FLOW OF VIBRATION ENERGY IN A GEAR PAIR

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Abstract :*The average vibration power in a gear pair is calculated for one-stage gear transmission. Only cinematic excitation is taken into consideration as the most dangerous. The analysis of the received equations for the input and output powers in a gear pair and the transmission coefficients between them and also their frequency characteristics demonstrated the following. The power radiated into the pinion support is considerably greater than the power radiated into the gear support. In the middle frequency range the resonance related to the pinion vibrations is more expressed. In low frequency range the magnitudes of the transmission coefficients are practically constant till the medium frequency range at 400 Hz. They have the extreme values at the natural frequencies of transverse vibrations of the pinion and the gear. In the high frequency range (above 6000 Hz) the values of transmission coefficients fall rapidly. The analysis also demonstrated that in the low frequency range power transmission coefficients depends only on loss factors and stiffness coefficients. To reduce vibration power radiated into the supports it is necessary to increase supports stiffness and decrease the stiffness of the tooth engagement.*

Key Words: *Vibration Energy, Gear Pair, Cinematic Excitation, Power Transmission Coefficient*

INTRODUCTION

An estimation of vibration energy in a gear pair and the flow of this energy from its main origin (the gear engagement) to the housing are necessary to know for the calculation of the acoustic power emitted by the gear mechanism. In some previous papers on the dynamics of gears attention has been paid to characteristics of vibration o dynamic forces (Kubo, Kiyono 1980, Mueller 1980). But gear noise depends not only on the forces but also on the velocity of vibration of radiating surfaces. So the vibration energy or vibration power is just the parameters in which we have both, the force and velocity.

DETERMINATION OF VIBRATION POWER IN A GEAR

The average vibration power input in a gear pair with respect to time can be determined considering force and cinematic excitation by the following equations (Lion 1975)

$$W_{in}^{F} = < F(t)\,\dot{x} >_t = 0.5\,\mathrm{Re}(\bar{F}\bar{V}) = 0.5\left|\bar{F}\right|^2 \mathrm{Re}(Y), \quad (1)$$

and for the cinematic excitation

$$W_{in}^{S} = 0.5\left|\dot{S}\right|^2 \mathrm{Re}(Z), \quad (2)$$

where

$\bar{F} = F_0\, e^{-j\psi_F}$ - complex amplitude of variable force,

$\bar{V} = V_0\, e^{-j\psi_V}$ - complex amplitude of vibration velocity F_0, V_0 -amplitude of force and velocity

ψ_F, ψ_V - phase shift of force and velocity, respectively.

Here and on sign $< >_t$ means average over time *t.*

Let us also determine the vibration power transmitted through the gear supports to the housing. The vibration power transmitted to *i*-th support is equal:

$$W_{Ci} = 0.5\left|V_{Ci}\right|^2 \mathrm{Re}(Z_{Ci}) \quad (3)$$

where

V_{Ci} - velocity of vibration in the i-th support,

Z_{Ci} - mechanical impedance of this support.

Let us consider the determination of vibration power for the example of one-stage gear with flexible supports and concentrated parameters (Fig.1).

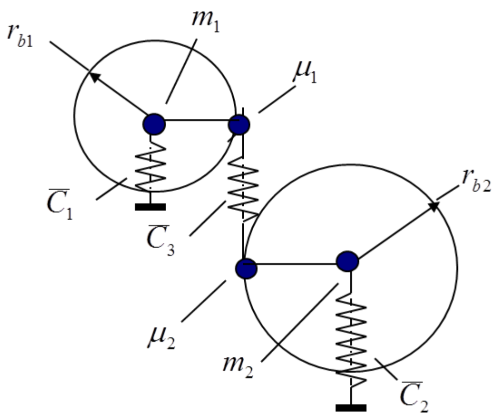

Fig -1: Dynamic model of one-stage gear transmission

Only cinematic excitation can be taken into account as the most dangerous. Then the input vibration power can be determined by the Eq. (2). In this equation the real part of the input impedance can be expressed through the input admittance

$$\mathrm{Re}(Z) = \mathrm{Re}(Y^{-1}). \qquad (4)$$

Following (Podzharov, 1983) let us find the total admittance of a gear with harmonic excitation as the sum of admittances of parallel branches of the equivalent electric circuit of the gear dynamic model (Fig. 2)

