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Application of Neutrosophic Rough Set in Multi Criterion Decision Making on two universal sets

C. Antony Crispin Sweety[1] & I. Arockiarani[2]

[1,2] Nirmala College for Women, Coimbatore- 641018 Tamilnadu, India.

Abstract -. The main objective of this study is to introduce a new hybrid intelligent structure called rough neutrosophic sets on the Cartesian product of two universe sets. Further as an application a multi criteria decision making problem is solved.

Key words: - Rough Set, Solitary Set, Relative Set, Accuracy, Neutrosophic Rough Set, Multi Criteria Decision Making.

I. INTRODUCTION

The rough sets theory introduced by Pawlak [10] is an excellent mathematical tool for the analysis of uncertain, inconsistency and vague description of objects. Neutrosophic sets and rough sets are two different topics, none conflicts the other. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness. By combining the Neutrosophic sets and rough sets the rough sets in neutrosophic approximation space [2] and Neutrosophic neutrosophic rough sets [4] were introduced Multi criterion decision making (MCDM) is a process in which decision makers evaluate each alternative according to multiple criteria. Many representative methods are introduced to solve MCDM problem in business and industry areas. However, a drawback of these approaches is that they mostly consider the decision making with certain information of the weights and decision values. This makes them much less useful when managing uncertain information. To this end, multi criteria fuzzy decision making has been studied in [4, 6, 7]. Several attempts have already been made to use the rough set theory to decision support . But, in many real life problems, an information system establishes relation between two universal sets. Multi criterion decision making on such information system is very challenging. This paper discusses how neutrosophic rough set on two universal sets can be employed on MCDM problems for taking decisions.

2. PRELIMINARIES

Definition 2.1[13] A Neutrosophic set A on the universe of discourse X is defined as A = $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$, Where $T, I, F: X \to]^-0,1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2.2:[12] Let U be any non empty set. Suppose R is an equivalence relation over U. For any non null subset X of U, the sets
$A_1(X) = \{x: [x]_R \subseteq X\}$, $A_2(X) = \{x: [x]_R \cap X \neq \emptyset\}$
are called lower approximation and upper approximation respectively of X and the pair
S= (U, R) is called approximation space. The equivalence relation R is called indiscernibility relation. The pair $A(X) = (A_1(X), A_2(X))$ is called the rough set of X in S. Here $[x]_R$ denotes the equivalence class of R containing x.

Definition 2.3[4]:
Let U be a non empty universe of discourse. For an arbitrary fuzzy neutrosophic relation R over $U \times U$ the pair (U, R) is called fuzzy neutrosophic approximation space. For any $A \in FN(U)$, we define the upper and lower approximation with respect to (U, R), denoted by $\overline{R}$ and $\underline{R}$ respectively.

$$\overline{R}(A) = \{< x, T_{\overline{R}(A)}(x), I_{\overline{R}(A)}(x), F_{\overline{R}(A)}(x) >/x \in U\}$$
$$\underline{R}(A) = \{< x, T_{\underline{R}(A)}(x), I_{\underline{R}(A)}(x), F_{\underline{R}(A)}(x) > x \in U\}$$

$$T_{\overline{R}(A)}(x) = \bigvee_{y\in U} [T_R(x,y) \wedge T_A(y)],\ I_{\overline{R}(A)}(x) = \bigvee_{y\in U} [I_R(x,y) \wedge I_A(y)],\ F_{\overline{R}(A)}(x) = \bigwedge_{y\in U} [F_R(x,y) \wedge T_A(y)]$$

$$T_{\underline{R}(A)}(x) = \bigwedge_{y\in U} [F_R(x,y) \wedge T_A(y)], I_{\underline{R}(A)}(x) = \bigwedge_{y\in U} [1 - I_R(x,y) \wedge I_A(y)],\ F_{\underline{R}(A)}(x) = \bigvee_{y\in U} [T_R(x,y) \wedge F_A(y)]$$

The pair ($\underline{R}$, $\overline{R}$) is fuzzy neutrosophic rough set of A with respect to (U,R) and $\overline{R}$, $\underline{R}$:FN(U)→FN(U) are refered to as upper and lower Fuzzy neutrosophic rough approximation operators respectively.

