SOME NEW OUTCOMES ON PRIME LABELING

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Abstract:

In this paper, we show that Mongolian tent graph $M(m,n)$,Umbrella graph $U(m,n)$, where $m = 2$ admits prime labeling. We also prove that the graph $C_n + K_1$ is a prime graph when n is even and is not a prime graph when n is odd.

Keywords:

Prime labeling, Mongolian tent, Umbrella graph, the graph $C_n + K_1$

Introduction:

We consider simple, finite, connected and undirected graph $G = (V, E)$ with p vertices and q edges. For all other standard terminology and notations, we refer to J.A. Bondy and U.S.R. Murthy [1]. We give a brief summary of definitions and other information which are useful for the present investigation. A current survey of various graph labeling problem can be found in [4] (Gallian J, 2015)

Following are the common features of any graph Labeling problem.

- ✓ A set of numbers from which vertex labels are assigned.
- ✓ A rule that assigns value to each edge.
- ✓ A condition that these values must satisfy.

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by A. Tout [8]. Many researchers have studied prime graph for example in H.C. Fu [3] have proved that path P_n on n vertices is a prime graph.

T. Deretsky [2] have proved that the cycle C_n on n vertices is a prime graph. S.M. Lee [5] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Entringer conjectured that all tress have prime labeling, which is not settled till today. The prime labeling for planner grid is investigated by M. Sundaram [7]. S.K. Vaidhya and K.K. Kanmani have proved that the prime labeling for some cycle related graphs [9]. S. Meena and K. Vaithilingam, Prime Labeling for some Helm related graphs [6].

Definition 1.1: If the vertices of the graph are assigned values subject to certain conditions then it is known as (vertex) graph labeling.

Definition 1.2: Let $G = \big(V(G),\ E(G)\big)$ be a graph with n vertices. A bijection $f : V(G) \to \{1, 2, \dots n\}$ is called a prime labeling if for each edge $e = uv$, $gcd\big(f(u), f(v)\big) = 1$. A graph which admits prime labeling is called a prime graph.

Definition 1.3: An independent set of vertices in a graph G is a set of mutually non-adjacent vertices.

Definition 1.4: A Mongolian tent as a graph obtained from $P_m \times P_n$ by adding one extra vertex above the grid and joining every other vertex of the top row of $P_m \times P_n$ to the new vertex.

Definition 1.5: For any integers $m > 2$ and $n > 1$, the Umbrella graph $U(m,n)$ whose vertex set and edge set is defined as

$$V\big(U(m,n)\big) = \{x_1, x_2, \dots \dots . x_m;\ y_1, y_2, \dots . . y_n\}$$

$$E\big(U(m,n)\big) = \begin{cases} (x_i, x_{i+1}), & for\ \ i = 1, 2, \dots . . m-1 \\ (y_i, y_{i+1}), & for\ \ i = 1, 2, \dots . . n-1 \\ (x_i, y_1), & for\ \ i = 1, 2, \dots . . m \end{cases}$$

Definition 1.6: The sum of two graphs C_n and K_1, $C_n + K_1$ is obtained by joining a vertex of K_1 with every vertex of C_n with an edge.

2. Main Results

Theorem 2.1:

For any integer $m = 2$ and $n > 2$, the Mongolian tent admits prime labeling.

Proof:

Let $M(m,n)$ be a Mongolian tent graph and let $m = 2$

Consider $M(2,n)$ with the vertex set $\{u, x_{1,1},\ x_{1,2}, \dots, x_{1,n};\ x_{2,1},\ x_{2,2}, \dots, x_{2,n}\}$

