Geometric coupling of scalar multiplets to *D*=4, *N*=1 pure supergravity

Paolo Di Sia[1,2,3]

[1] *Adjunct Professor, Department of Philosophy, Education and Psychology, University of Verona, Verona, Italy*
[2] *Professor, Higher Institute of Religious Science, Bolzano, Italy*
[3] *Member, ISEM, Institute for Scientific Methodology, Palermo, Italy*

---***---

Abstract - ***In this paper we consider the coupling of the scalar multiplets (multiplets of Wess-Zumino) to D=4, N=1 pure supergravity. We use the "geometric approach", that is all fields are considered as superforms in superspace and we use the concepts of supersymmetry, superspace and rheonomic principle. The Bianchi identities are analyzed and resolved, ending with the Bianchi identity of gravitino.***

Key Words: *Supergravity, Scalar (Wess-Zumino) Multiplets, Supersymmetry, Superspace, Rheonomic Principle, Geometric Coupling, Differential Geometry.*

1. INTRODUCTION

With *N*=1 supergravity (*N* is the number of supersymmetry generators) in the "geometric approach" we work with the vierbein V^a, the gravitino ψ and the spin connection ω^{ab}. From the particle point of view V^a, ψ and ω^{ab} describe the *N*=1 gravitational multiplet [2, 3/2] [1,2]. The main goal of this paper is the coupling of such multiplet to *n* multiplets of Wess-Zumino:

$$[1/2, 0^+, 0^-], \tag{1}$$

described by the set of 0-forms:

$$[\Lambda^i, A^i, B^i], \ (i = 1, \ldots, n). \tag{2}$$

A^i and B^i are a real scalar and a real pseudo-scalar respectively, Λ^i is a Majorana spinor. From a phenomenological point of view, the scalar multiplets contain quarks, leptons and Higgs particle together with their superpartners, i.e. squarks, sleptons, Higgsino.
It is possible to introduce a set of complex fields z^i :

$$z^i = A^i + i\,B^i\,;\ \bar{z}^{i^*} \overset{def}{=} (z^i)^* = A^i - i\,B^i\,, \tag{3}$$

and we can consider them as coordinates of a *n*-dimensional complex manifold $\mathscr{M}$ with Kählerian structure. On $\mathscr{M}$ the Kähler potential is introduced:

$$G = G(\,z^i, \bar{z}^{i^*}\,);\quad G = G^*\,. \tag{4}$$

We introduce also for later use the chiral projections of spinors Λ^i and ψ :

$$\Lambda^i = \chi^i + \chi^{i^*}\,;\ \chi^i = \frac{1+\gamma_5}{2}\Lambda^i;\ \chi^{i^*} = \frac{1-\gamma_5}{2}\Lambda^i\ \ ; \tag{5}$$

$$\gamma_5\,\chi^i = \chi^i;\ \gamma_5\,\chi^{i^*} = -\chi^{i^*};\ \chi^{i^*} = C\,\gamma_0{}^T\,(\chi^i)^*\,; \tag{6}$$

$$\psi = \psi_\bullet + \psi^\bullet;\ \psi_\bullet = \frac{1+\gamma_5}{2}\psi;\ \psi^\bullet = \frac{1-\gamma_5}{2}\psi\ \ ; \tag{7}$$

$$\gamma_5\,\psi_\bullet = \psi_\bullet;\ \gamma_5\,\psi^\bullet = -\psi^\bullet;\ \psi^\bullet = C\,\gamma_0{}^T\,(\psi_\bullet)^*\,; \tag{8}$$

$$\bar{\psi}^\bullet = (\psi_\bullet)^+\,\gamma_0 = \psi^+\,(\frac{1+\gamma_5}{2})\gamma_0 = \bar{\psi}\,(\frac{1-\gamma_5}{2})\ ; \tag{9}$$

$$\bar{\psi}_\bullet = (\psi^\bullet)^+\,\gamma_0 = \psi^+\,(\frac{1-\gamma_5}{2})\gamma_0 = \bar{\psi}\,(\frac{1+\gamma_5}{2})\ ; \tag{10}$$

$$\bar{\chi}^i = (\chi^{i^*})^+\,\gamma_0 = \bar{\Lambda}^i\,(\frac{1+\gamma_5}{2})\ ; \tag{11}$$

$$\bar{\chi}^{i^*} = (\chi^i)^+\,\gamma_0 = \bar{\Lambda}^i\,(\frac{1-\gamma_5}{2})\ . \tag{12}$$