3. NEUTROSOPHIC ROUGH SET ON TWO UNIVERSAL SETS

Now, we present the definitions, notations and results of neutrosophic rough set on two universal We define the basic concepts leading to neutrosophic rough set on two universal sets in which we denote for truth function T_{R_N} , indeterminacy I_{R_N} and falsity function F_{R_N} for non membership functions that are associated with an neutrosophic rough set on two universal sets.

Definition 3.1:[11] Let *U* and *V* be two non empty universal sets. An neutrosophic relation R_N from $U \to V$ is an neutrosophic set of (*U* x *V*) characterized by the truth value function T_{R_N} , indeterminacy function and falsity function F_{R_N} where $R_N = \{\langle (x,y), T_{R_N}(x,y), I_{R_N}(x,y), F_{R_N}(x,y), \rangle \mid \text{x} \in \text{U, y} \in \text{V}\}$ with $0 \le T_{R_N}(x,y) + I_{R_N}(x,y) + F_{R_N}(x,y) \le 3$ for every (x,y) ∈ U x V.

Definition 3.2 [13] Let U and V be two non empty universal sets and R_N is a neutrosophic relation from U to V. If for $x \in U$, $T_{R_N}(x,y) = 0$, $I_{R_N}(x,y) = 0$ and $F_{R_N}(x,y) = 1$ for all *y* ∈ V , then *x* is said to be a solitary element with respect to R_N . The set of all solitary elements with respect to the relation R_N is called the solitary set *S*. That is,

$$S = \{x \mid x \in U, T_{R_N}(x,y) = 0, I_{R_N}(x,y) = 0, F_{R_N}(x,y) = 1, \ \forall\ y \in V$$

Definition 3.3:[13] Let *U* and *V* be two non empty universal sets and R_N is a neutrosophic relation from *U* to *V*. Therefore, (U,V,R_N) is called a neutrosophic approximation space. For $Y \in N(V)$ an neutrosophic rough set is a pair ($\underline{R_N}Y$, $\overline{R_N Y}$) of neutrosophic set on *U* such that for every *x* ∈ U.

$$\underline{R_N}(Y) = \{\left\langle x, T_{\underline{R_N}(Y)}(x), I_{\underline{R_N}(Y)}(x), F_{\underline{R_N}(Y)}(x), \right\rangle \mid x \in U\}\} \qquad (11)$$

$$\overline{R_N}(Y) = \{\left\langle x, T_{\overline{R_N}(Y)}(x), I_{\overline{R_N}(Y)}(x), F_{\overline{R_N}(Y)}(x) \right\rangle \mid x \in U\} \qquad (12)$$

Where

$$T_{\overline{R}(A)}(x) = \bigvee_{y \in V} [T_R(x,y) \wedge T_A(y)] \quad I_{\overline{R}(A)}(x) = \bigvee_{y \in V} [I_R(x,y) \wedge I_A(y)] \quad F_{\overline{R}(A)}(x) = \bigwedge_{y \in V} [F_R(x,y) \wedge T_A(y)]$$

$$T_{\underline{R}(A)}(x) = \bigwedge_{y \in V} [F_R(x,y) \wedge T_A(y)] \quad I_{\underline{R}(A)}(x) = \bigwedge_{y \in V} [1 - I_R(x,y) \wedge I_A(y)] \quad F_{\underline{R}(A)}(x) = \bigvee_{y \in V} [T_R(x,y) \wedge F_A(y)]$$

The pair ($\underline{R_N}(Y)$, $\overline{R_N}(Y)$) is called the neutrosophic rough set of Y with respect to (U,V,R_N) where $\underline{R_N}(Y)$, $\overline{R_N}(Y)$: *N(U)* → *N(V)* are referred as lower and upper neutrosophic rough approximation operators on two universal sets.

4.ALGEBRAIC PROPERTIES:

In this section, we discuss the algebraic properties of neutrosophic rough set on two universal sets through solitary set .

