights are given below:

$$\sum_{i=1}^{2} w_{0i} \text{ in the objective function} = 1$$

(Normality condition) (6)

and for each primal variable x_j given n variables and k terms

$$\sum_{i=1}^{m}(w_i \text{ for each trms})\times(\text{exponent } on\ x_j \text{ in thatterm}) = 0$$

(Orthogonality Condition) (7)

$w_{0i} \geq 0$ and $w_i \geq 0$ (Positivity Condition)

Primal-dual relationship theorem: If w_{0i}^* is a maximizing point for dual problem (5), each minimizing points (x) for primal problem (3) satisfies the system of equations:

$$f_0(x) = \begin{cases} w_{0i}^* \, v(w^*), & i \in J[0], \\ \dfrac{w_{ij}}{v_L(w_{0i}^*)}, & i \in J[L], \end{cases} \qquad (8)$$

where L ranges over all positive integers for which $v_L(w_{0i}^*) > 0$.

4. FUZZY GEOMETRIC PROGRAMMING APPROACH

The solution procedure to solve the problem (5) consists of the following steps:

Step-1: Solve the MOGPP as a single objective problem using only one objective at a time and ignoring the others. These solutions are known as the ideal solution.

Step-2: From the results of step-1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated where the diagonal solutions are the best solutions and the off-diagonal solutions are the worst.

$$\begin{array}{c} \\ (x^{(1)}) \\ (x^{(2)}) \\ \vdots \\ (x^{(j)}) \\ \vdots \\ (x^{(p)}) \end{array} \begin{array}{c} \begin{array}{ccccc} f_{01}(x) & f_{02}(x) & \cdots f_{0j}(x) & \cdots & f_{0p}(x) \end{array} \\ \begin{bmatrix} f_{01}{}^*(x^{(1)}) & f_{02}(x^{(1)}) \cdots & f_{0j}(x^{(1)}) & \cdots f_{0p}(x^{(1)}) \\ f_{01}(x^{(2)}) & f_{02}^*(x^{(2)}) \cdots & f_{0j}(x^{(2)}) & \cdots f_{0p}(x^{(2)}) \\ \vdots & \vdots \ \cdots & \vdots & \cdots \ \vdots \\ f_{01}(x^{(j)}) & f_{02}(x^{(j)}) \cdots & f_{0j}^*(x^{(j)}) & \cdots f_{0p}(x^{(j)}) \\ \vdots & \vdots & \vdots & \vdots \\ f_{01}(x^{(p)}) & f_{02}(x^{(p)}) \cdots & f_{0j}(x^{(p)}) & \cdots f_{0p}^*(x^{(p)}) \end{bmatrix} \end{array}$$

Here $(x^{(1)}), (x^{(2)}), \cdots, (x^{(j)}), \cdots, (x^{(p)})$ are the ideal solutions of the objective functions

$f_{01}(x^{(1)}),\ f_{02}(x^{(2)}), \cdots,\ f_{0j}(x^{(j)}), \cdots,\ f_{0p}(x^{(p)})$.

So $U_j = Max\{f_{01}(x^{(1)}),\ f_{02}(x^{(2)}),\ \cdots,\ f_{0p}(x^{(p)})\}$

and $L_j = f_{0j}^*(x^{(j)}),\ j = 1,2,\ldots,p$.

$\left[U_j \text{ and } L_j \text{ be the upper and lower bonds of the } j^{th} \text{ objective function } f_{0j}(x),\ j = 1,2,..,p. \right]$

Step 3: The membership function for the given problem can be defined as:

$$\mu_j(f_{0j}(x)) = \begin{cases} 0\,, & if\ f_{0j}(x) \geq U_j \\ \dfrac{U_j(x) - f_{0j}(x)}{U_j(x) - L_j(x)}, & if\ L_j \leq f_{0j}(x) \geq U_j \\ 1, & if\ f_{0j}(x) \leq L_j \\ & j = 1, 2, \ldots, p \end{cases} \qquad (9)$$

Here $U_j(x)$ is a strictly monotonic decreasing function with respect to $f_{0j}(x)$.

Following figure 4.1 illustrates the graph of the membership function $\mu_j(f_{0j}(x))$, i.e., $\mu_j(f_{0j}(x))$, $j = 1, 2, \ldots, p$.