2. COUPLING OF SCALAR MULTIPLETS TO PURE SUPERGRAVITY

The coupling of scalar multiplets to pure supergravity corresponds to the construction of a "cross-section" of the fiber bundle $B(R^{4/4}, \mathscr{M})$, which has the *N*=1 superspace $R^{4/4}$ as support space and the Kähler manifold $\mathscr{M}$ as fiber [3]. The coordinate *z* is a superfield [4]:

$$z^i = z^i\,(x,\theta)\,, \tag{13}$$

therefore at every point $(x,\theta) \in R^{4/4}$ of the support it is associated a point $z^i \in \mathscr{M}$ of the fiber. Developing dz^i in the basis (V , ψ), we find:

$$dz^i = Z^i{}_a V^a + \bar{\chi}^i \psi_\bullet = Z^i{}_a V^a + \bar{\psi}_\bullet \chi^i ; \qquad (14)$$

$$d\bar{z}^{i^*} = \bar{Z}^{i^*}{}_a V^a + \bar{\chi}^{i^*} \psi^\bullet = \bar{Z}^{i^*}{}_a V^a + \bar{\psi}^\bullet \chi^{i^*} . \qquad (15)$$

In doing that we imposed the rheonomy, writing the "out" component of dz as χ, that is the field of spin 1/2. Therefore $Z^i{}_a$ is a vector field and $\bar{\chi}^i$ a left-handed spinor field. The action of the Kähler transformation on fermionic fields can be defined as a chiral rotation [5]. The Kähler connection is defined as:

$$Q = \frac{1}{2i}(\partial_i G\, dz^i - \partial_{i^*} G\, d\bar{z}^{i^*}) . \qquad (16)$$

The curvature of the Kähler connection is the 2-form K defined as:

$$K = dQ = i\, g_{ij^*}\, dz^i \wedge d\bar{z}^{j^*} . \qquad (17)$$

Using relations (14,15) and defining the quantities:

$$K_{ab} = i\, g_{ij^*} Z^i{}_{[a} \bar{Z}^{j^*}{}_{b]} , \qquad (18)$$

$$T_a = \bar{\chi}^i \gamma_a \chi^{j^*} g_{ij^*} , \qquad (19)$$

$$\Sigma^a{}_\bullet = i\, g_{ij^*} \chi^i \bar{Z}^{j^* a} , \qquad (20)$$

$$\Sigma^\bullet{}_a = (\Sigma_{\bullet a})^C = -i\, g_{i^* j} \chi^{i^*} Z^j{}_a , \qquad (21)$$

we can write:

$$K = K_{ab} \wedge V^a \wedge V^b + \frac{i}{2} T_a \bar{\psi}_\bullet \wedge \gamma^a \psi^\bullet + $$
$$+ \bar{\psi}_\bullet \Sigma^a{}_\bullet \wedge V_a + \bar{\psi}^\bullet \Sigma^\bullet{}_a \wedge V^a . \qquad (22)$$

The exterior derivatives of fields of matter z^i e χ^i are the correspondent of curvatures R^{ab}, R^a e ρ of the supergravity fields. More precisely, it is helpful to define as "curvature" of χ the covariant derivative $\nabla\chi^i$, which is covariant both with respect to Lorentz transformations, and with respect to Kähler transformations and to coordinates transformations on the Kähler manifold. In general, all covariant derivatives of fermions contain the Kähler connection, for being covariant under Kähler transformations. Therefore the set of supergravity curvatures of Wess-Zumino multiplets are given by:

$$R^a = \mathscr{D} V^a - i\bar{\psi}^\bullet \wedge \gamma^a \psi_\bullet , \qquad (23)$$

$$(\mathscr{D} V^a \equiv dV^a - \omega^{ab} \wedge V_b),$$

$$R^{ab} = d\omega^{ab} - \omega^a{}_c \wedge \omega^{cb} \equiv \mathscr{R}^{ab}, \qquad (24)$$

$$\rho_\bullet = \nabla \psi_\bullet , \qquad (25)$$

$$R(z)^i \overset{def}{=} dz^i , \qquad (26)$$

$$R(\chi)^i \overset{def}{=} \nabla \chi^i , \qquad (27)$$

with:

$$\nabla \psi_\bullet = d\psi_\bullet - \frac{1}{4}\omega^{ab} \wedge \gamma_{ab} \psi_\bullet + \frac{i}{2} Q \wedge \psi_\bullet , \qquad (28)$$

$$\nabla \chi^i = d\chi^i - \frac{1}{4}\omega^{ab} \wedge \gamma_{ab} \chi^i + \left\{ {i \atop j\,k} \right\} dz^j \chi^k + \frac{i}{2} Q \chi^i . \qquad (29)$$

