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Performance analysis of Piecewise linear Companding with various precoders for PAPR Reduction of OFDM Signals

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Abstract - *Orthogonal Frequency Division Multiplexing (OFDM) is an attractive technique for wireless communication applications. However, an OFDM signal has a large Peak-to-Average Power Ratio (PAPR), which can result in significant distortion when passed through a nonlinear device, such as a transmitter power amplifier. Number of techniques has been proposed for reducing the PAPR in OFDM systems. Among them Companding is a well-known technique for the PAPR reduction of OFDM signals. Recently, a piecewise linear companding technique is proposed to extenuate companding distortion. There are different precoder systems to reduce PAPR without degradation Bit Error Rate (BER). In this paper we present Performance analysis of Discrete Hartley Transform (DHT) precoder with Piecewise linear companding OFDM system. We compare the computer simulation results of DHT precoded Piecewise linear companding OFDM system with Discrete Cosine Transform (DCT) precoder, Walsh Hadamard Transform (WHT) precoder, conventional Piecewise Linear Companding (PLC) OFDM system. Simulation results show that the PAPR of DHT precoded Piecewise linear companding OFDM system is lower than WHT precoded PLC OFDM system, DCT precoded PLC OFDM system and conventional PLC OFDM while maintaining good performance in the Bit Error Rate (BER) and Power Spectral Density (PSD).*

Key Words: **OFDM, PAPR; Discrete Cosine Transform,; Companding Distortion, Walsh Hadamard Transform, Discrete Hartley Transform.**

1.INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been recently seen rising popularity in wireless applications. Keeping in view of the less complexity in implementing the OFDM using Fast Fourier Transform (FFT), high spectral efficiency and vigour to frequency selective fading channel OFDM has gained reputation in a number of applications such as Digital Video and Audio Broadcasting (DVAB), Wireless Local Area Networks (WLAN) and Wireless Metropolitan Area Networks (WMAN). Despite many advantages, OFDM affect from high fluctuation of the transmitted envelope signal. To characterize the envelope fluctuations of OFDM signals Peak to Average Power Ratio (PAPR) mostly used by relating peak value and mean value power. When non linear power amplifier (PA) is used, then high PAPR causes serious degradation in performance.

Several techniques are proposed to reduce PAPR in OFDM signals [1, 2]. Selective level mapping (SLM) partial transmit sequence (PTS), Hadamard transform, and companding techniques are proposed in [3-9, 12]. In these techniques, companding techniques gain more attention due to their flexibility and simplicity. The concept of companding technique was first introduced in [5], which uses the μ-law companding technique for the reduction PAPR by increasing the average power of the signal while keeping the peak power remains unchanged. Later on, exponential companding (EC) was developed in [6], which can improve PAPR reduction by altering the distribution of OFDM signals while keeping average power constant. In recent times, a nonlinear companding technique is proposed [7] with a linear function which changes the Gaussian distributed signal into distribution form. The nonlinear companding technique reduces the PAPR with the expense of high computational complexity. The two piecewise companding (TPWC) technique proposed in [8] compress large signal amplitudes and expand small ones by using two different piecewise functions. Thus companding techniques decrease PAPR by producing companding distortion. Recently, a piecewise linear companding technique was investigated in [9] to decrease the companding distortion alters the signals linearly with amplitudes close to the given companded peak amplitude and chopping the signals with amplitudes over a given companded amplitude. In this technique Zadoff-Chu based pre-coder is applied before the IFFT in the PLC OFDM system. There are different precoder systems to reduce PAPR without degradation Bit Error Rate (BER). Park et al. [12] proposes Hadamard transform technique may reduce the occurrence of the high peaks when compared to the original OFDM system. Discrete cosine transform may reduce PAPR of OFDM signal while the error probability of system is not increased. DHT is purely real transform. The calculation of DHT[10] involves only real multiplications and additions and it is having identical inverse.

In this paper, the authors analyzed performance of different precoder systems by combining with piecewise linear companding technique. The precoder operation can be applied to reduce the PAPR of OFDM signals before OFDM signal is modulated using an IFFT (Inverse Fast Fourier Transform). This further reduces the PAPR of the OFDM signal the piecewise linear companding technique is applied after the IFFT operation. This technique will be compared with the original system and PLC technique for reduction of PAPR.