Therefore the general aggregation function can be defined as

$$\mu_{\tilde{D}}(x) = \mu_{\tilde{D}}\{\mu_1(f_{01}(x)), \mu_2(f_{02}(x)), \ldots, \mu_p(f_{0p}(x))\}$$

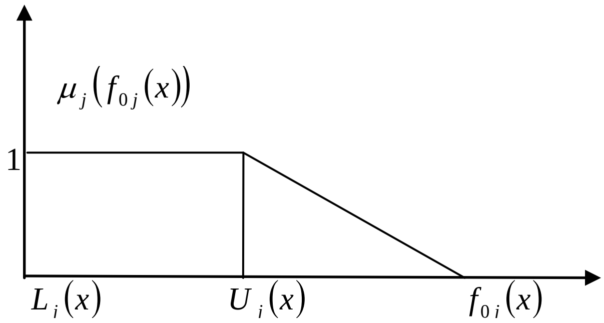

Fig.4.1: Membership function of min. prob.

The fuzzy multi-objective formulation of the problem can be defined as:

$$\left. \begin{array}{l} Max\ \mu_{\tilde{D}}(x) \\ Subject\ \ to\ \ \sum_{i=1}^{2} C_i x_i \leq C; x_i \geq 0\ and\ \ i = 1,2 \end{array} \right\} (10)$$

The problem is to find out the optimal values of (x^*) for this convex-fuzzy decision based on addition operator (like Tewari et al. (1987)). Therefore the problem (10) is reduced according to max-addition operator as:

$$\left. \begin{array}{l} Max\ \mu_D(x^*) = \sum_{j=1}^{p} \mu_j(f_{0j}(x)) = \sum_{j=1}^{p} \dfrac{U_j - (f_{0j}(x))}{U_j - L_j} \\ Subject\ to\ \sum_{i=1}^{2} C_i\, x_i \leq C;\ 0 \leq \mu_j(f_{0j}(x)) \leq 1; x_i \geq 0\ \ and\ i = 1,2. \end{array} \right\} (11)$$

The problem (9) reduces to

$$\left.\begin{array}{l} Max\ \mu_D\left(x^*\right)=\sum_{j=1}^{p}\left\{\dfrac{U_j}{U_j-L_j}-\dfrac{\left(f_{0j}(x)\right)}{U_j-L_j}\right\} \\ Subject\ \ to\ \ f_q(x)\le 1;\ x_i\ge 0\ \ and\ i=1,2 \end{array}\right\} \quad (12)$$

$$\text{where } f_q(x)=\sum_{i=1}^{2}C_i\,x_i\le C$$

The problem (12) maximizes if the function attains the minimum values.

$$F_{oj}(x)=\left\{\frac{\left(f_{0j}(x)\right)}{U_j-L_j}\right\}$$

The fuzzy multi-objective formulation of the standard primal problem can be defined as:

$$\left.\begin{array}{l} Min\qquad \sum_{j=0}^{p}F_{0j}(x) \\ Subject\ to\ \ f_q(x)\le 1,\ q=1,\ldots,p\ \ and\ \ x_i>0,\ i=1,2 \end{array}\right\} \quad (13)$$

$$\text{where }\ f_q(x)=\sum_{i=1}^{2}C_i\,x_i\le C$$

The dual form of the primal GPP which is stated in (13) can be given as:

$$\left.\begin{array}{ll} Max\ \ v(w)=\prod_{q=0}^{k}\prod_{i\in j[q]}\left\{\left(\dfrac{d_i}{w_i}\right)^{w_i}\right\}\prod_{q=1}^{k}\left(\sum_{i\in j[q]}w_i\right)^{\sum_{i\in j[q]}w_i} & (i) \\ Subject\ \ \sum_{i\in[0]}w_i=1 & (ii) \\ \qquad \sum_{q=0}^{k}\sum_{i\in j[q]}p_{ih}\ w_i=0 & (iii) \\ and\quad w_i\ge 0,q=o,1,\cdots,k\ \ and\ \ i=1,2,\ldots,m_k & (iv) \end{array}\right\} \quad (14)$$

The optimal values of the sample sizes of the problems n_h^* can be calculated with the help of the primal–dual relationship theorem (8).