3. BIANCHI IDENTITIES OF SUPERGRAVITY COUPLED TO SCALAR MULTIPLETS

The Bianchi identities of pure supergravity coupled to multiplets of Wess-Zumino are given by [6]:

$$\mathscr{D} R^a + R^{ab} \wedge V_b + i\bar{\rho}^\bullet \wedge \gamma^a \psi_\bullet - i\bar{\psi}^\bullet \wedge \gamma^a \rho_\bullet = 0 , \qquad (30)$$

$$\mathscr{D} R^{ab} = 0 , \qquad (31)$$

$$\nabla \rho_\bullet + \frac{1}{4} R^{ab} \wedge \gamma_{ab} \psi_\bullet - \frac{i}{2} K \wedge \psi_\bullet = 0 , \qquad (32)$$

$$d\, dz^i = 0 , \qquad (33)$$

$$\nabla\nabla \chi^i + \frac{1}{4} R^{ab} \wedge \gamma_{ab} \chi^i + \frac{i}{2} K \chi^i -$$
$$- R_{m^* j}{}^i{}_k\, d\bar{z}^{m^*} \wedge dz^j \chi^k = 0 . \qquad (34)$$

4. SOLUTIONS OF BIANCHI IDENTITIES AND AUXILIARY FIELDS

In Bianchi identities (30-34) we insert the following informations:

1) the equation of the torsion:

$$R^a = 0 ; \qquad (35)$$

2) the rheonomic condition:

$$dz^i = Z^i{}_a V^a + \bar{\chi}^i \psi_\bullet . \tag{36}$$

The first condition is a kinematic constraint that can be imposed redefining the spin connection, while the rheonomic condition defines who are the supersymmetric partners of z^i. To give these two conditions is equivalent to fix the supersymmetry transformations [7]. They set the supersymmetry transformation laws of V^a e z^i:

$$\delta_\varepsilon z^i = \bar{\chi}^i \varepsilon_\bullet , \tag{37}$$

$$\delta_\varepsilon V^a = i\bar{\varepsilon}^\bullet \gamma^a \psi_\bullet - i\bar{\psi}^\bullet \gamma^a \varepsilon_\bullet . \tag{38}$$

The fields $Z^i{}_a$ e $\bar{Z}^{i^*}{}_a$ which appear in relations (14,15) are the components along the vierbeins of dz^i e $d\bar{z}^{i^*}$. We undeline that $Z^i{}_a V^a{}_\mu$ is not equal to $\partial_\mu z^i$, as would happen if the theory was formulated only on space-time. Indeed, projecting relation (14) along the differentials of space-time coordinates, we get:

$$\partial_\mu z^i = Z^i{}_a V^a{}_\mu + \bar{\chi}^i \psi_{\bullet\mu} \Rightarrow$$

$$\Rightarrow Z^i{}_\mu = Z^i{}_a V^a{}_\mu = \partial_\mu z^i - \bar{\chi}^i \psi_{\bullet\mu} . \tag{39}$$

In the "component approach" $Z^i{}_\mu$ is also called "supercovariant derivative" of z^i. For solving the Bianchi identities (30-34) we must insert the most general rheonomic parametrizations of curvatures R^{ab} e ρ, which are given by:

$$R^{ab} = R^{ab}_{cd} V^c \wedge V^d + \bar{\theta}^{ab}{}_c \psi \wedge V^c + \bar{\psi} \wedge K^{ab} \psi , \tag{40}$$

$$\rho = \rho_{ab} V^a \wedge V^b + H_c \psi \wedge V^c + \Omega_{\alpha\beta} \psi^\alpha \wedge \psi^\beta . \tag{41}$$

For pure supergravity, values of $\bar{\theta}^{ab}{}_c$, K^{ab}, H_c, $\Omega_{\alpha\beta}$ have been fixed as [7]:

$$K^{ab} = K^a = \Omega_{\alpha\beta} = \bar{\theta}^a{}_c = H_c = 0 , \tag{42}$$

$$\bar{\theta}^{pq}_d = -\varepsilon^{pqrs} \bar{\rho}_{rs} \gamma_5 \gamma_d - \delta^{[p}_d \varepsilon^{q]mst} \bar{\rho}_{st} \gamma_5 \gamma_m . \tag{43}$$

With the presence of matter fields, values of previous tensors may also depend from the matter fields, because we can even use for their construction the "inner" fields $Z^i{}_a$ and $\nabla_a \chi^i$ [6]. It is therefore appropriate to solve the Bianchi identities of the supergravitational "off-shell" multiplet, i.e. without giving the explicit form of these tensors. Then, introducing the new general parameterizations in the complete Bianchi identities of "pure supergravity + Wess-Zumino multiplets" (30-34), we obtain the explicit form of $\bar{\theta}^{ab}{}_c$, K^{ab}, H_c, $\Omega_{\alpha\beta}$ in terms of the matter fields and supergravity fields.