This paper starts with the introduction of the topic in section I. Section II presents a typical OFDM system with a formulation of PAPR problem. Piecewise linear Companding technique and Different precoder systems are introduced in section III and section IV. Section V provides signal processing steps to implement a PAPR reduction technique by combing precoder system and PLC technique. Simulation results are presented in section VI and conclusion is given in VII.

2 . FORMULATION OF PAPR PROBLEM

Generally, N independent data symbols are modulated by using baseband modulation techniques like phase-shift keying (PSK) and quadrature amplitude modulation (QAM). OFDM signal is nothing but sum of those N independent modulated data symbols. In discrete-time domain, since the Nyquist rate samples might not represent the peaks of the continuous-time signal, so for getting better approximation of PAPR the researchers do oversampling of the OFDM signal. The oversampled time-domain OFDM symbols $X=[x_0,x_1,....,x_{LN-1}]^T$ can be calculated as

$$x_n=\frac{1}{\sqrt{NL}}\sum_{k=0}^{N-1}X_k.e^{j2\pi\frac{kn}{NL}},\quad 0\le n\le NL-1, \quad (1)$$

Where $n=0,1,...,LN-1$ time is index and L is the oversampling factor. Usually, $L\ge 4$ is used to accurately describe the PAPR of the continuous-time signal. For getting oversampling of OFDM signal by inserting $(L-1)N$ zeros in the middle of N length vector, i.e.

$$X_e=\left[X_0,X_1,...,X_{\frac{N}{2}-1},\underbrace{0,....,0}_{N(L-1)},X_{\frac{N}{2}},...,X_{N-1}\right]^T. \quad (2)$$

It is clear that x = $IFFT_{LN}\left\{X_e\right\}$. For a large N (e.g. $N\ge 64$), the real and imaginary parts are approximated as independent and identically distributed Gaussian random variables with zero mean and a variance σ_x^2. Based on this approximation, the signal amplitude $|x_n|$ follows a Rayleigh distribution with the probability density function (PDF) as

$$f_{|x_n|}(x)=\frac{2x}{\sigma_x^2}e^{-\frac{x^2}{\sigma_x^2}}\qquad x\ge 0. \quad (3)$$

The cumulative density function (CDF) of $|x_n|$ is therefore

$$F_{|x_n|}(x)=\Pr ob\left\{|x_n|\le x\right\}=\int_0^x\frac{2y}{\sigma_x^2}e^{-\frac{y^2}{\sigma_x^2}}dy=1-e^{-\frac{x^2}{\sigma_x^2}},\quad x\ge 0 \quad (4)$$

The PAPR of OFDM signal in a given frame is defined as

$$PAPR_X=\frac{\max_{n\in[0,LN-1]}\left\{|x_n|^2\right\}}{E\left\{|x_n|^2\right\}}. \quad (5)$$

It is more helpful to consider the PAPR as a random variable and utilize a statistical description given by the complementary cumulative density function (CCDF), defined as the probability that the PAPR of exceeds an assigned level $\gamma_0>0$, i.e.

$$CCDF_X(\gamma_0)=\Pr ob\left\{PAPR_X>\gamma_0\right\}=1-(1-e^{-\gamma_0})^N \quad (6)$$

The companding transform can be applied to the original signal x_n before it is converted into analog waveform and amplified by the HPA. The companded signal is denoted as $s_n=C(x_n)$, where $s_n=C(\cdot)$ is the companding function that only changes the amplitude of x_n. In the case of additive Gaussian white noise (AWGN) channel, the received signal $r_n=s_n+\upsilon_n$ can be recovered by the inverse companding function $C^{-1}(\cdot)$, namely, $x_n'=C^{-1}(s_n+\upsilon_n)=x_n+C^{-1}(\upsilon_n)$, where υ_n is channel noise.

3.PIECEWISE LINEAR COMPANDING TECHNIQUE

When the original signal x_n is companded with a given peak amplitude A_c, the piecewise linear companding technique shown in Fig. 1 clips the signals with amplitudes over A_c and linearly changing the signals with amplitudes close to A_c. Then, the companding function of the piecewise linear companding technique is

$$s(n)=C\left\{x(n)\right\}=\begin{cases}x(n) & |x(n)|\le A_i\\ mx(n)+(1-m)A_c & A_i<|x(n)|\le A_c,\\ sgn(x(n))A_c & |x(n)|>A_c\end{cases} \quad (7)$$

where *sgn(x)* is the sign function.