Proposition 4.1: Let U and V be two universal sets. Let R_N be an neutrosophic relation from U to V and further let S be the solitary set with respect to R_N . Then for *X, Y* ∈ *N(V),* the following properties holds:

(a) $\underline{R_N}$ *(V) = U and* $\overline{R_N}$ (ϕ) = ϕ

(b) If $X \subseteq Y$, then $\underline{R_N}$ (X) $\subseteq$ $\underline{R_N}$ (Y) and $\overline{R_N}$ (X) $\subseteq$ $\overline{R_N}$ (Y)

(c) $\underline{R_N}$ (X) = $\overline{R_N}$ (X′))′ and $\overline{R_N}$ (X)= $\underline{R_N}$ (X′))′

(d) $\underline{R_N}\phi \supseteq S$ and $R_N V \subseteq S'$, where S' denotes the complement of S in U.

(e) For any given index set J, $X_i \in N(V)$,

$$\underline{R_N}\ (\bigcup_{i \in J} X_i) \supseteq \bigcup_{i \in J} \underline{R_N} X_i \ \ and \ \ \overline{R_N}\ (\bigcap_{i \in J} X_i) \subseteq \bigcap_{i \in J} \overline{R_N} X_i$$

(f) For any given index set J,

$X_i \in N(V)$, $\underline{R_N}\ (\bigcap_{i \in J} X_i) = \bigcap_{i \in J} \underline{R_N} X_i$ and $\overline{R_N}\ (\bigcup_{i \in J} X_i) = \bigcup_{i \in J} \overline{R_N} X_i$.

Proof:

First note that V is a neutrosophic set satisfying $T_Y(x)=1$, $I_Y(x)=0$ and $F_Y(x)=1$ for all $x \in V$. Thus, V can be represented as $V = \{\langle x,1,1,0\rangle \mid x \in V\}$

Now, by definition we have

$$T_{\underline{R}(A)}(x) = \bigwedge_{y\in V} F_{R_N}(x,y) \vee T_Y(y) = 1 \ I_{\underline{R}(A)}(x) = \bigwedge_{y\in V} 1 - I_{R_N}(x,y) \vee I_Y(y) = 1 \ F_{\underline{R}(A)}(x) = \bigvee_{y\in V} T_{R_N}(x,y) \wedge F_Y(y) = 0$$

Therefore we get,

$$\underline{R_N}(V) = \{\left\langle x, T_{\underline{R_N}(V)}(x), I_{\underline{R_N}(V)}(x), F_{\underline{R_N}(V)}(x), \right\rangle \mid x \in U\} = \{\langle x,1,1,0\rangle \mid x \in U\}$$

Similarly, ϕ is a neutrosophic set satisfying $T_Y(x)=0$, $I_Y(x)=1$ and $F_Y(x)=0$ for all $x \in V$. Thus, ϕ can be represented as $\phi = \{\langle x,0,0,1\rangle \mid x \in V\}$

Now, by definition we have

$$T_{\overline{R}(A)}(x) = \bigvee_{y\in V} [T_R(x,y) \wedge T_A(y)] = 0, \ I_{\overline{R}(A)}(x) = \bigvee_{y\in V} [I_R(x,y) \wedge I_A(y)] = 0, \ F_{\overline{R}(A)}(x) = \bigwedge_{y\in V} [F_R(x,y) \wedge T_A(y)] = 1$$

Therefore we get, $\overline{R_N}(\phi) = \{\left\langle x, T_{\overline{R_N}(\phi)}, I_{\overline{R_N}(\phi)}, F_{\overline{R_N}(\phi)} \right\rangle \mid x \in U\}\} = \{\langle x,0,0,1\rangle \mid x \in U\} = \phi$.