Preliminarily we solve the gravitational "off-shell" Bianchi identities. Inserting equation (35) in (30) and considering the cancellation of $\psi \wedge V \wedge V$ terms, we obtain the same solution of pure supergravity [7].

The $\psi \wedge \psi \wedge V$ sector gives:

$$\bar{\psi} \wedge K^{ab} \psi \wedge V_b - i\bar{\psi} \wedge \gamma^a H^b \psi \wedge V_b = 0 . \tag{44}$$

This equation breaks in two parts:

$$\bar{\psi} \wedge \gamma^{\{a} H^{b\}} \psi = 0 , \tag{45}$$

$$\bar{\psi} \wedge \gamma^{[a} H^{b]} \psi = -i\bar{\psi} \wedge K^{ab} \psi . \tag{46}$$

Eq. (45) is solvable decomposing H^b in a complete Dirac basis:

$$H_b = H^{(+)}{}_b + i H^{(-)}{}_b \gamma_5 + H^{(-)}{}_{b/a} \gamma_5 \gamma^a +$$
$$+ i H^{(+)}{}_{b/a} \gamma^a - \frac{1}{2} H_b^{cd} \gamma_{cd} . \tag{47}$$

The general solution is given by:

$$H_m = i A_m \gamma_5 + \wp \gamma_5 \gamma_m + i \mathscr{S} \gamma_m + i \gamma_5 \gamma_m{}^u A'_u , \tag{48}$$

with A_m and A'_m two axial vectors, $\mathscr{S}$ and $\wp$ a scalar and a pseudo-scalar respectively. Replacing Eq. (48) in (46), we get:

$$K^{ab} = -i \wp \gamma_5 \gamma^{ab} - \mathscr{S} \gamma^{ab} - i \varepsilon^{abcd} A'_c \gamma_d . \tag{49}$$

The $\psi \wedge \psi \wedge \psi$ sector of (30) brings to:

$$-i\bar{\psi} \wedge \gamma^a \Omega = 0 \Leftrightarrow \bar{\Omega} \wedge \gamma^a \psi = 0 , \tag{50}$$

with:

$$\Omega = \Omega_{\alpha\beta} \psi^\alpha \wedge \psi^\beta . \tag{51}$$

Writing, in complete generality:

$$\Omega = i \zeta_a \bar{\psi} \wedge \gamma^a \psi - \frac{1}{2} \xi_{ab} \bar{\psi} \wedge \gamma^{ab} \psi , \tag{52}$$

and using the decomposition in irreducible representations [1]:

$$\zeta_a = \zeta_a{}^{(12)} + \frac{1}{4} \gamma_a \zeta^{(4)} , \tag{53}$$

International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395 -0056
Volume: 02 Issue: 09 | Dec-2015 www.irjet.net p-ISSN: 2395-0072

$$\xi_{ab} = \xi_{ab}{}^{(8)} - \gamma_{[a}\, \xi_{b]}{}^{(12)} - \frac{1}{12}\gamma_{ab}\, \xi^{(4)}, \tag{54}$$

we get:

$$\Omega = -\frac{1}{2}\gamma^{a}\, \zeta \overline{\psi} \wedge \gamma_{a} \psi + \frac{1}{2}\gamma_{ab}\, \zeta \overline{\psi} \wedge \gamma^{ab} \psi\,, \tag{55}$$

with ζ Majorana spinor.