Next, the inverse companding function at the receiver is

$$y(n)=C^{-1}(r(n))=\begin{cases} r(n) & |r(n)|\le A_i \\ \left(r(n)-(1-m)A_c\right)/m & (1-m)A_c<|r(n)|\le A_c, \\ sgn(r(n))A_c & |r(n)|>A_c \end{cases} \quad (8)$$

By using the A_c value remaining parameters A_i and m can be obtained by solving

$$\int_{A_i}^{A_c}\left(mx+(1-m)A_c\right)^2 f_{|x_n|}(x)dx+\int_{A_c}^{\infty}A_c^2 f(x)dx = \int_{A_i}^{\infty}x^2 f_{|x_n|}(x)dx. \quad (9)$$

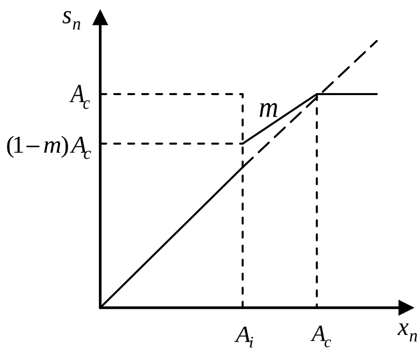

Fig. 1. Piecewise linear companding transform

4.PRECODER SYSTEMS

Fig. 2 shows the block diagram of Precoder based OFDM System. We implemented the Precoding matrix P of dimension N × N before the IFFT to reduce the PAPR.

The Precoding matrix P can be written as

$$\begin{pmatrix} P_{00} & P_{01} & \cdots & P_{0(N-1)} \\ P_{10} & P_{11} & & P_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ P_{(N-1)0} & P_{(N-1)1} & \cdots & P_{(N-1)(N-1)} \end{pmatrix}$$

where P is a Precoding Matrix of size N × N is shown in equation (4). The complex baseband OFDM signal with N subcarriers can be written as

$$x(t)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}P.X_k e^{j2\pi k\Delta ft}. \qquad 0\le t\le NT \quad (10)$$

We can express modulated OFDM vector signal with N subcarriers as

$$x_n=IFFT\{P.X_N\} \quad (11)$$

The PAPR of OFDM signal in (5) can be written as

$$PAPR=\frac{\max\left[|x(t)|^2\right]}{E\left[|x(t)|^2\right]} \quad (12)$$

4.1 Hadamard Transform:

Hadamard transform is used to reduce the autocorrelation between the input data sequence and the PAPR of the OFDM signal. The key point is that the Hadamard transform can be generated by a recursive procedure; we assume the Hadamard transform matrix of N orders. The Hadamard matrix has just two kinds of elements 1 or −1. The Hadamard matrix of order 2 is expressed as:

$$H_2=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \quad (13)$$

Hadamard matrix of order $2N$ can be constructed by:

$$H_{2N}=\frac{1}{\sqrt{2N}}\begin{bmatrix}H_N & H_N\\ H_N & -H_N\end{bmatrix} \quad (14)$$

Where $-H_N$ is the complementary of H_N. Hadamard matrix satisfy the relation

$$H_{2N}H_{2N}^T=H_{2N}^T H_{2N}=I_{2N} \quad (15)$$

Where H_{2N}^T is the transport matrix, I_{2N} is the unit matrix of 2N order.

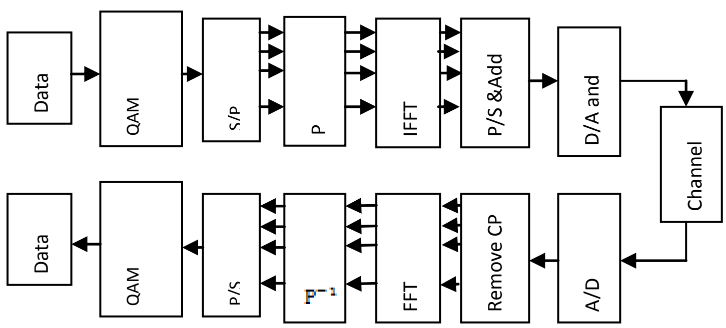

Fig. 2. Block diagram of Precoder based OFDM System

4.2 Discrete Cosine Transform:

The DCT decorrelates the data sequence such as the Hadamard transform. DCT is applied to decrease the autocorrelation of the input sequence before the IFFT operation is applied. The one-dimensional DCT of length N can be formulated as following:

$$X_c(k)=\alpha(k)\sum_{n=0}^{N-1}x(n)\cos\left[\frac{\pi(2n+1)k}{2N}\right], \quad (16)$$

for $k=0,....,N-1$

Similarly, the inverse transformation can be formulated as

$$x(n)=\sum_{k=0}^{N-1}\alpha(k)X_c(k)\cos\left[\frac{\pi(2n+1)k}{2N}\right], \quad (17)$$

for $n=0,....,N-1$

For both Equations (10) and (11) $\alpha(k)$ is defined as

$$\alpha(k) = \begin{cases} \frac{1}{\sqrt{N}}, & for \quad k = 0 \\ \frac{2}{\sqrt{N}}, & for \quad k \neq 0 \end{cases} \tag{18}$$

Equation (10) can be expressed in matrix form as

$$X_c = C_N x \tag{19}$$

where X_c and ***x*** are both vectors of dimension $N \times 1$, and C_N is a DCT matrix of dimension $N \times N$. The rows (or column) of the DCT matrix, C_N are orthogonal matrix vectors. The authors can reduce the peak power of OFDM signals by using orthogonal property of the DCT matrix.

4.3 Discrete Hartley Transform:

The DHT is a linear transform. In DHT N real numbers $x_0, x_1, \ldots\ldots x_{N-1}$ are transformed in to N real number $H_1, H_2, \ldots\ldots H_{N-1}$. According to [15] the N point DHT can be defined a

$$H_K = \sum_{n=0}^{N-1} x_n \left[\cos\left(\frac{2\pi nk}{N}\right) + \sin\left(\frac{2\pi nk}{N}\right) \right] \tag{20}$$

$$= \sum_{n=0}^{N-1} x(n).cas\left(\frac{2\pi nk}{N}\right)$$

$cas = \cos\theta + \sin\theta$ and $k = 0,1.....N-1$.

$$P_{m,n} = cas\left(\frac{2\pi nk}{N}\right) \tag{21}$$

P is precoding matrix of size N× N shown in equation (4), mand n are integers from 0 to N-1.The DHT is also invertibleinverse can be obtained by simply multiplying DHT H_K by $\frac{1}{N}$.

5. PROPOSED SCHEME

In this section, we analyzed performance of Piecewise Linear Companding (PLC) with various precoder systems to reduce the PAPR of OFDM signal by combining Piecewise Linear Companding Transform and Precoder.

The coming input data stream is firstly transformed by Precoder matrix, and then the transformed data stream is given as input to IFFT signal processing unit. The OFDM system with proposed scheme is shown at Fig.3.

The proposed scheme processing steps are given below:

Step1: firstly Precoder matrix is applied to the sequence X, i.e. $Y = PX$

Step2: apply IFFT to Precoded signal, **y**=IFFT (**Y**), where $y = [y(1) \;\; y(2) \;\; .... \;\; y(N)]^T$

Step3: apply PLC to y, i.e. $s(n) = C\{y(n)\}$

Step4: apply inverse PLC to the received signal *r* (*n*), i.e. $\hat{y}(n) = C^{-1}\{y(n)\}$.

Step5: apply FFT transform to the signal $\hat{y}(n)$, i.e. $\hat{Y} = FFT(\hat{y})$, where $\hat{y} = [\hat{y}(1) \;\; \hat{y}(2) \;\; .... \;\; \hat{y}(N)]^T$

Setp6: apply inverse Precoder to the signal $\hat{Y}$, i.e. $\hat{X} = P^{-1}\hat{Y}$. Then the signal $\hat{x}$ is demapped to bit stream.

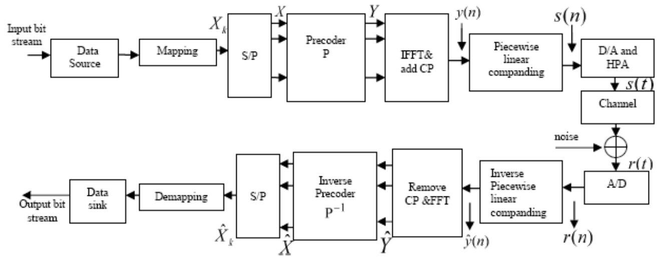

Fig.3. Block diagram of a OFDM system with Precoder and PLC.