5. NUMERICAL

Data from a sample survey using a double sampling technique were collected from northeastern Colorado in 1979. The cost to obtain one direct and one indirect sample were estimated at \$5.00 and \$0.50, respectively. The total cost of the sample survey for the specified variances is \$ 550.00. The values of the variances and standard errors for each species were from Ahmad and Bonham (1987).

Using the above values the primal problem can be written as:

$$\left.\begin{array}{l} Min\quad \dfrac{6}{x_1}+\dfrac{100}{x_2} \\ Subject\ to\ \ 0.009090\,x_1+0.0009090\,x_2\le 1 \text{ and } x_1>x_2>0. \end{array}\right\} \quad (15)$$

The dual GPP of the above problem (18) is as follows:

$$\left.\begin{array}{ll} Max\ \ v(w_{0i}^*)=\left(\left(6/w_{01}\right)^{w_{01}}\right)\times\left(\left(100/w_{02}\right)^{w_{02}}\right)\times & \\ \qquad\left(\left(0.00909\right)^{w_{11}}\right)\times\left(\left(0.000909\right)^{w_{12}}\right) & \\ \qquad\times(w_{11}+w_{12})\wedge(w_{11}+w_{12}) & (i) \\ Subject\ \ to & \\ \qquad w_{01}+w_{02}=1\ \ \text{(normality condition)} & (ii) \\ \left.\begin{array}{l}-w_{01}-w_{11}=0\\ -w_{02}-w_{12}=0\end{array}\right\}\text{(orthogonality condition)} & (iii) \\ w_{01},w_{02},w_{11},w_{12}\ge 0\}\ \ \text{(positivity condition)} & (iv) \end{array}\right\} \quad (16)$$

Solving the above formulated dual problem (16), we have the corresponding solution as:

$$w_{01}=0.4364917,\ \ w_{02}=0.5635083 \text{ and } v(w_{01}^*)=0.2862617$$

The optimal values x_i^* of the sample sizes of the standard primal problems can be calculated with the help of the primal–dual relationship theorem (8).

$$f_{0j}(x)=w_{0i}^*\ v\left(w^*\right)$$

$$f_{01}(x_1)=w_{01}^*\ v\left(w^*\right)\Rightarrow x_1\cong 48,\ \ f_{02}(x_2)=w_{02}^*\ v\left(w^*\right)\Rightarrow x_2\cong 620$$

and value of objective function is 0.2862617.

Using the above values the primal problem can be written as:

$$\left.\begin{array}{l} Min\quad \dfrac{40}{x_1}+\dfrac{175}{x_2} \\ Subject\ to\ \ 0.009090\,x_1+0.0009090\,x_2\le 1 \text{ and } x_1>x_2>0. \end{array}\right\} \quad (17)$$

The dual GPP of the above problem (17) is as follows:

$$\left.\begin{array}{ll} Max\ \ v(w_{0i}^*)=\left(\left(40/w_{01}\right)^{w_{01}}\right)\times\left(\left(175/w_{02}\right)^{w_{02}}\right)\times & \\ \left(\left(0.00909\right)^{w_{11}}\right)\times\left(\left(0.000909\right)^{w_{12}}\right) & \\ \qquad\times(w_{11}+w_{12})\wedge(w_{11}+w_{12}) & (i) \\ Subject\ \ to & \\ \quad w_{01}+w_{02}=1\quad \text{(normality condition)} & (ii) \\ \left.\begin{array}{l}-w_{01}-w_{11}=0\\ -w_{02}-w_{12}=0\end{array}\right\}\ \text{(orthogonality condition)} & (iii) \\ w_{01},w_{02},w_{11},w_{12}\ge 0\}\ \text{(positivity condition)} & (iv) \end{array}\right\} \quad (18)$$

Solving the above formulated dual problem (18), we have the corresponding solution as:

$$w_{01}=0.6018883\ \ w_{02}=0.3981117 \text{ and } v(w_{01}^*)=1.003673$$

The optimal values x_i^* of the sample sizes of the standard primal problems can be calculated with the help of the primal–dual relationship theorem (8).

$$f_{0j}(x)=w_{0i}^*\ v\left(w^*\right)$$

$$f_{01}(x_1)=w_{01}^*\ v\left(w^*\right)\Rightarrow x_1\cong 66,\ \ f_{02}(x_2)=w_{02}^*\ v\left(w^*\right)\Rightarrow x_2\cong 438$$

and value of objective function is 1.003673.