Introducing the complex field:

$$S = \wp + i\, \mathscr{S}, \tag{56}$$

and using the chiral notation, we obtain:

$$R^{a} = 0\,; \tag{57}$$

$$\rho_{\bullet} = \rho_{\bullet}^{ab} V_{a} \wedge V_{b} + i A_{a}\, \psi_{\bullet} \wedge V^{a} + i A'_{a} \gamma^{ab} \psi_{\bullet} \wedge V_{b} + + S \gamma_{a}\, \psi^{\bullet} \wedge V^{a} + \psi_{\bullet} \wedge \overline{\psi}_{\bullet} \zeta_{\bullet} - \psi_{\bullet} \wedge \overline{\psi}^{\bullet} \zeta^{\bullet}\,; \tag{58}$$

$$R^{ab} = R^{ab}_{cd} V^{c} \wedge V^{d} - (2i\overline{\psi}^{\bullet} \gamma^{[a}\, \rho_{\bullet}{}^{b]c} + 2i\overline{\psi}_{\bullet} \gamma^{[a} \rho^{b]c} - i\overline{\psi}^{\bullet} \gamma^{c}\, \rho_{\bullet}{}^{ab} - i\overline{\psi}_{\bullet} \gamma^{c}\, \rho^{ab\bullet}) \wedge V_{c} - i S^{*}\, \overline{\psi}_{\bullet} \wedge \gamma^{ab} \psi_{\bullet} + + i S \overline{\psi}^{\bullet} \wedge \gamma^{ab} \psi^{\bullet} - 2i A'_{c} \overline{\psi}^{\bullet} \wedge \gamma_{d}\, \psi_{\bullet}\, \varepsilon^{abcd}\,. \tag{59}$$

The case of pure supergravity is recoverable from the previous if:

$$A_{a} = A'_{a} = S = \zeta_{\bullet} = 0 \;\Rightarrow\; \zeta^{\bullet} = 0\,. \tag{60}$$

It follows that in the theory coupled to matter the auxiliary fields A_a, A'_a, S and $\zeta_{\bullet}$ must be identified with convenient functions of the matter fields z^i and χ^i. In pure supergravity the Bianchi identities are invariant with respect to the scale transformations [1,7]:

$$\omega^{ab} \;\to\; \omega^{ab}, \tag{61}$$

$$V^{a} \;\to\; w V^{a}, \tag{62}$$

$$\psi \;\to\; \sqrt{w}\, \psi\,. \tag{63}$$

These transformations are extended in a consistent way to the matter fields by placing:

$$z^{i} \;\to\; z^{i}, \tag{64}$$

$$\chi^{i} \;\to\; \frac{1}{\sqrt{\lambda}} \chi^{i}, \tag{65}$$

with λ real constant parameter.

Naming $w(\phi)$ the scaling power of every field ϕ, we can rewrite relations (61-63) as:

$$w(V^{a}) = 1, \quad w(\psi) = 1/2, \quad w(\omega^{ab}) = 0\,. \tag{66}$$

From the rheonomic conditions (14,15) we get:

$$w(z^{i}) = w(Z^{i}{}_{a}) + w(V^{a}) = w(Z^{i}{}_{a}) + 1\,, \tag{67}$$

$$w(z^{i}) = w(\chi^{i}) + w(\psi) = w(\chi^{i}) + 1/2\,. \tag{68}$$

On the other hand, since z^i are coordinates of the Riemannian manifold of scalars, they are scale invariant. This leads to:

$$w(z^{i}) = 0 \;\Leftrightarrow\; w(Z^{i}{}_{a}) = -1\,, \tag{69}$$

$$w(\chi^{i}) = -1/2\,. \tag{70}$$

The remaining values are:

$$w(A_{a}) = w(A'_{a}) = -1, \tag{71}$$

$$w(S) = -1, \tag{72}$$

$$w(\zeta) = -1/2\,. \tag{73}$$

5. BIANCHI IDENTITY OF GRAVITINO

The solution of Bianchi identities is completed with the analysis of gravitino, the supersymmetric partner of graviton. The parameterization of the gravitino curvature is given by relation (58). We preliminarily observe that in (58) we can put $\zeta_{\bullet} = \zeta^{\bullet} = 0$, since the gravitino curvature $\rho_{\bullet}$ contains the Kähler connection Q, which was not in the definition of $\rho_{\bullet}$ in absence of matter. The additional term $i/2\, Q \wedge \psi_{\bullet}$ is a 2-form whose component along $\psi \wedge \psi$ can be identified with ζ. Therefore there is no restriction putting $\zeta = 0$. We can so rewrite relation (58) as:

$$\rho_{\bullet} = \rho_{\bullet}^{ab} V_{a} \wedge V_{b} + i A_{a}\, \psi_{\bullet} \wedge V^{a} + i A'_{a} \gamma^{ab} \psi_{\bullet} \wedge V_{b} + + S \gamma_{a}\, \psi^{\bullet} \wedge V^{a}\,. \tag{74}$$

