BALANCED BIPOLAR INTUITIONISTIC FUZZY GRAPHS

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Abstract:

In this paper, we discuss balanced bipolar intuitionistic fuzzy graphs and study some of their properties

1.Introduction

Graph theory is developed when Euler gave the solution to the famous Konigsberg bridge problem in 1736. Graph theory is very useful as a branch of combinatorics in the field of geometry, algebra, number theory, topology, operations research, optimization and computer science. Rosenfeld[2] developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Later on, Bhattacharya[3] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and Peng[4]. The complement of a fuzzy graph was defined by Mordeson and Nair[5] and further studied by Sunitha and Vijayakumar [6]. Akram[7] has introduced Bipolar fuzzy graphs and investigated their properties. Balanced graph first arose in the study of random graphs and balanced IFG defined here is based on density functions. A graph with maximum density is complete and graph with minimum density is a null graph. There are several papers written on balanced extension of graph [8] which has tremendous applications in artificial intelligence, signal processing, robotics, computer networks and decision making Al-Hawary [9] introduced the concept of balanced fuzzy graphs and studied some operations of fuzzy graphs. Shannon and Atanassov[10] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs, and investigated some of their properties. Parvathi et al[11].defined operations on intuitionistic fuzzy graphs.Karunambigai et al [2] introduced balanced intuitionistic fuzzy graphs and studied some of their properties.In 2015, D.Ezhilmaran and K.Sankar[16] have introduced bipolar intuitionistic fuzzy graphs. In this paper, we discussed balanced bipolar intuitionistic fuzzy graphs and study some of their properties.

2.Preliminaries

In this section, we first review some definitions of undirected graphs that are necessary for this paper.

Definition2.1[1].

Recall that a graph is an ordered pair $G^*=(V,E)$, where V is the set of vertices of $G^*$and E is the set of edges of G^*. Two vertices x and y in an undirected graph G^* are said to be adjacent in G^* if $\{x,y\}$ is an edge of G^*. A simple graph is an undirected graph that has no loops and no more than one edge between any two different vertices.

Definition 2.2[1].

A subgraph of a graph$G^*=(V,E)$ is a graph $H=(W,F)$ where $W\subseteq V$and $F\subseteq E$.

Definition 2.3[1].

The complementary graph $\bar{G}^*$of a simple graph has the same vertices as G^*. Two vertices are adjacent in $\bar{G}^*$ if and only if they are not adjacent in G^*.

Definition 2.4[1].

Consider the Cartesian product $G^* = G_1^* \times G_2^* = (V,E)$of graphs G_1^* and G_2^*. Then $V = V_1 \times V_2$ and

$$E = \{(x,x_2)(x,y_2)|x \in V_1, \quad x_2y_2 \in E_2\} \cup \{(x_1,z)(y_1,z)|z \in V_2, \quad x_1y_1 \in E_1\}.$$

Definition 2.5[1].

Let $G_1^* = (V_1,E_1)$ and $G_2^* = (V_2,E_2)$ be two simple graphs. Then the composition of graphs G_1^* and G_2^* is denoted by $G_1^* o G_2^*=(V_1 \times V_2, E^0)$, where $E^0 = E \cup$

$\{(x_1, x_2)(y_1, y_2) | x_1 y_1 \in E_1, x_2 \neq y_2\}$ and E is defined in $G_1^* \times G_2^*$. Note that $G_1^* o G_2^* \neq G_2^* o G_1^*$.

Definition 2.6[1].

The union of two simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ is the simple graph with the vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$. The union of G_1^* and G_2^* is denoted by $G^* = G_1^* \cup G_2^* = (V_1 \cup V_2, E_1 \cup E_2)$.

Definition 2.7[1].

The join of two simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ is the simple graph with the vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2 \cup E'$, where E' is the set of all edges joining the nodes of V_1 and V_2 and assume that $V_1 \cap V_2 \neq \emptyset$. The join of G_1^* and G_2^* is denoted by $G^* = G_1^* + G_2^* = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$.

Definition 2.8[13].

A fuzzy subset μ on a set X is a map $\mu : X \to [0,1]$. A map $\vartheta : X \times X \to [0,1]$ is called a fuzzy relation on X if $\vartheta(x, y) \leq \mu(x) \wedge \mu(y)$ for all $x,y \in X$. A fuzzy relation ϑ is symmetric if $\vartheta(x, y) = \vartheta(y, x)$ for all $x,y \in X$.

Definition 2.9[14].

Let X be a non empty set. A bipolar fuzzy set B in X is an object having the form $B = \{(x, \mu^P(x), \mu^N(x)) | x \in X\}$ where $\mu^P : X \to [0,1]$ and $\mu^N : X \to [-1,0]$ are mappings.

Definition 2.10[15].

Let X be a non empty set. A intuitionistic fuzzy set $B = \{(x, \mu(x), \gamma(x)) | x \in X\}$ Where $\mu : X \to [0,1]$ and $\gamma : X \to [0,1]$ are mapping such that $0 \leq \mu(x) + \gamma(x) \leq 1$.