Using the above values the primal problem can be written as:

$$\left.\begin{array}{l} Min \quad \frac{6}{x_1} + \frac{30}{x_2} \\ Subject\ to\ 0.009090\, x_1 + 0.0009090\, x_2 \le 1 \text{ and } x_1 > x_2 > 0. \end{array}\right\} \quad (19)$$

The dual GPP of the above problem (19) is as follows:

$$\left.\begin{array}{l} Max\ v(w_{0i}^*) = \left((6/w_{01})^{w_{01}}\right)\times\left((30/w_{02})^{w_{02}}\right)\times \\ \left((0.00909)^{w_{11}}\right)\times\left((0.000909)^{w_{12}}\right) \\ \times(w_{11}+w_{12})\wedge(w_{11}+w_{12}) \qquad (i) \\ Subject\ to \\ w_{01}+w_{02}=1 \quad \text{(normality condition)} \quad (ii) \\ \left.\begin{array}{l} -w_{01}-w_{11}=0 \\ -w_{02}-w_{12}=0 \end{array}\right\} \text{(orthogonality condition)} \quad (iii) \\ w_{01}, w_{02}, w_{11}, w_{12} \ge 0\} \quad \text{(positivity condition)} \quad (iv) \end{array}\right\} \quad (20)$$

Solving the above formulated dual problem (20), we have the corresponding solution as:

$$w_{01} = 0.5857864, \quad w_{02} = 0.4142136$$

$$\text{and } v(w_{01}^*) = 0.1589412$$

The optimal values x_i^* of the sample sizes of the standard primal problems can be calculated with the help of the primal–dual relationship theorem (8).

$$f_{0j}(x) = w_{0i}^*\, v(w^*)$$

$$f_{01}(x_1) = w_{01}^*\, v(w^*) \Rightarrow x_1 \cong 64, \quad f_{02}(x_2) = w_{02}^*\, v(w^*) \Rightarrow x_2 \cong 456$$

and value of objective function is 0.1589412

Using the above values the primal problem can be written as:

$$\left.\begin{array}{l} Min \quad \frac{2}{x_1} + \frac{17}{x_2} \\ Subject\ to\ 0.009090\, x_1 + 0.0009090\, x_2 \le 1 \text{ and } x_1 > x_2 > 0. \end{array}\right\} \quad (21)$$

The dual GPP of the above problem (21) is as follows:

$$\left.\begin{array}{l} Max\ v(w_{0i}^*) = \left((2/w_{01})^{w_{01}}\right)\times\left((17/w_{02})^{w_{02}}\right)\times \\ \left((0.00909)^{w_{11}}\right)\times\left((0.000909)^{w_{12}}\right) \\ \times(w_{11}+w_{12})\wedge(w_{11}+w_{12}) \qquad (i) \\ Subject\ to \\ w_{01}+w_{02}=1 \quad \text{(normality condition)} \quad (ii) \\ \left.\begin{array}{l} -w_{01}-w_{11}=0 \\ -w_{02}-w_{12}=0 \end{array}\right\} \text{(orthogonality condition)} \quad (iii) \\ w_{01}, w_{02}, w_{11}, w_{12} \ge 0\} \quad \text{(positivity condition)} \quad (iv) \end{array}\right\} \quad (22)$$

Solving the above formulated dual problem (22), we have the corresponding solution as:

$$w_{01} = 0.5203037, \quad w_{02} = 0.4796963 \text{ and } v(w_{01}^*) = 0.0671552$$

The optimal values x_i^* of the sample sizes of the standard primal problems can be calculated with the help of the primal–dual relationship theorem (8).

$$f_{0j}(x) = w_{0i}^*\, v(w^*)$$

$$f_{01}(x_1) = w_{01}^*\, v(w^*) \Rightarrow x_1 \cong 57, \quad f_{02}(x_2) = w_{02}^*\, v(w^*) \Rightarrow x_2 \cong 527$$

and value of objective function is 0.0671552

Using the above values the primal problem can be written as:

$$\left.\begin{array}{l} Min \quad \frac{3}{x_1} + \frac{21}{x_2} \\ Subject\ to\ 0.009090\, x_1 + 0.0009090\, x_2 \le 1 \text{ and } x_1 > x_2 > 0. \end{array}\right\} \quad (23)$$