The corresponding Bianchi identity is:

$$\mathscr{D} \rho_{\bullet} + \frac{1}{4} R^{ab} \wedge \gamma_{ab} \psi_{\bullet} + \frac{1}{2} g_{ij^{*}}\, dz^{i} \wedge d\overline{z}^{j^{*}} \wedge \psi_{\bullet} = 0\,. \tag{75}$$

Introducing in (75) the parametrizations (74) and (59) for R^{ab}, in $\psi\wedge\psi\wedge\psi$ sector we find:

$$A_a\,\psi_\bullet\wedge\bar\psi_\bullet\wedge\gamma^a\psi^\bullet + A'_a\gamma^{ab}\psi_\bullet\wedge\bar\psi_\bullet\wedge\gamma_b\psi^\bullet -$$
$$iS\gamma_a\,\psi^\bullet\wedge\bar\psi_\bullet\wedge\gamma^a\psi^\bullet + \frac{1}{4}\gamma_{ab}\psi_\bullet\wedge(-iS^*\bar\psi_\bullet\wedge\gamma^{ab}\psi_\bullet +$$
$$+iS\bar\psi^\bullet\wedge\gamma^{ab}\psi^\bullet - 2iA'_c\bar\psi^\bullet\wedge\gamma_d\psi_\bullet\varepsilon^{abcd}) +$$
$$+\frac{1}{2}g_{ij*}\psi_\bullet\wedge\left[(\bar\chi^i\psi_\bullet)\wedge(\bar\chi^{j^*}\psi^\bullet)\right]=0. \qquad (76)$$

Considering the validity of Fierz identities [1,7]:

$$\gamma_{ab}\psi_\bullet\wedge\bar\psi_\bullet\wedge\gamma^{ab}\psi_\bullet=0, \qquad (77)$$

$$\gamma_a\psi\wedge\bar\psi\wedge\gamma^a\psi=0, \qquad (78)$$

and that it holds:

$$\gamma_{ab}\psi_\bullet\wedge\bar\psi^\bullet\wedge\gamma^{ab}\psi^\bullet=0, \qquad (79)$$

for self-duality, we note that the field S does not give contribution. Considering that:

$$\psi\wedge\bar\psi\wedge\gamma_a\psi=\Xi_a^{(12)}, \qquad (80)$$

$$2\psi_\bullet\wedge\bar\psi_\bullet\wedge\gamma_a\psi^\bullet=\Xi_{a\bullet}^{(12)}, \qquad (81)$$

$$\gamma_{ab}\psi\wedge\bar\psi\wedge\gamma^b\psi=-\Xi_a^{(12)}, \qquad (82)$$

$$2\gamma_{ab}\psi_\bullet\wedge\bar\psi_\bullet\wedge\gamma^b\psi^\bullet=-\Xi_{a\bullet}^{(12)}, \qquad (83)$$

from relation (76) we get:

$$\frac{1}{2}A_a\,\Xi_\bullet^{a(12)} - \frac{1}{2}A'_a\,\Xi_\bullet^{a(12)} - \frac{1}{2}A'_a\,\Xi_\bullet^{a(12)} +$$
$$+\frac{1}{4}g_{ij*}\psi_\bullet\wedge\bar\chi^i(\bar\psi_\bullet\wedge\gamma_a\psi^\bullet)\gamma^a\chi^{j^*}=0. \qquad (84)$$

Therefore:

$$(\frac{1}{2}A_a - A'_a)\,\Xi_\bullet^{a(12)} + \frac{1}{8}\,\Xi_\bullet^{a(12)}\,(g_{ij*}\bar\chi^i\gamma_a\chi^{j^*})=0, \qquad (85)$$

from which it is:

$$A'_a=\frac{1}{2}A_a+\frac{1}{8}g_{ij*}\bar\chi^i\gamma_a\chi^{j^*}. \qquad (86)$$

Considering that the expression on the right side of relation (86) is the only object with correct weight scale, to which may be identified the fields A_a and A'_a, they are on fact not independent. In particular we choose $A_a=0$.