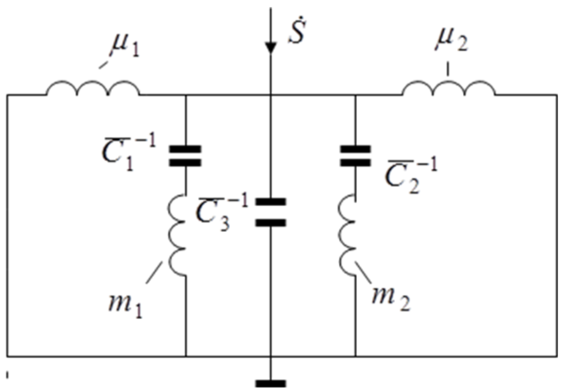

Fig -2: Equivalent electric cirquit of the gear dynamic model

$$Y = Y_0 + Y_1 + Y_2 + Y_3, \qquad (5)$$

where

Y_i is the admittance of the *i*-th branch of the equivalent electric circuit (Fig. 2):

$$Y_0 = (j\omega\mu)^{-1},$$
$$Y_1 = (j\omega m_1 + C_1(1 + j\eta_1)/j\omega)^{-1},$$
$$Y_2 = (j\omega m_2 + C_2(1 + j\eta_2)/j\omega)^{-1}$$
$$Y_3 = j\omega/C_3(1 + j\eta_3) \qquad (6)$$

$$\mu = \mu_1\mu_2/(\mu_1 + \mu_2), \quad \mu_1 = I_1/r_{b1}^2, \quad \mu_2 = I_2/r_{b2}^2$$

where I_1, I_2 are moments of inertia and r_{b1}, r_{b2} - base radius of the pinion and gear respectively, C_1, C_2, C_3 are the stiffness coefficients of the pinion and gear supports and of the gear engagement respectively, m_1, m_2 are the masses of the pinion and the gear, η_i is the loss factor in the elastic element with the stiffness C_i, $j = \sqrt{-1}$ and ω is the angular frequency.
It may be represented by

$$\mathrm{Re}(1/Y) = \mathrm{Re}\frac{1}{\mathrm{Re}(Y) + j\,\mathrm{Im}(Y)} = \frac{\mathrm{Re}(Y)}{[\mathrm{Re}(Y)]^2 + [\mathrm{Im}(Y)]^2}. \qquad (10)$$

Inserting this equation in Eq. (2), we have

$$W_{in} = 0.5|\dot{S}|^2 \frac{\mathrm{Re}(Y)}{[\mathrm{Re}(Y)]^2 + [\mathrm{Im}(Y)]^2}. \qquad (11)$$

Amplitudes of shaft vibration velocity can be found using the work (Lion, 1975), as

$$|V_{Ci}|^2 = \left|\frac{\dot{S}\,Y_i}{Y}\right|^2 = \frac{|\dot{S}|^2\,|Y_i|^2}{|Y|^2}, \qquad (12)$$

where

$$|Y_i|^2 = \frac{\omega^2}{C_i^2\left[\left(1 - \omega^2/\omega_i^2\right)^2 + \eta_i^1\right]}, \qquad (13)$$

$$|Y|^2 = [\mathrm{Re}(Y)]^2 + [\mathrm{Im}(Y)]^2, \qquad (14)$$

and

$$\mathrm{Re}(Z_{Ci}) = C_i\,\eta_i/\omega. \qquad (15)$$

Solving the Eqs. (12) – (15) together with Eq. (3), we find

$$W_{Ci} = 0.5\frac{|\dot{S}|^2\,\eta_i\,\omega}{C_i\left[\left(1 - \frac{\omega^2}{\omega_i^2}\right)^2 + \eta_i^2\right]\left[(\mathrm{Re}(Y))^2 + (\mathrm{Im}(Y))^2\right]} \qquad (16)$$

It is also interesting to find the relation between the power radiated to the supports and the input power, which is called the power transmission coefficient τ

$$\tau_i = \frac{W_{Ci}}{W_{in}} = \frac{\eta_i\,\omega}{C_i\left[\left(1-\omega^2/\omega_i^2\right)^2+\eta_i^2\right]\mathrm{Re}(Y)}. \qquad (17)$$

Inserting the Eqs. (5) and (6) in (17), we find

$$\tau_1 = \frac{\frac{\eta_1}{C_1}\left(1+\eta_3^2\right)\left[\left(1-\frac{\omega^2}{\omega_2^2}\right)+\eta_2^2\right]}{\frac{\eta_1}{C_1}\left(1+\eta_3^2\right)\left[\left(1-\frac{\omega^2}{\omega_2^2}\right)+\eta_2^2\right]+\left[\frac{\eta_2}{C_2}\left(1+\eta_3^2\right)+\frac{\eta_3}{C_3}\right]\left[\left(1-\frac{\omega^2}{\omega_1^2}\right)+\eta_1^2\right]} \qquad (18)$$

One can get similar expressions for τ_2. It follows from the presented equation, that if $(1-\omega^2/\omega_2^2)^2+\eta_2^2=0$ and $\eta_2^2 \ll 1$, $\omega \cong \omega_2$, then $\tau_1 = \tau_{1min}$, $\tau_2 = \tau_{2max}$, and when $\omega \cong \omega_1$, on the contrary $\tau_1 = \tau_{1min}$, $\tau_2 = \tau_{2min}$. Computer calculations confirm these conclusions. In the Fig. 3 are presented the frequency characteristics of W_{in}, and W_{C2}, calculated using Eqs. (2) – (13) for an example of a gear with the following parameters μ = 0.8125 kg, m_1 = 1.2 kg, m_2 = 4.2 kg, C_1 = 16 MN/m, C_2 = 64.8 MN/m, C_3 = 201 MN/m. The calculation was conducted with harmonic cinematic excitation with a constant amplitude of the velocity of cinematic excitation $\dot{S}_0$ = 1 cm/s.