3.Bipolar intuitionistic fuzzy graphs

Definition 3.1[16]

Let X be a non empty set. A bipolar intuitionistic fuzzy set $B = \{(x, \mu^P(x), \mu^N(x), \gamma^P(x), \gamma^N(x)) | x \in X\}$ where $\mu^P : X \to [0,1], \mu^N : X \to [-1,0] \gamma^P : X \to [0,1], \gamma^N : X \to [-1,0]$ are the mappings such that $0 \leq \mu^P(x) + \gamma^P(x) \leq 1, -1 \leq \mu^N(x) + \gamma^N(x) \leq 0$. We use the positive membership degree $\mu^P(x)$ to denote the satisfaction degree of an element x to the property crossponding to a bipolar intuitionistic fuzzy set B and the negative membership degree $\mu^N(x)$ to denote the satisfaction degree of an element x to some implicit counter property corresponding to a bipolar intuitionistic fuzzy set. Similarly we use the positive nonmembership degree $\gamma^P(x)$ to denote te satisfaction degree of an element x to the property corresponding to a bipolar intuitionistic fuzzy set and the negative nonmembership degree $\gamma^N(x)$ to denote the satisfaction degree of an element x to some implicit counter property crossponding to a bipolar intuitionistic fuzzy set. If $\mu^P(x) \neq 0, \mu^N(x) = 0$ and $\gamma^P(x) = 0, \gamma^N(x) = 0$ it is the situation that x regarded as having only the positive membership property of a bipolar intuitionistic fuzzy set. If $\mu^P(x) = 0, \mu^N(x) \neq 0$ and $\gamma^P(x) = 0, \gamma^N(x) = 0$ it is the situation that x regarded as having only the negative membership property of a bipolar intuitionistic fuzzy set. $\mu^P(x) = 0, \mu^N(x) = 0$ and $\gamma^P(x) \neq 0, \gamma^N(x) = 0$ it is the situation that x regarded as having only the positive nonmembership property of a bipolar intuitionistic fuzzy set. $\mu^P(x) = 0, \mu^N(x) = 0$ and $\gamma^P(x) = 0, \gamma^N(x) \neq 0$ it is the situation that x regarded as having only the negative nonmembership property of a bipolar intuitionistic fuzzy set. It is possible for an element x to be such that $\mu^P(x) \neq 0, \mu^N(x) \neq 0$ and $\gamma^P(x) \neq 0, \gamma^N(x) \neq 0$ when the membership and nonmembership function of the property overlaps with its counter properties over some portion of X.

Definition 3.2[16].

Let X be a non empty set. Then we call a mapping $(\mu_A^P, \mu_A^N, \gamma_A^P, \gamma_A^N) : X \times X \to [0,1] \times [-1,0] \times [0,1] \times [-1,0]$ a bipolar intuitionistic fuzzy relation on X such that $\mu_A^P(x, y) \in [0,1]$, $\mu_A^N(x, y) \in [-1,0]$, $\gamma_A^P(x, y) \in [0,1]$, $\gamma_A^N(x, y) \in [-1,0]$

Definition 3.3[16].

Let $A = (\mu_A^P(x), \mu_A^N(x), \gamma_A^P(x), \gamma_A^N(x))$ and $B = (\mu_B^P(x), \mu_B^N(x), \gamma_B^P(x), \gamma_B^N(x))$ be bipolar intuitionistic fuzzy sets on a set X. If $A = (\mu_A^P(x), \mu_A^N(x), \gamma_A^P(x), \gamma_A^N(x))$ is a bipolar intuitionistic fuzzy relation on $B = (\mu_B^P(x), \mu_B^N(x), \gamma_B^P(x), \gamma_B^N(x))$ if $\mu_A^P(x, y) \leq \mu_B^P(x) \wedge \mu_B^P(y)$, $\mu_A^N(x, y) \geq \mu_B^N(x) \vee \mu_B^N(y)$, $\gamma_A^P(x, y) \geq \gamma_B^P(x) \vee \gamma_B^P(y)$, $\gamma_A^N(x, y) \leq \gamma_B^N(x) \wedge \gamma_B^N(x)$ for all $x, y \in X$. A bipolar intuitionistic fuzzy relation A on X is called symmetric if $\mu_A^P(x, y) = \mu_A^P(y, x)$, $\mu_A^N(x, y) = \mu_A^N(y, x)$ and $\gamma_A^P(x, y) = \gamma_A^P(y, x)$, $\gamma_A^N(x, y) = \gamma_A^N(y, x)$ for all $x, y \in X$.

Definition 3.4[16].

For any two bipolar intuitionistic fuzzy sets $A = (\mu_A^P(x), \mu_A^N(x), \gamma_A^P(x), \gamma_A^N(x))$ and

$$B = (\mu_B^P(x), \mu_B^N(x), \gamma_B^P(x), \gamma_B^N(x))$$
$$(A \cap B)(x) = (\mu_A^P(x) \wedge \mu_B^P(x), \mu_A^N(x) \vee \mu_B^N(x))$$
$$(A \cup B)(x) = (\mu_A^P(x) \vee \mu_B^P(x), \mu_A^N(x) \wedge \mu_B^N(x))$$
$$(A \cap B)(x) = (\gamma_A^P(x) \vee \gamma_B^P(x), \gamma_A^N(x) \wedge \gamma_B^N(x))$$
$$(A \cup B)(x) = (\gamma_A^P(x) \wedge \gamma_B^P(x), \gamma_A^N(x) \vee \gamma_B^N(x))$$

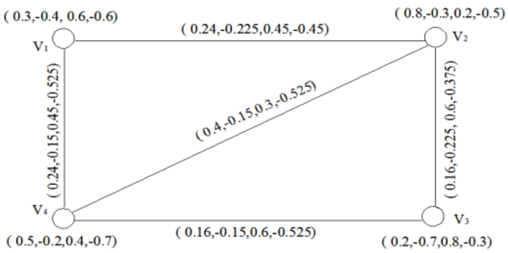
Definition 3.5[16].