We consider now the $V\wedge\psi\wedge\psi$ sector of the same equation:

$$2i\rho_{\bullet ab}\bar\psi^\bullet\wedge\gamma^a\psi_\bullet\wedge V^b - i\gamma^a{}_b\psi_\bullet\wedge(\bar\psi_\bullet\nabla_{(0,1)\bullet}A'_a +$$
$$+\bar\psi^\bullet\nabla^\bullet{}_{(0,1)}A'_a)\wedge V^b - \gamma_b\psi^\bullet\wedge(\nabla_m S\bar\psi_\bullet\chi^m +$$
$$+\nabla_{m^*}S\bar\psi^\bullet\chi^{m^*})\wedge V^b + \frac{1}{4}\gamma_{ac}\psi_\bullet\wedge(\bar\psi_\bullet\theta^{ac}{}_{\bullet b} + \bar\psi^\bullet\theta^{\bullet ac}{}_b)\wedge$$
$$\wedge V^b + \frac{1}{2}g_{ij*}\psi_\bullet\wedge(\bar\psi_\bullet\chi^i\bar Z^{j^*}{}_b - \bar\psi^\bullet\chi^{j^*}Z^i{}_b)\wedge V^b=0, \qquad (87)$$

where with the symbol $\nabla_{(0,1)}$ we mean the coefficient along ψ of the exterior derivative. The cancellation of the coefficient of the current $\bar\psi^\bullet\wedge\gamma^{lm}\psi^\bullet$ brings to:

$$\nabla_{m^*}S=0, \qquad (88)$$

with:

$$\nabla_{m^*}=\partial_{m^*}-\frac{p'}{2}G_{m^*};\quad (G_{m^*}\equiv\partial_{m^*}G). \qquad (89)$$

In this case it is:

$$\nabla_{m^*}S=\partial_{m^*}S-\frac{1}{2}G_{m^*}S=0,\quad (p'=1) \qquad (90)$$

and this is equivalent to write:

$$\nabla_{m^*}S=e^{G/2}\partial_{m^*}(e^{-G/2}S)=0, \qquad (91)$$

which implies:

$$\partial_{m^*}(e^{-G/2}S)=0 \Rightarrow e^{-G/2}S=f(z), \qquad (92)$$

with $f(z)$ an arbitrary analytical function. Therefore:

$$S=f(z)\,e^{G/2}. \qquad (93)$$

On the other hand, the S field must be pure imaginary for the Majorana conditions, then the analytic function $f(z)$ must be equal to "i" times a constant, denoted by "e". The final solution is therefore:

$$S=iee^{G/2}. \qquad (94)$$

In the $\bar\psi_\bullet\wedge\gamma^{lm}\psi_\bullet$ sector we have:

International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395 -0056
Volume: 02 Issue: 09 | Dec-2015 www.irjet.net p-ISSN: 2395-0072

$$\frac{i}{8}\gamma^a{}_b\,\gamma_{lm}\,(\nabla_{(0,1)\bullet}\,A'_a)\,\overline{\psi}_\bullet\wedge\gamma^{lm}\,\psi_\bullet - \frac{1}{32}\gamma_{ac}\,\gamma_{lm}\,\theta^{ac}{}_{\bullet b}\,\overline{\psi}_\bullet\wedge$$
$$\wedge\gamma^{lm}\,\psi_\bullet - \frac{1}{16}g_{ij^*}\,\gamma_{lm}\,\chi^i\,\overline{Z}^{j^*}{}_b\,\overline{\psi}_\bullet\wedge\gamma^{lm}\,\psi_\bullet = 0. \tag{95}$$

Multiplying both sides of (95) for γ^{lm} and considering the relations:

$$\gamma^{ab}\,\gamma_{lm} = i\,\varepsilon^{ab}{}_{lm}\,\gamma_5 - 4\,\delta^{[a}{}_{[l}\,\gamma^{b]}{}_{m]} - 2\,\delta^{ab}{}_{lm}\,, \tag{96}$$

$$\gamma^{ab}\,\gamma^{cd}\,\gamma_{ab} = 4\gamma^{cd}\,, \tag{97}$$

we get:

$$\frac{i}{2}\gamma^a{}_b\,(\nabla_{(0,1)\bullet}\,A'_a) - \frac{1}{8}\gamma_{ac}\,\theta^{ac}{}_{\bullet b} + \frac{3}{4}g_{ij^*}\,\chi^i\,\overline{Z}^{j^*}{}_b = 0\,. \tag{98}$$

In the sector of current with one index of Eq. (75) we finally have:

$$2i\,\rho_{\bullet ab} - \frac{i}{2}\gamma^l{}_b\,\gamma_a\,(\nabla^\bullet{}_{(0,1)}\,A'_l) - \frac{ie}{4}\gamma_b\,\gamma_a\,\chi^m\,(\partial_m\,G)\,e^{G/2} +$$
$$+\frac{1}{8}\gamma_{lm}\,\gamma_a\,\theta^{\bullet\,lm}{}_b - \frac{1}{4}g_{ij^*}\,Z^i{}_b\,\gamma_a\,\chi^{j^*} = 0\,. \tag{99}$$