6. SIMULATION RESULTS

Computer simulation results are presented in this section to evaluate the performance of the proposed technique i.e. hybrid companding transform with respect to the PAPR reduction, BER and PSD performance. The number of subcarrier *N* = 256 and oversampling factor *L* is 4 considered as per IEEE 802.16 standards. The baseband modulation techniques 4-QAM and 16-QAM are considered in the simulations. Solid State Power Amplifier (SSPA) is considered as a model for input-output characteristics of the nonlinear region by passing companded signals through HPA. This model is formulated by

$$|z(t)| = \frac{|y(t)|}{(1 + (\frac{|y(t)|}{A_{sat}})^{2p})^{\frac{1}{2p}}} \tag{22}$$

where A_{sat} is the saturation level, and *p* = 2 is selected for this paper.

6.1 Performance in PAPR Reduction

Fig. 4 plots the simulated Complementary Cumulative Distribution Function (CCDF) of PAPR of PLC technique with different precoder matrices and PLC technique. The authors observed from Fig. 4, proposed scheme can draw good PAPR reduction and among three different precoder. matrices, DHT precoder provides efficient PAPR reduction. Given that CCDF=10^{-3}, the PLC with DHT precoder at $PAPR_{preset} = 4$dB is 0.9dB, 1dB and 1.1dB superior over the PLC with DCT, PLC with Hadamard and PLC schemes respectively