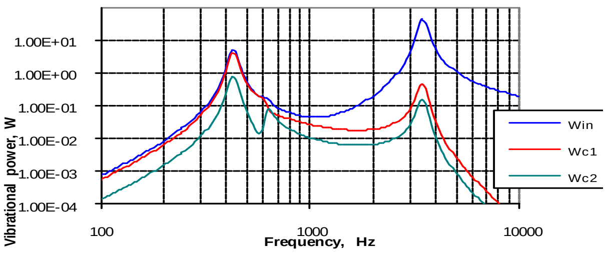

Fig -3: *Frequency characteristics of vibration power*

This figure demonstrates that there are peaks on the frequencies 428 Hz, 617 Hz and 3448 Hz, which are natural frequencies of the combined torsion and transverse vibrations of the gear.
The power radiated into the pinion support (W_{C1}) is considerably greater than the power radiated into the gear support (W_{C2}). In the middle frequency range the resonance related to the pinion vibrations is more expressed.

The results of the calculations of the power transmissions coefficients by Eq. (13) are presented in Fig. 4. They confirm the presented analysis of Eq. (18). Besides, it is necessary to note that in low frequency range the magnitudes of τ_1 and τ_2 are practically constant till the frequency 400 Hz.

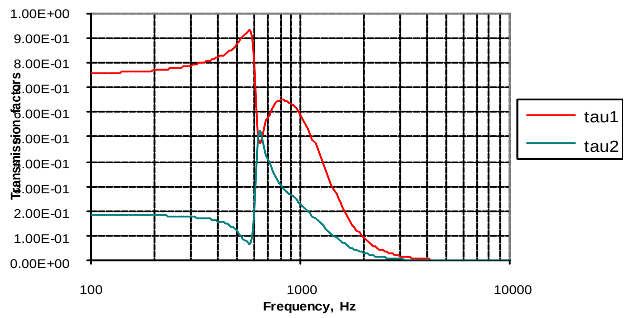

Fig -4: Frequency characteristics of transmission coefficients

They have the extreme values on the natural frequencies of transverse vibrations of the pinion and the gear, which are 581 Hz and 625 Hz respectively. In the high frequency range (above 6000 Hz) the values of τ_1 and τ_2 fall rapidly. The independence of these values from the frequency in the low frequency range can be explained in the following way. In Eq. (18), when $\omega^2 \ll \omega_1^2$ and $\omega^2 \ll \omega_2^2$, we have $\omega^2/\omega_{1,2}^2 \ll 1$. Then omitting the last values in Eq. (18) one can obtain

$$\tau_1 = 1/\left[1+\frac{\eta_2 C_1}{\eta_1 C_2}\left(\frac{1+\eta_1^2}{1+\eta_2^2}\right)+\frac{\eta_3 C_1}{\eta_1 C_3}\left(\frac{1+\eta_1^2}{1+\eta_3^2}\right)\right]. \qquad (19)$$

When the values of $\eta_1 = \eta_2 = 0.07$, which was accepted in the calculation, we receive τ_1 = 0.754 and τ_2 = 0.186. The same value we find from the graphics in the Fig. 4 in the range near 400 Hz. So that, in the low frequency range power transmission coefficients depends only on loss factors and stiffness coefficients. When $\eta_1 = \eta_2 = \eta_3$ then τ_1 and τ_2 are proportional to admittances of the supports and vice versa proportional to the total admittance of the dynamical system. So that it is necessary to increase supports stiffness and decrease the stiffness of the tooth engagement for reducing vibration power radiated into the supports.

CONCLUSIONS

The results obtained shows that the power radiated into the pinion support is considerably greater than the power radiated into the gear support. In the middle frequency range the resonance related to the pinion vibrations is more expressed. In low frequency range the magnitudes of the transmission coefficients are practically constant till the medium frequency range at 400 Hz. They have the extreme values at the natural frequencies of transverse vibrations of the pinion and the gear. In the high frequency range (above 6000 Hz) the values of transmission coefficients fall rapidly. The analysis also demonstrated that in the low frequency range power transmission coefficients depends only on loss factors and stiffness coefficients. For reducing vibration power radiated into the supports it is necessary to increase supports stiffness and decrease the stiffness of the tooth engagement.

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BIOGRAPHIES

He graduated in 1968 as mechanical engineer and in 1972 with Ph.D from Moscow Peoples Friendship University. In 1993 he earned Doctor of Science degree in Moscow Textile University. Since 1997 he is professor of Engineering Mechanics in the University of Guadalajara

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