(ii) First note that $X \subseteq Y$ if and only $T_X(x) \le T_Y(x)$, $I_X(x) \le I_Y(x)$ and $F_X(x) \le F_Y(x)$ for all $x \in V$. Therefore, we have

$$T_{\underline{R_N}(X)}(x) = \bigwedge_{y\in V} [F_{R_N}(x,y) \vee T_X(y)] \le \bigwedge_{y\in V} [F_{R_N}(x,y) \vee T_Y(y)] = T_{\underline{R_N}(Y)}(x)$$

$$I_{\underline{R_N}(X)}(x) = \bigwedge_{y\in V} [1 - I_{R_N}(x,y) \vee I_X(y)] \le \bigwedge_{y\in V} [I_{R_N}(x,y) \vee I_Y(y)] = I_{\underline{R_N}(Y)}(x)$$

$$F_{\underline{R_N}(X)}(x) = \bigvee_{y\in V} [T_{R_N}(x,y) \wedge F_X(y)] \ge \bigvee_{y\in V} [T_{R_N}(x,y) \wedge F_Y(y)] = F_{\underline{R_N}(Y)}(x)$$

Therefore, $\underline{R_N}$ (X) $\subseteq$ $\underline{R_N}$ (Y). Similarly, we have

$$T_{\overline{R_N}(x)}(x) = \bigvee_{y\in V} [T_{R_N}(x,y) \wedge T_X(y)] \le \bigvee_{y\in V} [I_{R_N}(x,y) \wedge T_Y(y)] = T_{\overline{R_N}(Y)}(x)$$

$$I_{\overline{R_N}(x)}(x) = \bigvee_{y\in V} [I_{R_N}(x,y) \wedge I_X(y)] \leq \bigvee_{y\in V} [I_{R_N}(x,y) \wedge I_Y(y)] = I_{\overline{R_N}(Y)}(x)$$

$$F_{\overline{R_N}(x)}(x) = \bigwedge_{y\in V} [F_{R_N}(x,y) \vee F_X(y)] \geq \bigwedge_{y\in V} [F_{R_N}(x,y) \vee F_Y(y)] = F_{\overline{R_N}(Y)}(x)$$

Therefore, $\overline{R_N}$ *(X)* $\subseteq$ $\overline{R_N}$ *(Y).*

(iii) We know that $\overline{R_N}(X') = \{\langle x, T_{\overline{R_N}(X')}(x), I_{\overline{R_N}(X')}(x), F_{\overline{R_N}(X')}(x)\rangle / x \in U\}$, *where*

$$T_{\overline{R_N}(x')}(x) = \bigvee_{y\in V} [T_{R_N}(x,y) \wedge T_{X'}(y)] = \bigvee_{y\in V} T_{R_N}(x,y) \wedge F_Y(y)] = F_{\underline{R_N}(X)}(x)$$

$$I_{\overline{R_N}(x')}(x) = \bigvee_{y\in V} [I_{R_N}(x,y) \wedge I_{X'}(y)] = \bigvee_{y\in V} I_{R_N}(x,y) \wedge 1 - I_Y(y)] = 1 - I_{\underline{R_N}(X)}(x) = I_{\underline{R_N}(X)}(x)$$

$$F_{\overline{R_N}(x)}(x) = [F_{R_N}(x,y) \vee F_{X'}(y)] = [F_{R_N}(x,y) \vee T_X(y)] = T_{\underline{R_N}(X)}(x)$$

Therefore, we have

$$\overline{R_N}(X') = \{\langle x, T_{\overline{R_N}(X')}(x), I_{\overline{R_N}(X')}(x), F_{\overline{R_N}(X')}(x)\rangle / x \in U\}$$

$$= \{\langle x, F_{\underline{R_N}(X')}(x), 1 - I_{\underline{R_N}(X')}(x), T_{\underline{R_N}(X')}(x)\rangle / x \in U\}$$

indicates that $\left(\overline{R_N}(X')\right)' = \{\langle x, T_{\underline{R_N}(X)}(x), I_{\underline{R_N}(X)}(x), F_{\underline{R_N}(X)}(x), \rangle \mid x \in U$ and consequently

$$\left(\overline{R_N}(X')\right)' = \underline{R_N}(X)$$

$$\underline{R_N}(X') = \{\langle x, T_{\underline{R_N}(X')}(x), I_{\underline{R_N}(X')}(x), F_{\underline{R_N}(X')}(x), \rangle \mid x \in U$$