A bipolar intuitionistic fuzzy graph of a graph $G^*=(V,E)$ is a pair $G(A,B)$ where $A=(\mu_A^P,\mu_A^N,\gamma_A^P,\gamma_A^N)$ is a bipolar intuitionistic fuzzy set in V and $B=(\mu_B^P,\mu_B^N,\gamma_B^P,\gamma_B^N)$ is a bipolar intuitionistic fuzzy set in $V\times V$ such that

$$\mu_B^P(xy)\le\left(\mu_A^P(x)\wedge\mu_A^P(y)\right) for all xy\in V\times V,$$
$$\mu_B^N(xy)\ge\left(\mu_A^N(x)\vee\mu_A^N(y)\right) for all xy\in V\times V,$$
$$\gamma_B^P(xy)\le(\gamma_A^P(x)\vee\gamma_A^P(y)) for all xy\in V\times V,$$

$\gamma_B^N(xy)\ge(\gamma_A^N(x)\wedge\gamma_A^N(y)) for all xy\in V\times V$ and
$$\mu_B^P(xy)=\mu_B^N(xy)=0\ for all xy\in V\times V-E,$$
$$\gamma_B^P(xy)=\gamma_B^N(xy)=0\ for all xy\in V\times V-E.$$

Example 3.6.



G

A bipolar intuitionistic fuzzy graph (BIFG) is of the form $G=(V,E)$ said to be mini-max BIFG if

1. $V=\{v_0,v_1,\dots v_n\}$ such that $\mu_1^P:V\to[0,1]$, $\mu_1^N:V\to[-1,0]$ and $\gamma_1^P:V\to[0,1]$, $\mu_1^N:V\to[-1,0]$ denotes the degree of positive membership, negative membership and degree of positive nonmembership, negative nonmembership of the element $v_i\in V$ respectively and $0\le\mu_1^P+\gamma_1^P\le1, -1\le\mu_1^N+\gamma_1^N\le0$ for every $v_i\in V(i=1,2,3\dots n)$.

2. $E\subseteq V\times V$ where $\mu_2^P:V\times V\to[0,1]$, $\mu_2^N:V\times V\to[-1,0]$ and $\gamma_2^P:V\times V\to[0,1]$, $\gamma_2^N:V\times V\to[-1,0]$ are such that $\mu_2^P(v_i,v_j)\le\left(\mu_1^P(v_i)\wedge\mu_1^P(v_j)\right)$,

$\mu_2^N(v_i,v_j)\ge\left(\mu_1^N(v_i)\vee\mu_1^N(v_j)\right)$ and

$\gamma_2^P(v_i,v_j)\ge\left(\gamma_1^P(v_i)\vee\gamma_1^P(v_j)\right)$,

$\gamma_2^N(v_i,v_j)\le\left(\gamma_1^N(v_i)\wedge\gamma_1^N(v_j)\right)$ denotes the degree of positive, negative membership and degree of positive, negative non membership of the edge $(v_i,v_j)\in E$ respectively, where $0\le\mu_2^P(v_i,v_j)+\gamma_2^P(v_i,v_j)\le1$, $-1\le\mu_2^N(v_i,v_j)+\gamma_2^N(v_i,v_j)\le0$ for every $(v_i,v_j)\in E$.

A BIFG $H=(V',E')$ is said to be BIF sub graph of $G=(V,E)$ if

(i) $V'\subseteq V$ where $\mu_1^P(v_i')=\mu_1^P(v_i)$, $\mu_1^N(v_i')=\mu_1^N(v_i)$ and $\gamma_1^P(v_i')=\gamma_1^P(v_i)$, $\gamma_1^N(v_i')=\gamma_1^N(v_i)$ for all $v_i'\in V'$, $v_i'=v_i\in V$ $i=1,2,3\dots n$.

(ii) $\mu_2^P(v_i',v_j')=\mu_2^P(v_i,v_j)$, $\mu_2^N(v_i',v_j')=\mu_2^N(v_i,v_j)$ and $\gamma_2^P(v_i',v_j')=\gamma_2^P(v_i,v_j)$, $\gamma_2^N(v_i',v_j')=\gamma_2^N(v_i,v_j)$ for all $(v_i',v_j')\in E'$, $(v_i',v_j')=(v_i,v_j)\in E$ $i,j=1,2,3\dots n$.

A BIFG, $G=(V,E)$ is said to be complete BIFG if
$\mu_2^P(v_i,v_j)=\left(\mu_1^P(v_i)\wedge\mu_1^P(v_j)\right)$,
$\mu_2^N(v_i,v_j)=\left(\mu_1^N(v_i)\vee\mu_1^N(v_j)\right)$ and

$\gamma_2^P(v_i,v_j)=\left(\gamma_1^P(v_i)\vee\gamma_1^P(v_j)\right)$,
$\gamma_2^N(v_i,v_j)=\left(\gamma_1^N(v_i)\wedge\gamma_1^N(v_j)\right)$ for every $v_i,v_j\in V$.