The dual GPP of the above problem (23) is as follows:

$$\left.\begin{array}{l} Max\ v(w_{0i}^*) = \left((3/w_{01})^{w_{01}}\right)\times\left((21/w_{02})^{w_{02}}\right)\times \\ \left((0.00909)^{w_{11}}\right)\times\left((0.000909)^{w_{12}}\right) \\ \times(w_{11}+w_{12})\wedge(w_{11}+w_{12}) \qquad (i) \\ Subject\ to \\ w_{01}+w_{02}=1 \quad \text{(normality condition)} \quad (ii) \\ \left.\begin{array}{l} -w_{01}-w_{11}=0 \\ -w_{02}-w_{12}=0 \end{array}\right\} \text{(orthogonality condition)} \quad (iii) \\ w_{01}, w_{02}, w_{11}, w_{12} \ge 0\} \quad \text{(positivity condition)} \quad (iv) \end{array}\right\} \quad (24)$$

Solving the above formulated dual problem (24), we have the corresponding solution as:

$$w_{01} = 0.5444665, \quad w_{02} = 0.4555335$$

$$\text{and } v(w_{01}^*) = 0.0919044$$

The optimal values x_i^* of the sample sizes of the standard primal problems can be calculated with the help of the primal–dual relationship theorem (8).

$$f_{0j}(x) = w_{0i}^*\, v(w^*)$$

$$f_{01}(x_1) = w_{01}^*\, v(w^*) \Rightarrow x_1 \cong 60, \quad f_{02}(x_2) = w_{02}^*\, v(w^*) \Rightarrow x_2 \cong 501$$

and value of objective function is 0.09199044

Now the pay-off matrix of the above problems is given below:

	$f_{01}(x)$	$f_{02}(x)$	$f_{03}(x)$	$f_{04}(x)$	$f_{05}(x)$
$x^{(1)}$	0.2862617	1.115591	0.1733871	0.06908602	0.09637097
$x^{(2)}$	0.3192196	1.003673	0.1599402	0.06911582	0.09339975
$x^{(3)}$	0.3130482	1.008772	0.1589412	0.06853070	0.09292763
$x^{(4)}$	0.2950165	1.033823	0.1621892	0.06726547	0.09247978
$x^{(5)}$	0.2996008	1.015968	0.1598802	0.09199044	0.09199044

The lower and upper bond of $f_{01}(x), f_{02}(x), f_{03}(x), f_{04}(x)$ and $f_{05}(x)$ can be obtained from the pay-off matrix as:

$$0.22862617 \le f_{01}(x) \le 0.3192196,\ 1.003673 \le f_{02}(x) \le 1.115591,$$

$$0.1589412 \le f_{03}(x) \le 0.1733871, 0.0671552 \le f_{04}(x) \le 0.6911582$$

$$\text{and } 0.199044 \le f_{05}(x) \le 0.09637097.$$

Let $\mu_1(x), \mu_2(x), \mu_3(x), \mu_4(x)$, and $\mu_5(x)$ be the fuzzy membership function of the objective function $f_{01}(x), f_{02}(x), f_{03}(x), f_{04}(x)$ and $f_{05}(x)$ respectively and they are defined as:

$$\mu_1(x)=\begin{cases}1 & ,if\ f_{01}(x)\le 0.2862617\\ \dfrac{0.3192196-f_{01}(x)}{0.0329579} & ,if\ 0.2862617\le f_{01}(x)\le 0.3192196\\ 0 & ,if\ f_{01}(x)\ge 0.3192196\end{cases}$$

$$\mu_2(x)=\begin{cases}1 & ,if\ f_{02}(x)\le 1.003673\\ \dfrac{1.115591-f_{02}(x)}{0.111918} & ,if\ 1.003673\le f_{02}(x)\le 1.115591\\ 0 & ,if\ f_{02}(x)\ge 1.115591.\end{cases}$$

$$\mu_3(x)=\begin{cases}1 & ,if\ f_{03}(x)\le 0.1589412\\ \dfrac{0.1733871-f_{03}(x)}{0.0144459} & if\ 0.1589412\le f_{03}(x)\le 0.1733871\\ 0 & if\ f_{03}(x)\ge 0.1733871\end{cases}$$