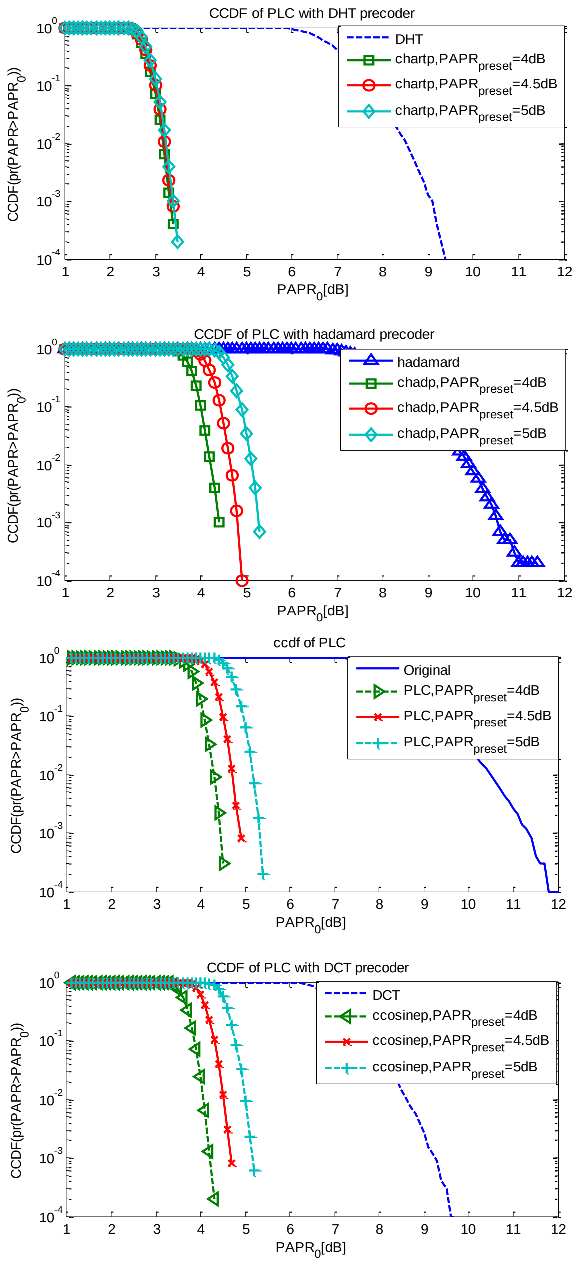

Fig.4. CCDFs of PLC and PLC with different Precoder schemes

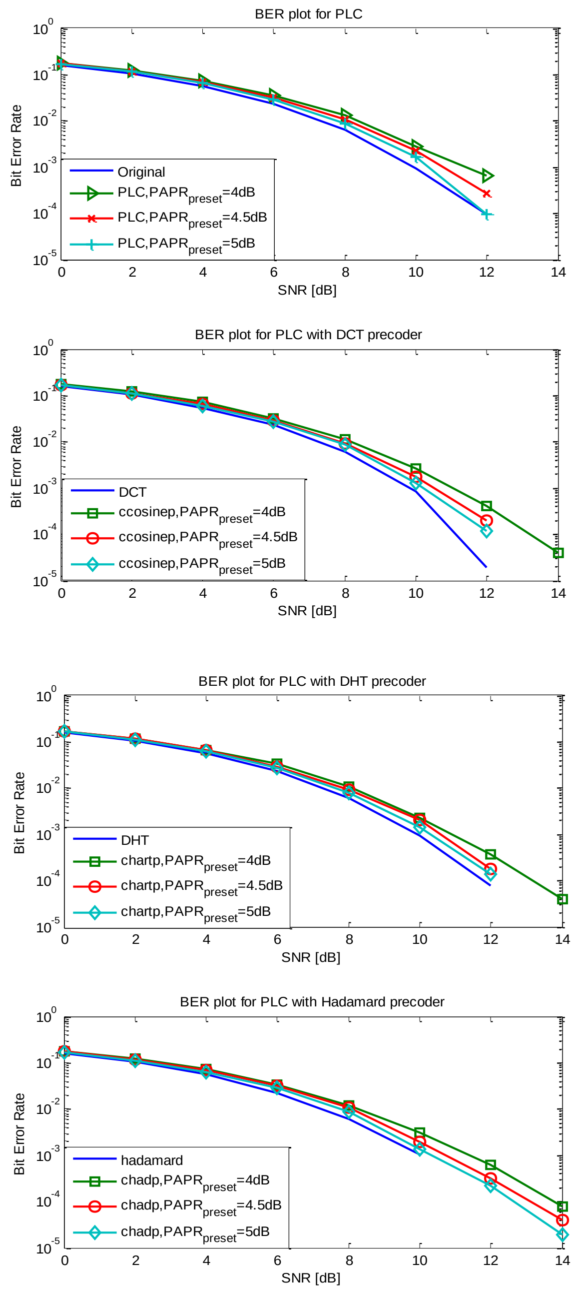

Fig.5. BER Performance of PLC and PLC with different Precoder schemes over AWGN channel with 4-QAM modulation.

6.2 BER Performance

The impact of PLC with three precoder systems on the BER performance is discussed in this section. BER versus Signal to Noise Ratio (SNR) curves for PLC with different precoder systems under an AWGN channel using 4-QAM and 16-QAM are shown in Fig. 5 and 6 respectively. It is observed that BER performance is improved in 4-QAM modulation, with the proposed technique. At BER level of , the PLC with DHT precoder at $PAPR_{preset}$ = 4dB surpasses the PLC with DCT, PLC with Hadamard and PLC schemes by 0.5dB, 0.85dB and 1dB respectively.

It is also observed that in 16-QAM, the BER performance of proposed technique has performance floor at high SNR because of the output of the proposed companding function is not continuous. At a BER level of 10^{-2}, the PLC with DHT precoder at $PAPR_{preset}$ = 4dB is surpasses the PLC with DCT, PLC with Hadamard and PLC schemes by 0.2dB, 0.6dB and 1.2dB, respectively.

Fig. 7 depicts the BER performance using 4-QAM modulation with SSPA passing through an AWGN channel. It can be seen that the BER performance of the proposed technique with SSPA model is also sufficient. At a BER level of10^{-3}, the PLC with DHT precoder at $PAPR_{preset}$ = 4dB is surpasses the PLC with DCT, PLC with Hadamard and PLC schemes by 0.2dB, 0.6dB and 0.6dB, respectively.