$$T_{\underline{R_N}(X')}(x) = \bigwedge_{y\in V} [F_{R_N}(x,y) \vee T_{X'}(y)] = \bigwedge_{y\in V} [F_{R_N}(x,y) \vee F_X(y)] = F_{\overline{R_N}(X)}(x)$$

$$I_{\underline{R_N}(X')}(x) = \bigwedge_{y\in V} [1 - I_{R_N}(x,y) \vee I_{X'}(y)] = \bigwedge_{y\in V} [1 - I_{R_N}(x,y) \vee 1 - I_X(y)] = [I_{R_N}(x,y) \wedge I_Y(y)]$$

$$= I_{\overline{R_N}(X)}(x)$$

$$F_{\underline{R_N}(X)}(x) = \bigvee_{y\in V} [T_{R_N}(x,y) \wedge F_{X'}(y)] \geq \bigvee_{y\in V} [T_{R_N}(x,y) \wedge T_Y(y)] = T_{\overline{R_N}(X)}(x).$$

$$\left(\underline{R_N}(X')\right)' = \{\langle x, T_{\underline{R_N}(X')}(x), I_{\underline{R_N}(X')}(x), F_{\underline{R_N}(X')}(x), \rangle \mid x \in U$$

$$= \{\langle x, F_{\underline{R_N}(X)}(x), I_{\underline{R_N}(X)}(x), T_{\underline{R_N}(X)}(x), \rangle \mid x \in U$$

It indicates that $\left(\underline{R_N}(X')\right)' = \{\langle x, T_{\overline{R_N}(X)}(x), I_{\overline{R_N}(X)}(x), F_{\overline{R_N}(X)}(x)\rangle / x \in U\}$ and consequently

$$\left(\underline{R_N}(X')\right)' = \overline{R_N}(X).$$

(iv) First note that, ϕ is a neutrosophic set satisfying $T_Y(x) = 0$, $I_Y(x) = 0$ and $F_Y(x) = 1$ for all $x \in V$. Thus, ϕ can be represented as

$\phi = \{\langle x,0,0,1\rangle \mid x \in V\}$

Also note that, S is a solitary set. This indicates that

Therefore, we have for all $x \in S$

$$T_{\underline{R_N(\phi)}}(x) = \bigwedge_{y\in V} [F_{R_N}(x,y) \vee T_\phi(y)] = \bigwedge_{y\in V} [1 \vee 0] = 1$$

$$I_{\underline{R_N(\phi)}}(x) = \bigwedge_{y\in V} [1 - I_{R_N}(x,y) \vee I_\phi(y)] = \bigwedge_{y\in V} [1 \vee 0] = 1$$

$$F_{\underline{R_N(\phi)}}(x) = \bigvee_{y\in V} [T_{R_N}(x,y) \vee F_\phi(y)] = \bigvee_{y\in V} [0 \wedge 1] = 0$$

Hence, it is clear that $T_{\underline{R_N(\phi)}}(x) \geq T_{R_N}(x,y)$, $I_{\underline{R_N(\phi)}}(x) \geq I_{R_N}(x,y)$ and $F_{\underline{R_N(\phi)}}(x) \leq F_{R_N}(x,y)$ for $x \in S$.

Therefore, by proposition (ii) we have $\underline{R_N}(\phi) \supseteq S$.

Similarly, by proposition (iii) we have $\overline{R_N}(X) = \left(\underline{R_N}(X')\right)'$.

On taking $x \in V$ we get $\overline{R_N}(V) = \left(\underline{R_N}(V')\right)'$.

But $V' = \phi$.Again by proposition (iv), we have $\underline{R_N}(\phi) \supseteq S$. It implies that $(\underline{R_N}(\phi))' \subseteq S'$.

Therefore, we get $\overline{R_N}(V) \subseteq S'$.