$$\mu_4(x)=\begin{cases}1 & ,if\ f_{04}(x)\le 0.0671552\\ \dfrac{0.06911582-f_{04}(x)}{0.00196062} & ,if\ 0.06715520\le f_{04}(x)\le 0.06911582\\ 0 & ,if\ f_{04}(x)\ge 0.06911582.\end{cases}$$

$$\mu_5(x)=\begin{cases}1 & ,if\ f_{05}(x)\le 0.09199044\\ \dfrac{0.09637097-f_{05}(x)}{0.00438053} & ,if\ 0.09199044\le f_{05}(x)\le 0.09637097\\ 0 & ,if\ f_{05}(x)\ge 0.09637097.\end{cases}$$

On applying the max-addition operator, the MOGPP, reduces to the crisp problem and the final problem is given as:

$$\left.\begin{array}{l} Min\ \dfrac{2659.5934}{x_1}+\dfrac{320502.411}{x_2}\\ Subject\ \ to\ 0.00909\,x_1+0.000909\,x_2\le 1\ \ and\ \ x_1,x_2\ge 0.\end{array}\right\} \qquad (25)$$

Degree of Difficulty of the problem (26) is =(4-(2+1) =1

Hence the dual problem of the above problem (26) is given as:

$$\left.\begin{array}{l} Max\ \ v(w_{0i}^{*})=\left((2659.5934/w_{01})^{w_{01}}\right)\\ \times\left((320502.411/w_{02})^{w_{02}}\right)\times\\ \left(\left(\dfrac{0.00909}{w_{11}}\right)^{w_{11}}\right)\times\left(\left(\dfrac{0.000909}{w_{12}}\right)^{w_{12}}\right)\\ ((w_{11}+w_{12})\wedge(w_{11}+w_{12})); \qquad (i)\\ Subject\ \ to\ w_{01}+w_{02}=1;\ (\text{normality condition})\ (ii)\\ \left.\begin{array}{l}-w_{01}+w_{11}=0\\ -w11+w_{12}=0\end{array}\right\}(\text{orthogonality condition})\ (iii)\\ w_{01},w_{02},w_{11},w_{12},\ge 0\}(\text{positivity condition})\ (iv)\end{array}\right\} \qquad (26)$$

For orthogonality condition defined in expression 26 (iii) are evaluated with the help of the payoff matrix which is defined below:

$$\begin{pmatrix}-1 & 0 & 1 & 0\\ 0 & -1 & 0 & 1\end{pmatrix}\begin{pmatrix}w_{01}\\ w_{02}\\ w_{11}\\ w_{12}\end{pmatrix}=\begin{Bmatrix}-w_{01}+w_{11}=0\\ -w_{02}+w_{12}=0\end{Bmatrix}$$

After solving the formulated dual problem (26) using lingo software we obtain the following values of the dual variables which are given as:

$w_{01}=0.2236423;\ w_{02}=0.7763577$ and $v(w_{0i}^{*})=483.3608$

The optimal values x_i^{*} of the sample sizes of the standard primal problems can be calculated with the help of the primal–dual relationship theorem (11).

$$f_{0j}(x)=w_{0i}^{*}\,v(w^{*})$$

$$f_{01}(x_1)=w_{01}^{*}\,v(w^{*})\Rightarrow x_1\cong 25$$

$$f_{02}(x_2)=w_{02}^{*}\,v(w^{*})\Rightarrow x_2\cong 854$$

The optimal values and the objective function value are given as: $x_1^{*}=25, x_2^{*}=854$ and 483.3608.

6. CONCLUSIONS

This paper gives the insightful study of the problem of Multivariate Double sampling Design which is formulated as a convex MOGPP with non-linear objective function and linear constraints. The fuzzy programming approach is used for converting the (MOGPP) into Single Objective Geometric Programming Problem (SOGPP) with the help of membership function. The formulated SOGPP has been solved with the help of LINGO Software and the dual solution is obtained. The optimum allocations are obtained with the help of primal-dual relationship theorem along with corresponding dual variables. A numerical example is illustrated to establish the practical utility of the given method in multivariate two-stage stratified sample surveys. The researcher can adopt this method for obtaining the solution of very complicated convex programming problem even with multi-stage sample survey problems.

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