A BIFG, $G=(V,E)$ is said to be strong BIFG if
$\mu_2^P(v_i,v_j)=\left(\mu_1^P(v_i)\wedge\mu_1^P(v_j)\right)$,
$\mu_2^N(v_i,v_j)=\left(\mu_1^N(v_i)\vee\mu_1^N(v_j)\right)$ and

$\gamma_2^P(v_i,v_j)=\left(\gamma_1^P(v_i)\vee\gamma_1^P(v_j)\right)$,
$\gamma_2^N(v_i,v_j)=\left(\gamma_1^N(v_i)\wedge\gamma_1^N(v_j)\right)$ for every $(v_i,v_j)\in E$.

The complement of a BIFG, $G=(V,E)$ is a BIFG $\bar{G}=(\bar{V},\bar{E})$ where

(i) $\bar{V}=V$

(ii) $\overline{\mu_1^P}(v_i)=\mu_1^P(v_i)$,

$\overline{\mu_1^N(v_i)}=\mu_1^N(v_i)$,

$\overline{\gamma_1^P}(v_i)=\gamma_1^P(v_i)$,

$\overline{\gamma_1^N}(v_i)=\gamma_1^N(v_i)$ for all $v_i, i=1,2,3\dots,n$.

(iii) $\overline{\mu_2^P}(v_i,v_j)=\left(\mu_1^P(v_i)\wedge\mu_1^P(v_j)\right)-\mu_2^P(v_i,v_j)$,

$\overline{\mu_2^N}(v_i,v_j)=\left(\mu_1^N(v_i)\vee\mu_1^N(v_j)\right)-\mu_2^N(v_i,v_j)$,

$\overline{\gamma_2^P}(v_i,v_j)=\left(\gamma_1^P(v_i)\wedge\gamma_1^P(v_j)\right)-\gamma_2^P(v_i,v_j)$,

$$\overline{\gamma_2^N}(v_i, v_j) = \left(\gamma_1^N(v_i) \wedge \gamma_1^N(v_j)\right) - \gamma_2^N(v_i, v_j)$$

for all v_i, v_j $i, j = 1,2,3 \ldots, n$.

A BIFG, $G = (V, E)$ is said to be regular BIFG if all the vertices have the same closed neighbourhood degree.

The density of a complete fuzzy graph $G(\sigma, \mu)$ is $D(G) = 2\left(\frac{\sum_{u,v \in V} \mu(u,v)}{\sum_{u,v \in V} \wedge(\sigma(u), \sigma(v))}\right)$.

Consider the two BIFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. An isomorphism between two BIFGs G_1 and G_2, denoted by $G_1 \cong G_2$, is a bijective map $h: V_1 \to V_2$ which satisfies the following $\mu_1^P(v_i) = \mu_1^P(h(v_i))$, $\mu_1^N(v_i) = \mu_1^N(h(v_i))$, $\gamma_1^P(v_i) = \gamma_1^P(h(v_i))$, $\gamma_1^N(v_i) = \gamma_1^N(h(v_i))$ and $\mu_2^P(v_i, v_j) = \mu_2^P\left(h(v_i), h(v_j)\right)$,

$\mu_2^N(v_i, v_j) = \mu_2^N\left(h(v_i), h(v_j)\right)$,

$\gamma_2^P(v_i, v_j) = \gamma_2^P\left(h(v_i), h(v_j)\right)$,

$\gamma_2^N(v_i, v_j) = \gamma_2^N\left(h(v_i), h(v_j)\right)$ for every $v_i, v_j \in V$.

4. Balanced Bipolar Intuitionistic Fuzzy Graphs

Definition 4.1.

The density of a BIFG $G = (V, E)$ is $D(G) = \left(D_\mu^P(G), D_\mu^N(G), D_\gamma^P(G), D_\gamma^N(G)\right)$ where $D_\mu^P(G)$ is defined by $D_\mu^P(G) = \frac{2\sum_{u,v \in V}\left(\mu_2^P(u,v)\right)}{\sum_{(u,v)\in E}\left(\mu_1^P(u) \wedge \mu_1^P(v)\right)}$, for $u, v \in V$

$D_\mu^N(G)$ is defined by $D_\mu^N(G) = \frac{2\sum_{u,v \in V}\left(\mu_2^N(u,v)\right)}{\sum_{(u,v)\in E}\left(\mu_1^N(u) \vee \mu_1^N(v)\right)}$,

for $u, v \in V$

$D_\gamma^P(G)$ is defined by $D_\gamma^P(G) = \frac{2\sum_{u,v \in V}\left(\gamma_2^P(u,v)\right)}{\sum_{(u,v)\in E}\left(\gamma_1^P(u) \vee \gamma_1^P(v)\right)}$,

for $u, v \in V$ and

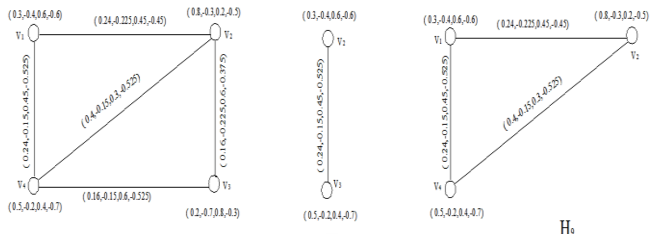
$D_\gamma^N(G)$ is defined by $D_\gamma^N(G) = \frac{2\sum_{u,v \in V}\left(\gamma_2^N(u,v)\right)}{\sum_{(u,v)\in E}\left(\gamma_1^N(u) \wedge \gamma_1^N(v)\right)}$,

for $u, v \in V$

Definition 4.2.