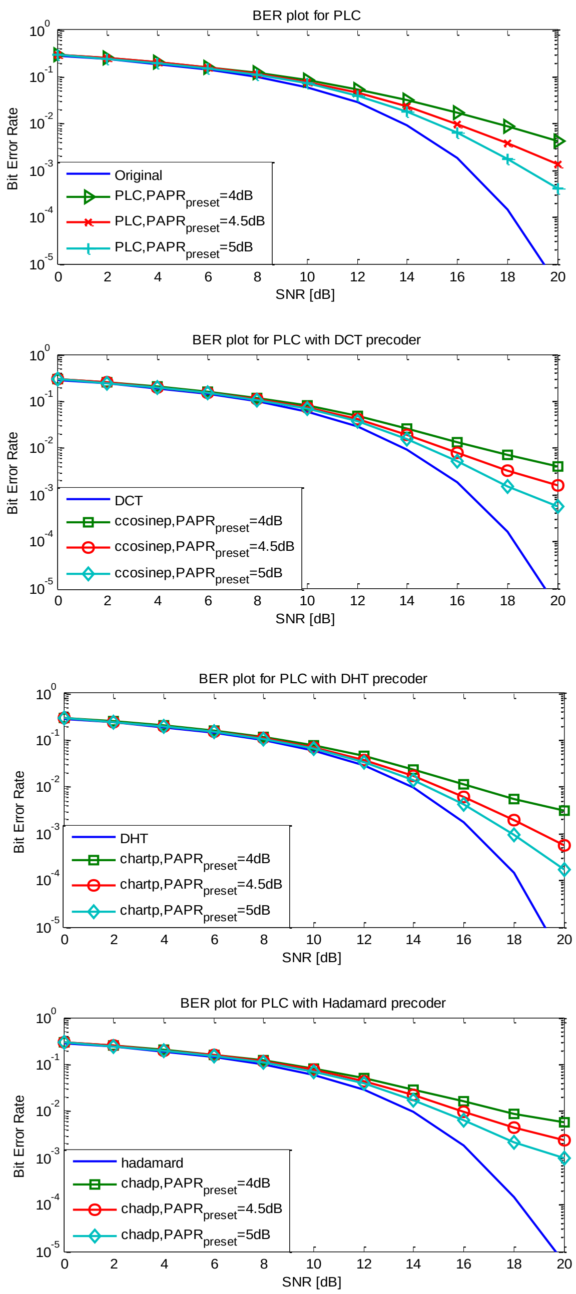

Fig. 6. BER Performance of PLC and PLC with different Precoder schemes over AWGN channel with 16-QAM modulation.

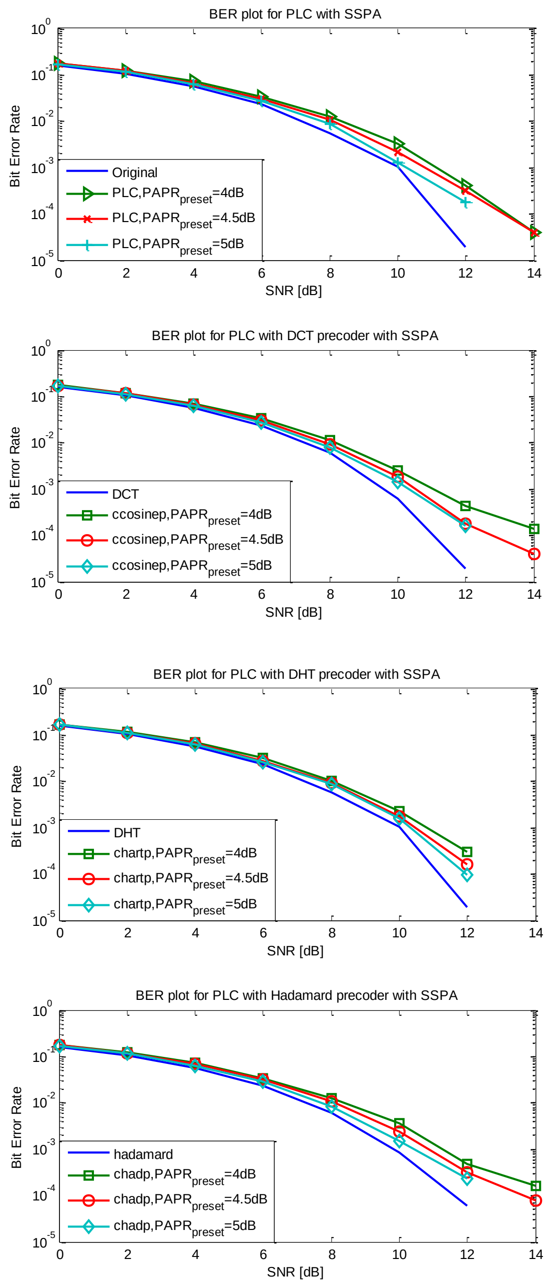

Fig. 7. BER Performance of PLC and PLC with different Precoder schemes with SSPA over AWGN channel with 4-QAM modulation.