(v) From the properties of union, for any index set J={1,2,3….,n}

$$X_1 \subseteq \bigcup_{i\in J} X_i,\ X_2 \subseteq \bigcup_{i\in J} X_i, X_3 \subseteq \bigcup_{i\in J} X_i, \ldots\ldots\ldots\ldots X_n \subseteq \bigcup_{i\in J} X_i.$$

Therefore, by proposition (ii) we have

$$\underline{R_N}(X_1) \subseteq \underline{R_N}(\bigcup_{i\in J} X_i),\ \underline{R_N}(X_2) \subseteq \underline{R_N}(\bigcup_{i\in J} X_i), \ldots\ldots\ldots \underline{R_N}(X_n) \subseteq \underline{R_N}(\bigcup_{i\in J} X_i)$$

It indicates that,

$$\bigcup_{i\in J} \underline{R_N}(X_i) \subseteq \underline{R_N}(\bigcup_{i\in J} X_i), \text{ ie } \underline{R_N}(\bigcup_{i\in J} X_i) \supseteq \bigcup_{i\in J} \underline{R_N}(X_i).$$

Similarly, for any index set J={1,2,3….,n}

$$\bigcap_{i\in J} X_i \subseteq X_1, \bigcap_{i\in J} X_i \subseteq X_2, \bigcap_{i\in J} X_i \subseteq X_3, \ldots\ldots\ldots\ldots \bigcap_{i\in J} X_i \subseteq X_n.$$

Therefore, by proposition (ii) we have

$$\overline{R_N}(\bigcap_{i\in J} X_i) \subseteq \overline{R_N}(X_1),\ \overline{R_N}(\bigcap_{i\in J} X_i) \subseteq \overline{R_N}(X_2), \ldots\ldots\ldots \overline{R_N}(\bigcap_{i\in J} X_i) \subseteq \overline{R_N}(X_n)$$

It indicates that, $\overline{R_N}(\bigcap_{i\in J} X_i) \subseteq \bigcap_{i\in J} \overline{R_N}(X_i)$.

(vi) For any index set J ={1, 2, 3,…., n}, $X_i \in N(V)$,

$\underline{R_N}(\bigcap_{i\in J} X_i) = \{\langle x, T_{\underline{R_N}(\bigcap_{i\in J} X_i)}(x), I_{\underline{R_N}(\bigcap_{i\in J} X_i)}(x), F_{\underline{R_N}(\bigcap_{i\in J} X_i)}(x)\rangle / x \in U\}$. But,

$$T_{\underline{R_N}(\bigcap_{i\in J} X_i)}(x) = \bigwedge_{y\in V} [F_{R_N}(x,y) \vee T_{(\bigcap_{i\in J} X_i)}(y)]$$

$$= \bigwedge_{y\in V} [F_{R_N}(x,y) \vee (T_{X_1}(y) \wedge T_{X_2}(y) \wedge T_{X_3}(y) \wedge \ldots\ldots\ldots \wedge T_{X_{n-1}}(y) \wedge T_{X_n}(y))]$$

$$= \bigwedge_{y\in V} [(F_{R_N}(x,y) \vee T_{X_1}(y) \wedge (F_{R_N}(x,y) \vee T_{X_2}(y)) \wedge \ldots\ldots \wedge (F_{R_N}(x,y) \vee T_{X_n}(y))]$$

$$= T_{\underline{R_N}(X_1)}(x) \wedge T_{\underline{R_N}(X_2)}(x) \wedge ..... \wedge T_{\underline{R_N}(X_n)}(x)$$

$$= \text{Min}\{ T_{\underline{R_N}(X_1)}(x) \}$$

Similary, we can prove for indeterrminacy and false value functions.

Again, for any index set J ={1, 2, 3....., n}, $X_i \in N(V)$,

$$\underline{R_N}(X_1) = \{ \left\langle x, T_{\underline{R_N}(X_1)}(x), I_{\underline{R_N}(X_1)}(x), F_{\underline{R_N}(X_1)}(x), \right\rangle \mid x \in U$$

$$\underline{R_N}(X_2) = \{ \left\langle x, T_{\underline{R_N}(X_2)}(x), I_{\underline{R_N}(X_2)}(x), F_{\underline{R_N}(X_2)}(x), \right\rangle \mid x \in U$$