A BIFG $G = (V, E)$ is balanced if $D(H) \leq D(G)$, that is $D_\mu^P(H) \leq D_\mu^P(G)$, $D_\mu^N(H) \leq D_\mu^N(G)$, $D_\gamma^P(H) \leq D_\gamma^P(G)$, $D_\gamma^N(H) \leq D_\gamma^N(G)$ for all subgraphs of G.

Example 3.3. Consider a BIFG $G = (V, E)$ such that



G H₃ H₉

$V = \{v_1, v_2, v_3, v_4,\}$ and

$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_2, v_4)\}$ $D(G) = (1.6, 1.5, 1.5, 1.5)$

$H_1 = \{v_1, v_2\}$ $D(H_1) = (1.6, 1.5, 1.5, 1.5)$
$H_2 = \{v_1, v_3\}$ $D(H_2) = (0,0,0,0)$
$H_3 = \{v_1, v_4\}$ $D(H_3) = (1.6, 1.5, 1.5, 1.5)$
$H_4 = \{v_2, v_3\}$ $D(H_4) = (1.6, 1.5, 1.5, 1.5)$
$H_5 = \{v_2, v_4\}$ $D(H_5) = (1.6, 1.5, 1.5, 1.5)$
$H_6 = \{v_3, v_4\}$ $D(H_6) = (1.6, 1.5, 1.5, 1.5)$
$H_7 = \{v_1, v_2, v_3\}$ $D(H_7) = (1.6, 1.5, 1.5, 1.5)$
$H_8 = \{v_1, v_3, v_4\}$ $D(H_8) = (1.6, 1.5, 1.5, 1.5)$
$H_9 = \{v_1, v_2, v_4\}$ $D(H_9) = (1.6, 1.5, 1.5, 1.5)$
$H_{10} = \{v_2, v_3, v_4\}$ $D(H_{10}) = (1.6, 1.5, 1.5, 1.5)$
$H_{11} = \{v_1, v_2, v_3, v_4\}$ $D(H_{11}) = (1.6, 1.5, 1.5, 1.5)$

μ^P density $D_\mu^P(G) = 2\left(\frac{0.24+0.16+0.16+0.24+0.4}{0.3+0.2+0.2+0.3+0.5}\right) = 1.6$

μ^N density $D_\mu^N(G) = 2\left(\frac{-0.225-0.225-0.15-0.15-0.15}{-0.3-0.3-0.2-0.2-0.2}\right) = 1.5$

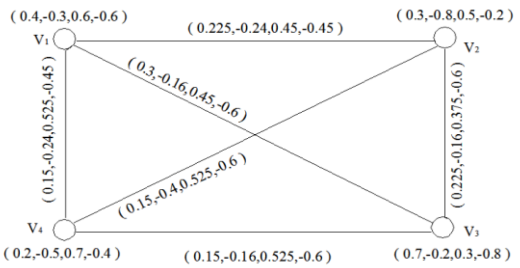
γ^P density $D_\gamma^P(G) = 2\left(\frac{0.45+0.6+0.6+0.45+0.3}{0.6+0.8+0.8+0.6+0.4}\right) = 1.5$

γ^N density $D_\gamma^N(G) = 2\left(\frac{-0.45-0.375-0.525-0.525-0.525}{-0.6-0.5-0.7-0.7-0.7}\right) = 1.5$

$D(G) = \left(D_\mu^P(G), D_\mu^N(G), D_\gamma^P(G), D_\gamma^N(G)\right) = (1.6, 1.5, 1.5, 1.5)$
Let $H_1 = \{v_1, v_2\}, H_2 = \{v_1, v_3\}, \ldots H_{11} = \{v_1, v_2, v_3, v_4\}$ be nonempty subgraphs of G. Density $\left(D_\mu^P(H), D_\mu^N(H), D_\gamma^P(H), D_\gamma^N(H)\right)$is $(H_1) = (1.6, 1.5, 1.5, 1.5)$, $D(H_2) = (0,0,0,0), \ldots D(H_{11}) = (1.6, 1.5, 1.5, 1.5)$. So $D(H) \leq D(G)$ for all subgraphs H of G. Hence G is balanced BIFG.

Definition 4.4. A BIFG $G = (V, E)$ is strictly balanced if for every $u, v \in V$, $D(H) = D(G)$ such that

$V = \{v_1, v_2, v_3, v_4,\}$ and $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3), (v_2, v_4)\}$



G

$D(G) = \left(D_\mu^P(G), D_\mu^N(G), D_\gamma^P(G), D_\gamma^N(G)\right) = (1.5,1.6,1.5,1.5)$. Let $H_1 = \{v_1, v_2\}, H_2 = \{v_1, v_3\}, \ldots H_{11} = \{v_1, v_2, v_3, v_4\}$ be nonempty subgraphs of G. Density $\left(D_\mu^P(H), D_\mu^N(H), D_\gamma^P(H), D_\gamma^N(H)\right)$ is