where u is the apex vertex. Then $|V(M(m,n))| = 2n + 1$

The ordinary labeling for $M(2,4)$ is given below

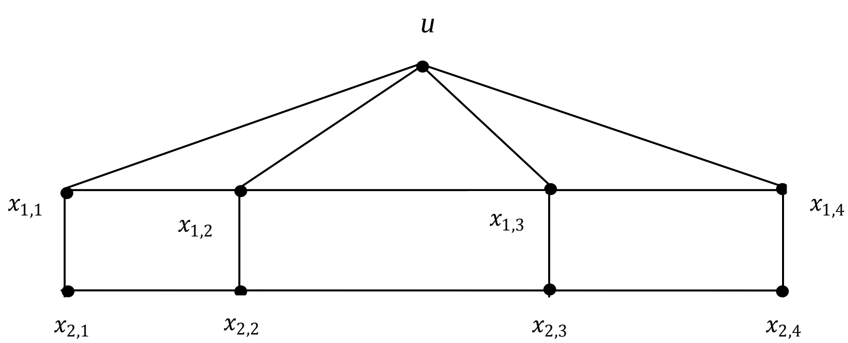

Figure 1: Monglian tent $M(2,4)$

Define a bijection $f: V(M(m,n)) \to \{1, 2, \dots, 2n+1\}$ by

Case (i): $n \neq 3k$ for any integer k

Let $f(u) = 1$

$f\big(x_{1,i}\big) = i + 1$; for $1 \leq i \leq n$

and $f(x_{2,i}) = (2n+2) - i$; for $1 \leq i \leq n$

Case (ii): $n = 3k$ for any integer k

Let $f(u) = 1$

$f(x_{1,1}) = 2n+1$ and $f(x_{1,i}) = i$ for $2 \leq i \leq n$

and $f(x_{2,i}) = (2n+1) - i$; for $1 \leq i \leq n$

In view of above defined labeling pattern, f satisfy the condition of prime labeling

Then $M(2,n)$ admits prime labeling

Hence, Mongolian tent $M(2,n)$ is a prime graph.

Illustration:

Case (i): Let $n = 7$, for $n \neq 3k$ where k is any integer

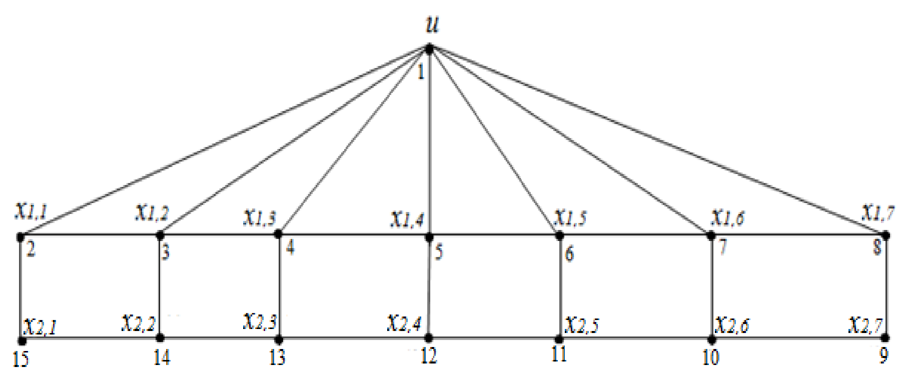

Figure 2: Mongolian tent $M(2,7)$ and its prime labeling

Case (ii): Let $n = 9$, for $n = 3k$ where k is any integer

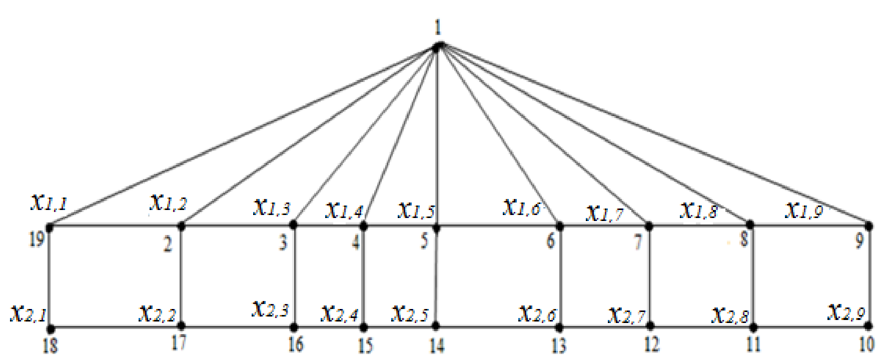

Figure 3: Mongolian tent $M(2,9)$ and its prime labeling

Special case

Let $n = 12$

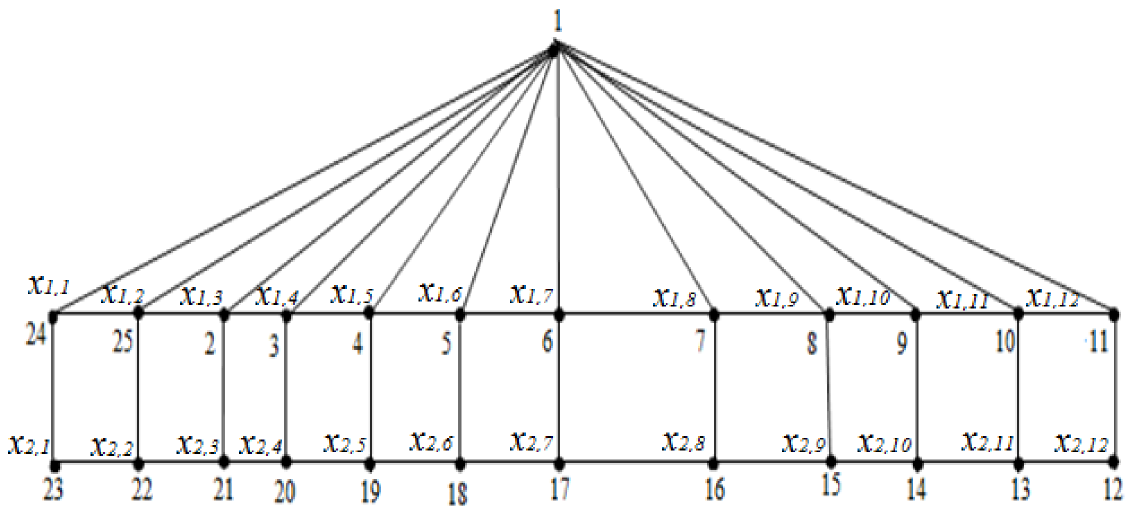

Figure 4: Mongolian tent $M(2, 12)$ and its prime labeling

Theorem 2.2:

For any integer $m > 2, n > 1$ the Umbrella graph $U(m, n)$ admits prime labeling.

Proof:

The graph $U(m, n)$ has $m + n$ vertices and $2m + n - 2$ edges

Define a bijection $f: V\big(U(m, n)\big) \rightarrow \{1, 2, \dots, m + n\}$ by

$f(y_i) = i, \quad\quad for \quad 1 \leq i \leq n$

and $f(x_i) = n + i, \quad for \quad 1 \leq i \leq m$

In view of above defined labeling pattern, f satisfy the condition of prime labeling

Therefore, $U(m, n)$ admits prime labeling.

Hence, $U(m, n)$ is a prime graph.

Illustration:

The following figure exhibit prime labeling for $U(6,4)$

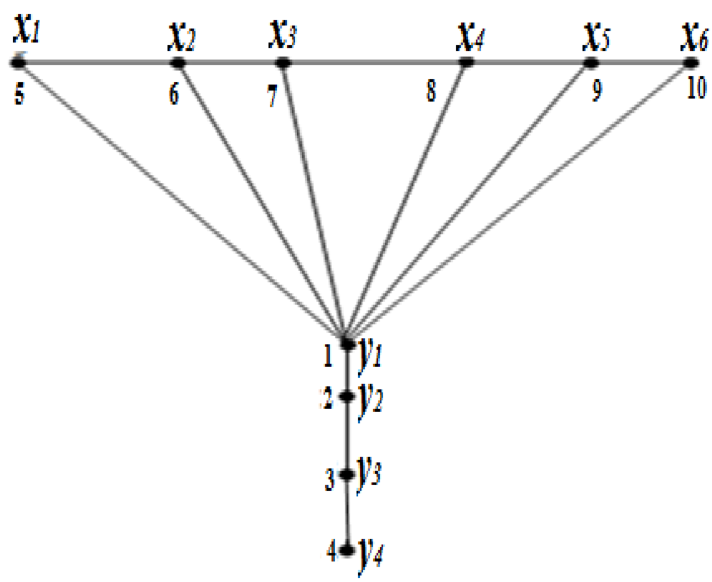

Figure 5: Umbrella graph $U(6,4)$ and its prime labeling

Theorem 2.3:

The graph $C_n + K_1$ admits prime labeling when n is even

Proof:

The graph $C_n + K_1$ has $n+1$ vertices and $2n$ edges

Let v be vertex of K_1 and $v_1, v_2, \ldots \ldots . v_n$ be the vertices of the cycle C_n

Define a bijection $f: V(C_n + K_1) \rightarrow (1, 2, \ldots \ldots n+1)$ by

$$f(u) = 1$$

$$f(v_i) = i + 1, \quad \text{for i} \leq i \leq n$$

In view of above defined labeling pattern, f satisfy the condition of prime labeling.

Therefore, $C_n + K_1$ admits prime labeling when n is even.

∴ The sum of two graphs C_n and K_1, $C_n + K_1$ is a prime graph.

Illustration:

The Prime labeling for $C_8 + K_1$ is given in following figure 6

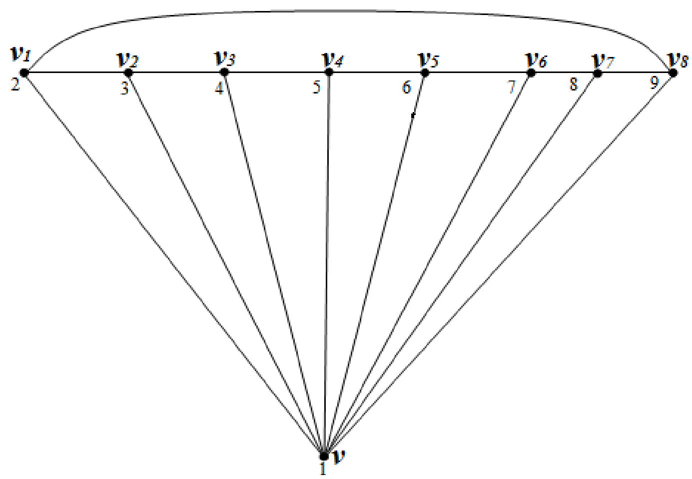

Figure 6: The graph $C_8 + K_1$ and its prime labeling

Theorem 4:

The graph $C_n + K_1$ is not a prime graph when n is odd.

Proof:

Let K_1 is a graph with single vertex u and C_n be a cycle with n vertices $v_1, v_2, \ldots \ldots v_n$ and n is odd. Let $G = K_1 + C_n$ be a graph obtained by joining the vertex of K_1 to all the vertices of C_n. Then $|V(G)| = |V(K_1) \cup V(C_n)| = n + 1$.

We have to label $n + 1$ vertices of G with the integers $1, 2, \ldots \ldots n + 1$. Since the vertex of u be adjacent to all vertices of $v_1, v_2, \ldots \ldots v_n$ of C_n. We shall label the vertex u by 1. Remaining integers $2, 3, \ldots \ldots n + 1$ to be assigned to the vertices $v_1, v_2, \ldots \ldots v_n$ in the linear ordering (i.e.) $2, 3, \ldots \ldots n + 1$. Label the vertices $v_1, v_2, \ldots \ldots v_n$ by $2, 3, \ldots \ldots n + 1$ respectively

Since $v_n v_1 \in G$ then $\gcd(f(v_1), f(v_n)) = \gcd(2, n + 1)$

$= \gcd(2, even\ integer)$ since n is odd, $n + 1$ is even

$\neq 1$

Therefore, v_1 and v_n are not relatively prime

Hence $G = K_1 + C_n$ is not a prime graph when n is odd.

3.Conclusion:

As all graphs are not prime graphs it is very interesting to investigate graphs which admits prime labeling. It is possible to investigate similar results for other graph families in the context of different labeling techniques is an open problem for further research.

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