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$$\underline{R_N}(X_n) = \{ \left\langle x, T_{\underline{R_N}(X_n)}(x), I_{\underline{R_N}(X_n)}(x), F_{\underline{R_N}(X_n)}(x), \right\rangle \mid x \in U$$

Therefore, we have

$$\bigcap_{i\in J} \underline{R_N}(X_i) = \{ \left\langle x, Min\{T_{\underline{R_N}(X_n)}(x)\}, Min\{I_{\underline{R_N}(X_n)}(x)\}, Max\{F_{\underline{R_N}(X_n)}(x)\} \right\rangle \mid x \in U$$

Hence, it is clear that $\underline{R_N}(\bigcap_{i\in J} X_i) = \bigcap_{i\in J} \underline{R_N}(X_i)$. Similarly for any index set J ={1, 2, 3,...., n} , $X_i \in N(V)$,

$$\overline{R_N}(\bigcup_{i\in J} X_i) = \{ \left\langle x, T_{\overline{R_N}(\bigcup_{i\in J} X_i)}(x), I_{\overline{R_N}(\bigcup_{i\in J} X_i)}(x), F_{\overline{R_N}(\bigcup_{i\in J} X_i)}(x) \right\rangle / x \in U \}.$$

$T_{\overline{R_N}(\bigcap_{i\in J} X_i)}(x)$ = Max{ $T_{\overline{R_N}(X_1)}(x)$ }, $I_{\overline{R_N}(\bigcap_{i\in J} X_i)}(x)$ =Max{ $I_{\overline{R_N}(X_1)}(x)$ }, $F_{\overline{R_N}(\bigcap_{i\in J} X_i)}(x)$ = Min{ $F_{\underline{R_N}(X_1)}(x)$ }

Again, for any index set J={1, 2, 3,...., n}, $X_i \in N(V)$,

$$\underline{R_N}(X_1) = \{ \left\langle x, T_{\underline{R_N}(X_1)}(x), I_{\underline{R_N}(X_1)}(x), F_{\underline{R_N}(X_1)}(x), \right\rangle \mid x \in U$$

$$\underline{R_N}(X_2) = \{ \left\langle x, T_{\underline{R_N}(X_2)}(x), I_{\underline{R_N}(X_2)}(x), F_{\underline{R_N}(X_2)}(x), \right\rangle \mid x \in U$$

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$$\underline{R_N}(X_n) = \{ \left\langle x, T_{\underline{R_N}(X_n)}(x), I_{\underline{R_N}(X_n)}(x), F_{\underline{R_N}(X_n)}(x), \right\rangle \mid x \in U$$

Therefore, we have

$$\bigcap_{i\in J} \underline{R_N}(X_i) = \{ \left\langle x, Min\{T_{\underline{R_N}(X_n)}(x)\}, Min\{I_{\underline{R_N}(X_n)}(x)\}, Max\{F_{\underline{R_N}(X_n)}(x)\} \right\rangle \mid x \in U$$

Hence, it is clear that $\underline{R_N}(\bigcap_{i\in J} X_i) = \bigcap_{i\in J} \underline{R_N}(X_i)$.

4. AN APPLICATION TO MULTI CRITERION DECISION MAKING

In this section, we depict a real life application of neutrosophic rough set on two universal sets to multi criterion decision making. The model application is explained as neutrosophic rough set upper approximation. Let us consider the multi criteria decision making in the case of a online shop. However, it is observed that due to several factors such as, on time delivery, offers and discounts, quality of the product, Easy returns, Diccreet shoping. Therefore, from customer behaviour, clear review of the particular online shop can be obtained. Hence, neutrosophic relation better depicts the relation between the customers and supermarkets.

Let us set the criteria ***U={U₁, U₂, U₃, U₄, U₅, U₆}*** in which **U_1** denotes the convenience in shopping, **U_2** denotes on time delivery of the products, **U_3** denotes offers and discounts given, **U_4** denote quality of the product, **U_5** denotes easy returns of the products, **U_6** denotes the discreetness in shoping. Let us consider the review rating, such as 5*,4*,3*,2*,1*,nil.
Several variety of customers and professionals are invited to the survey that only focuses on the criterion of best Easy returns in the online shop X.