6.3 PSD Performance

The PSD performances for the PLC scheme and PLC with three precoder schemes are given in Fig.8. We observed PLC with DHT precoder scheme at $PAPR_{preset}$ = 4dB can produce about 5 dB to 8 dB lower out-of-band interference than other referred methods at normalized frequency 0.2 Hz.

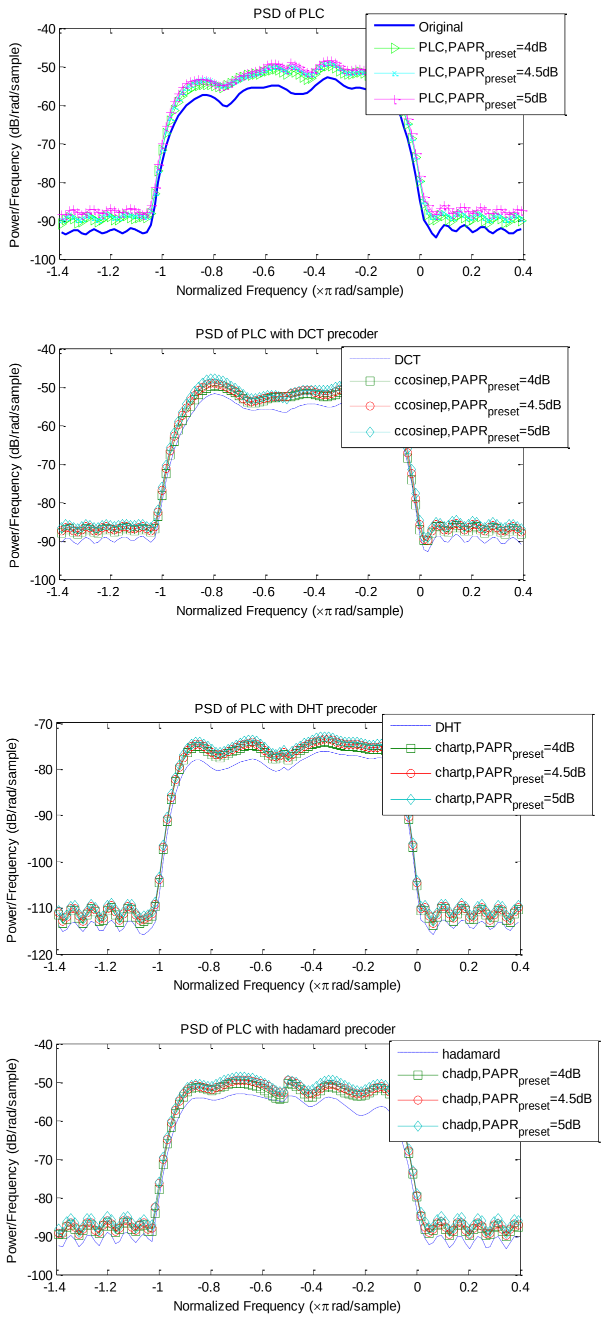

Fig. 8. PSDs of PLC and PLC with different Precoder schemes.

7. CONCLUSIONS

In this paper, the Piecewise Linear Companding (PLC) is procoded with three precoder schemes viz. DCT, Hadamard and DHT. The performance of these schemes are analysed in terms of PAPR, BER and PSD. From the It is evident that concluded that among PLC with three precoder schemes, PLC with DHT precoder performs better performance. In simulation results we compare PLC with three precoder schemes with PLC technique. By using PLC with DHT precoder scheme the PAPR is reduced by 0.9dB, 1dB and 1.1dB over the PLC with DCT, PLC with Hadamard and PLC schemes respectively. The BER performance under an AWGN channel using 4-QAM is improved by 0.2dB, 0.6dB and 0.6dB over the PLC with DCT, PLC with Hadamard and PLC schemes respectively and maintaining the same performance in 16QAM. In PSD performance it can produce about 5 dB to 8 dB lower out-of-band interference than other referred methods.

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