$H_1 = \{v_1, v_2\}$ $D(H_1) = (1.5,1.6,1.5,1.5)$
$H_2 = \{v_1, v_3\}$ $D(H_2) = (1.5,1.6,1.5,1.5)$
$H_3 = \{v_1, v_4\}$ $D(H_3) = (1.5,1.6,1.5,1.5)$
$H_4 = \{v_2, v_3\}$ $D(H_4) = (1.5,1.6,1.5,1.5)$
$H_5 = \{v_2, v_4\}$ $D(H_5) = (1.5,1.6,1.5,1.5)$
$H_6 = \{v_3, v_4\}$ $D(H_6) = (1.5,1.6,1.5,1.5)$
$H_7 = \{v_1, v_2, v_3\}$ $D(H_7) = (1.5,1.6,1.5,1.5)$
$H_8 = \{v_1, v_3, v_4\}$ $D(H_8) = (1.5,1.6,1.5,1.5)$
$H_9 = \{v_1, v_2, v_4\}$ $D(H_9) = (1.5,1.6,1.5,1.5)$
$H_{10} = \{v_2, v_3, v_4\}$ $D(H_{10}) = (1.5,1.6,1.5,1.5)$
$H_{11} = \{v_1, v_2, v_3, v_4\}$ $D(H_{11}) = (1.5,1.6,1.5,1.5)$. Hence $D(H) = D(G)$ for all nonempty subgraphs H of G. Hence G is strictly balanced BIFG.

Theorem 4.5.

Every complete bipolar intuitionistic fuzzy graph is balanced.

Proof.

Let $G = (V, E)$ be a complete BIFG, then by the definition of complete BIFG, we have $\mu_2^P(u,v) = \left(\mu_1^P(u)\wedge\mu_1^P(v)\right), \mu_2^N(u,v) = \left(\mu_1^N(u)\vee\mu_1^N(v)\right)$ and $\gamma_2^P(u,v) = \left(\gamma_1^P(u)\vee\gamma_1^P(v)\right), \gamma_2^N(u,v) = \left(\gamma_1^N(u)\wedge\gamma_1^N(v)\right)$ for every $u, v \in V$.

Therefore

$\sum_{u,v\in V} \mu_2^P(u,v) = \sum_{(u,v)\in E}\left(\mu_1^P(u)\wedge\mu_1^P(v)\right)$

$\sum_{u,v\in V} \mu_2^N(u,v) = \sum_{(u,v)\in E}\left(\mu_1^N(u)\vee\mu_1^N(v)\right)$

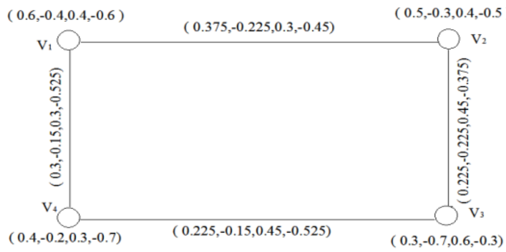
$\sum_{u,v\in V} \gamma_2^P(u,v) = \sum_{(u,v)\in E}\left(\gamma_1^P(u)\vee\gamma_1^P(v)\right)$

$\sum_{u,v\in V} \gamma_2^N(u,v) = \sum_{(u,v)\in E}\left(\gamma_1^N(u)\wedge\gamma_1^N(v)\right)$. Now

$$D(G) = \left(\frac{2\sum_{u,v\in V}\left(\mu_2^P(u,v)\right)}{\sum_{(u,v)\in E}\left(\mu_1^P(u)\wedge\mu_1^P(v)\right)}, \frac{2\sum_{u,v\in V}\left(\mu_2^N(u,v)\right)}{\sum_{(u,v)\in E}\left(\mu_1^N(u)\vee\mu_1^N(v)\right)}, \frac{2\sum_{u,v\in V}\left(\mu_2^N(u,v)\right)}{\sum_{(u,v)\in E}\left(\mu_1^N(u)\vee\mu_1^N(v)\right)}, \frac{2\sum_{u,v\in V}\left(\gamma_2^N(u,v)\right)}{\sum_{(u,v)\in E}\left(\gamma_1^N(u)\wedge\gamma_1^N(v)\right)}\right)$$

$D(G) = (2,2,2,2)$. Let H be a nonempty subgraph of G then, $D(H) = (2,2,2,2)$ for every $H \subseteq G$. Thus G is balanced.

Note 4.6. The converse of the above theorem is need not be true. Every balanced BIFG need not be complete.



G

$D(G) = \left(D_\mu^P(G), D_\mu^N(G), D_\gamma^P(G), D_\gamma^N(G)\right)$ where

$$D_\mu^P(G) = 2\left(\frac{0.375+0.225+0.225+0.31}{0.5+0.3+0.3+0.4}\right) = 1.5,$$

$$D_\mu^N(G) = 2\left(\frac{-0.225-0.225-0.15-0.15}{-0.3-0.3-0.2-0.2}\right) = 1.5,$$

$$D_\gamma^P(G) = 2\left(\frac{0.3+0.45+0.45+0.3}{0.4+0.6+0.6+0.4}\right) = 1.5,$$

$$D_\gamma^N(G) = 2\left(\frac{-0.45-0.375-0.525-0.525}{-0.6-0.5-0.7-0.7}\right) = 1.5.$$

That is, $D(G) = (1.5,1.5,1.5,1.5)$.