. DECISIONS		YES(%)	NEUTRAL(%)	NO(%)
*****	D_5	15	30	47
****	D_4	28	46	23
***	D_3	46	27	43
**	D_2	40	38	56
*	D_1	26	67	70
-----	D_0	34	40	14

then the vector can be obtained as

$[(0.15,0.30,0.28),(0.28,0.46,0.23),(0.46,0.27,0.23), (0.40,0.38,0.34),(0.26,0.67,0.35) ,(0.34,0.40,0.14)]^T$

where T represents the transpose.

Similarly, the decisions based on other criteria are obtained as follows:
$[(0.10,0.20,0.30), (0.30,0.30,0.21),(0.25,0.20,0.15),(0.10,0.40,0.8),(0.20,0.30,0.7),(0.25,0.20,0.25)]^T$
$[(0.55, 0.55, 0.41), (0.55,0.55,0.1), (0.20,0.30, 0.15), (0.50,0.40,0.6),(0.30,0.10,0.3),(0.20,0.20, 0.4)]^T$
$[(0.1,0.10, 0.7),(0.10,0.10,0.4), (0.40,0.45,0.20) ,(0.20,0.25,0.30), (0.20,0.22,0.10),(0.35,0.10,0.10)]^T$
$[(0.24,0.10,0.20), (0.20,0.12,0.67) ,(0.15,0.13,0.59) ,(0.35,0.30,0.36),(0.5,0.40,0.36),(0.20,0.22,0.54)]^T$
$[(0.25,0.28,0.31),(0.25,0.23,0.10),(0.25,0.23,0.20),(0.20,0.34,0.40),(0.10,0.35,0.40),(0.20,0.14,0.30)]^T$

Based on the decision vectors, the neutrosophic relation from U to V is presented by the following matrix. We define the neutrosophic relation by the following matrix.

$$R_N = \begin{pmatrix} (0.15,0.3,0.30) & (0.28,0.46,0.23) & (0.46,0.27,.23) & (0.4,0.38,0.34) & (0.26,0.67,0.35) & (0.34,0.4,0.14) \\ (0.1,0.2,0.3) & (0.30,0.30,0.21 & (0.25,0.20,0.15) & (0.1,0.40,0.80) & (0.20,.30,0.70) & (0.20,0.25,0.40) \\ (0.55,0.55,0.41) & (0.55,0.55,0.1) & 0.2,0.3,0.15) & (0.50,0,.40,0.60) & (0.30,0.10,0.30) & (0.20,0.20,0.40) \\ (0.10,0.10,0.70) & (0.10,0.10.40) & (0.40,0.45,0.20) & (0.20,0.25,0.30) & (0.20,0.22,0.10)] & (0.35,0.10,0.10) \\ (0.24,0.10,0.20) & (0.20,0.12,0.67) & 0.15,0.13,0.59 & (0.35,0.30,0.36) & (0.50,0.4,0.36) & (0.20,0.22,0.54) \\ (0.25,0.28,0.31) & (0.25,0.23,0.10) & (0.25,0.23,0.20) & (0.2,0.30,0.40) & (0.10,0.35,0.40) & (0.20,0.14,0.30) \end{pmatrix}$$

It is assumed that there are two categories of customers, where right weights for each criterion in V are

For, $Y_1 = (\langle d_1.0.34,0.43.0.2\rangle, \langle d_2,0.23,0.45,0.67\rangle, \langle d_3,34,23,56\rangle, \langle d_4,54,43,39\rangle, \langle d_5,45,27,39\rangle, \langle d_6,17,28,34\rangle)$
We can calculate,

$$\overline{R_N} Y_1 = \begin{pmatrix} \langle d_1.0.40,0.45.0.28\rangle, \langle d_2,0.25,0.30,0.30\rangle, \langle d_3,34,34,36\rangle, \\ \langle d_4,34,25,36\rangle, \langle d_5,35,30,36\rangle, \langle d_6,34,30,31\rangle \end{pmatrix}$$

and according to the principle of maximum membership, the decision for the first category of customers is 5*.

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