Let $H_1 = \{v_1, v_2\}, H_2 = \{v_1, v_3\}, H_3 = \{v_1, v_4\}, H_4 = \{v_2, v_3\}, H_5 = \{v_2, v_4\}, H_6 = \{v_3, v_4\}, H_7 = \{v_1, v_2, v_3\}, H_8 = \{v_1, v_3, v_4\}, H_9 = \{v_1, v_2, v_4\}, H_{10} = \{v_2, v_3, v_4\}, H_{11} = \{v_1, v_2, v_3, v_4\}$ be nonempty subgraphs of G. Density $\left(D_\mu^P(H), D_\mu^N(H), D_\gamma^P(H), D_\gamma^N(H)\right)$ is $D(H_1) = (1.5,1.5,1.5,1.5)$, $D(H_2) = (0,0,0,0)$, $D(H_3) = (1.5,1.5,1.5,1.5)$, $D(H_4) = (1.5,1.5,1.5,1.5)$, $D(H_5) = (0,0,0,0)$, $D(H_6) = (1.5,1.6,1.5,1.5)$, $D(H_7) = (1.5,1.5,1.5,1.5)$, $D(H_8) = (1.5,1.5,1.5,1.5)$, $D(H_9) = (1.5,1.5,1.5,1.5)$, $D(H_{10}) = (1.5,1.5,1.5,1.5)$, $D(H_{11}) = (1.5,1.5,1.5,1.5)$. Hence $D(H) = D(G)$ for all nonempty subgrap. Hence $D(H) \leq D(G)$ for all subgraphs H of G. So G is balanced IBFG. From the above graph it is easy to see that $\mu_2^P(u,v) \neq \left(\mu_1^P(u)\wedge\mu_1^P(v)\right), \mu_2^N(u,v) \neq \left(\mu_1^N(u)\vee\mu_1^N(v)\right)$ and $\gamma_2^P(u,v) \neq \left(\gamma_1^P(u)\vee\gamma_1^P(v)\right)$, $\gamma_2^N(u,v) \neq \left(\gamma_1^N(u)\wedge\gamma_1^N(v)\right)$. Hence G is balanced but not complete.

Corollary 4.7.

Every strong BIFG is balanced.

Theorem 4.8.

Let $G = (V, E)$ be a self complementary BIFG. Then $D(G) = (1,1,1,1)$.

Theorem 4.9.

Let $G = (V, E)$ be a strictly balanced BIFG and $\bar{G} = (\bar{V}, \bar{E})$ be its complement then $D(G) + D(\bar{G}) = (2,2,2,2)$.

Proof.

Let $G = (V, E)$ be a strictly balanced BIFG and $\bar{G} = (\bar{V}, \bar{E})$ be its complement. Let H be a nonempty sub graph of G, since G is strictly balanced $D(G) = D(H)$ for every $H \subseteq G$ and $u, v \in V$. In $\bar{G}$,

$\overline{\mu_2^P(u,v)} = \left(\mu_1^P(u) \wedge \mu_1^P(v)\right) - \mu_2^P(u,v)$ -------(1)

$\overline{\mu_2^N(u,v)} = \left(\mu_1^N(u) \vee \mu_1^N(v)\right) - \mu_2^N(u,v)$ ------ (2)

$\overline{\gamma_2^P(u,v)} = \left(\gamma_1^P(u) \vee \gamma_1^P(v)\right) - \gamma_2^P(u,v)$ -------(3)

$\overline{\gamma_2^N(u,v)} = \left(\gamma_1^N(u) \wedge \gamma_1^N(v)\right) - \gamma_2^N(u,v)$ ------ (4)

for every $u, v \in V$.

Dividing (1) by $\left(\mu_1^P(u) \wedge \mu_1^P(v)\right)$ gives

$$\frac{\overline{\mu_2^P(u,v)}}{\left(\mu_1^P(u) \wedge \mu_1^P(v)\right)} = 1 - \frac{\mu_2^P(u,v)}{\left(\mu_1^P(u) \wedge \mu_1^P(v)\right)}$$

Dividing (2) by $\left(\mu_1^N(u) \vee \mu_1^N(v)\right)$ gives

$$\frac{\overline{\mu_2^N(u,v)}}{\left(\mu_1^N(u) \vee \mu_1^N(v)\right)} = 1 - \frac{\mu_2^N(u,v)}{\left(\mu_1^N(u) \vee \mu_1^N(v)\right)}$$

Dividing (3) by $\left(\gamma_1^P(u) \vee \gamma_1^P(v)\right)$ gives

$$\frac{\overline{\gamma_2^P(u,v)}}{\left(\gamma_1^P(u) \vee \gamma_1^P(v)\right)} = 1 - \frac{\gamma_2^P(u,v)}{\left(\gamma_1^P(u) \vee \gamma_1^P(v)\right)} \text{ and}$$

Dividing (4) by $\left(\gamma_1^N(u) \wedge \gamma_1^N(v)\right)$ gives

$$\frac{\overline{\gamma_2^N(u,v)}}{\left(\gamma_1^N(u) \wedge \gamma_1^N(v)\right)} = 1 - \frac{\gamma_2^N(u,v)}{\left(\gamma_1^N(u) \wedge \gamma_1^N(v)\right)}.$$

Then

$$\sum_{u,v \in V} \frac{\overline{\mu_2^P(u,v)}}{\left(\mu_1^P(u) \wedge \mu_1^P(v)\right)} = 1 - \sum_{u,v \in V} \frac{\mu_2^P(u,v)}{\left(\mu_1^P(u) \wedge \mu_1^P(v)\right)}$$

$$\sum_{u,v \in V} \frac{\overline{\mu_2^N(u,v)}}{\left(\mu_1^N(u) \vee \mu_1^N(v)\right)} = 1 - \sum_{u,v \in V} \frac{\mu_2^N(u,v)}{\left(\mu_1^N(u) \vee \mu_1^N(v)\right)}$$

$$\sum_{u,v \in V} \frac{\overline{\gamma_2^P(u,v)}}{\left(\gamma_1^P(u) \vee \gamma_1^P(v)\right)} = 1 - \sum_{u,v \in V} \frac{\gamma_2^P(u,v)}{\left(\gamma_1^P(u) \vee \gamma_1^P(v)\right)} \text{ and}$$

$$\sum_{u,v \in V} \frac{\overline{\gamma_2^N(u,v)}}{\left(\gamma_1^N(u) \wedge \gamma_1^N(v)\right)} = 1 - \sum_{u,v \in V} \frac{\gamma_2^N(u,v)}{\left(\gamma_1^N(u) \wedge \gamma_1^N(v)\right)}$$

These implies

$$2\sum_{u,v \in V} \frac{\overline{\mu_2^P(u,v)}}{\left(\mu_1^P(u) \wedge \mu_1^P(v)\right)} = 2 - 2\sum_{u,v \in V} \frac{\mu_2^P(u,v)}{\left(\mu_1^P(u) \wedge \mu_1^P(v)\right)},$$

$$2\sum_{u,v \in V} \frac{\overline{\mu_2^N(u,v)}}{\left(\mu_1^N(u) \vee \mu_1^N(v)\right)} = 2 - 2\sum_{u,v \in V} \frac{\mu_2^N(u,v)}{\left(\mu_1^N(u) \vee \mu_1^N(v)\right)},$$

$$2\sum_{u,v \in V} \frac{\overline{\gamma_2^P(u,v)}}{\left(\gamma_1^P(u) \vee \overline{\gamma_1^P}(v)\right)} = 2 - 2\sum_{u,v \in V} \frac{\gamma_2^P(u,v)}{\left(\gamma_1^P(u) \vee \gamma_1^P(v)\right)} \text{ and}$$

$$2\sum_{u,v \in V} \frac{\overline{\gamma_2^N(u,v)}}{\left(\gamma_1^N(u) \wedge \overline{\gamma_1^N}(v)\right)} = 2 - 2\sum_{u,v \in V} \frac{\gamma_2^N(u,v)}{\left(\gamma_1^N(u) \wedge \gamma_1^N(v)\right)}$$

Therefore $D_\mu^P(\bar{G}) = 2 - D_\mu^P(G)$,
$D_\mu^N(\bar{G}) = 2 - D_\mu^N(G)$,
$D_\gamma^P(\bar{G}) = 2 - D_\gamma^P(G)$ and
$D_\gamma^N(\bar{G}) = 2 - D_\gamma^N(G)$.

Now

$D(G) + D(\bar{G}) =$

$$\left(D_\mu^P(G), D_\mu^N(G), D_\gamma^P(G), D_\gamma^N(G)\right) + \left(D_\mu^P(\bar{G}), D_\mu^N(\bar{G}), D_\gamma^P(\bar{G}), D_\gamma^N(\bar{G})\right)$$

$D(G) + D(\bar{G}) =$

$\left(D_\mu^P(G) + D_\mu^P(\bar{G}), D_\mu^N(G) + D_\mu^N(\bar{G}), D_\gamma^P(G) +\right.$

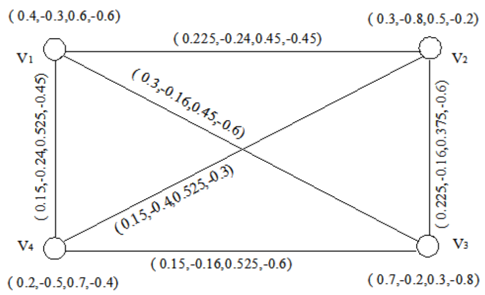
DγPG,DγNG+DγNG=2,2,2,2.

Theorem 4.10.

The complement of strictly balanced BIFG is strictly balanced.

Example 4.11.

Consider a BIFG, $G = (V, E)$ such that $V = \{v_1, v_2, v_3, v_4,\}$ and $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3), (v_2, v_4)\}$ and its complement $\bar{G} = (\bar{V}, \bar{E})$ such that $\bar{V} = \{v_1, v_2, v_3, v_4,\}$ and $\bar{E} = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3), (v_2, v_4)\}$.



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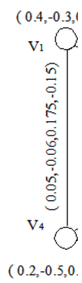

$$D(G) = \left(D_\mu^P(G), D_\mu^N(G), D_\gamma^P(G), D_\gamma^N(G)\right) = (1.5,1.6,1.5,1.5),$$

$$D(\bar{G}) = \left(D_\mu^P(\bar{G}), D_\mu^N(\bar{G}), D_\gamma^P(\bar{G}), D_\gamma^N(\bar{G})\right) = (0.5,0.4,0.5,0.5).$$

Therefore

$$D(G) + D(\bar{G}) = \left(D_\mu^P(G), D_\mu^N(G), D_\gamma^P(G), D_\gamma^N(G)\right) + \left(D_\mu^P(\bar{G}), D_\mu^N(\bar{G}), D_\gamma^P(\bar{G}), D_\gamma^N(\bar{G})\right)$$

$$= (1.5 + 0.5, 1.6 + 0.4, 1.5 + 0.5, 1.5 + 0.5,)$$

$$= (2, 2, 2, 2).$$

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