Multiplying both sides of (99) for γ^a, we arrive to the equation of motion of gravitino. The Bianchi identity (33) associated to the scalar field z^i can be used for the determination of the parametrization of the covariant derivative $\nabla\chi^i$. We write, in full generality:

$$\nabla\chi^i = \nabla_a\,\chi^i\,V^a + \mathscr{H}^i{}_a\,\gamma^a\,\psi^\bullet +$$
$$+\,\mathscr{H}^i{}_{ab}\,\gamma^{ab}\,\psi_\bullet + \mathscr{H}^i\,\psi_\bullet\,, \tag{100}$$

where the coefficients $\mathscr{H}^i{}_{..}$ depend by the field. Considering the exterior derivative of the rheonomic parametrization (14) and inserting relation (100), from $\psi\wedge\psi$ sector we have:

$$i\,Z^i{}_a\,\overline{\psi}^\bullet\wedge\gamma^a\,\psi_\bullet - \overline{\psi}^\bullet\wedge\gamma^a\,\psi_\bullet\,\mathscr{H}^i{}_a -$$
$$-\overline{\psi}_\bullet\wedge\gamma^{ab}\,\psi_\bullet\,\mathscr{H}^i{}_{ab} = 0, \tag{101}$$

which brings to:

$$\mathscr{H}^i{}_a = i\,Z^i{}_a\,;\quad \mathscr{H}^i{}_{ab} = 0;\quad \mathscr{H}^i = \text{free}. \tag{102}$$

The free function $\mathscr{H}^i$ is the auxiliary field of the scalar multiplet. Its Kähler weight is $p = 1/2$, therefore the explicit definition of its covariant derivative is:

$$\nabla\mathscr{H}^i = d\mathscr{H}^i + \begin{Bmatrix} i \\ j\,k \end{Bmatrix} dz^j\,\mathscr{H}^k - i\,Q\,\mathscr{H}^i\,. \tag{103}$$

The $\psi\wedge\psi$ sector of the Bianchi identity (34) connects $\mathscr{H}^i$, which is assumed as funcion of the scalar fields $(z^i, \overline{z}^{i^*})$, to the auxiliary field of supergravity S:

$$i\nabla_a\,\chi^i\,\overline{\psi}^\bullet\wedge\gamma^a\,\psi_\bullet - i\,\gamma^a\,\psi^\bullet\wedge\nabla Z^i{}_a - \psi_\bullet\wedge(\overline{\psi}_\bullet\,\chi^m\,\nabla_m\,\mathscr{H}^i +$$
$$+\overline{\psi}^\bullet\,\chi^{m^*}\,\nabla_{m^*}\,\mathscr{H}^i) = -\frac{1}{4}\gamma_{ab}\,\chi^i\,(i\,S\,\overline{\psi}^\bullet\wedge\gamma^{ab}\,\psi^\bullet -$$
$$-i\,S^*\,\overline{\psi}_\bullet\wedge\gamma^{ab}\,\psi_\bullet - i\,\nu\,\varepsilon^{abcd}\,T_c\;\overline{\psi}_\bullet\wedge\gamma_d\,\psi^\bullet) +$$
$$R_{m^*}{}^i{}_{j\,k}\,\overline{\psi}_\bullet\,\chi^{m^*}\wedge\overline{\psi}^\bullet\,\chi^j\,\chi^k - \frac{i}{2}\,p\,\chi^i\,T_a\,\overline{\psi}_\bullet\wedge\gamma^a\,\psi^\bullet = 0\,. \tag{104}$$

In the $\psi_\bullet\wedge\psi_\bullet$ sector, Eq. (103) becomes:

$$\frac{1}{8}\gamma_{ab}\,\chi^m\,\overline{\psi}_\bullet\wedge\gamma^{ab}\,\psi_\bullet\,\nabla_m\,\mathscr{H}^i = \frac{i}{4}\gamma_{ab}\,\chi^i\,\overline{\psi}_\bullet\wedge$$
$$\wedge\gamma^{ab}\,\psi_\bullet\,S^*\,, \tag{105}$$

that is:

$$\nabla_m\,\mathscr{H}^i = 2i\,\delta^i{}_m\,S^*\,, \tag{106}$$

with:

$$\nabla_m\,\mathscr{H}^i = \partial_m\,\mathscr{H}^i + \begin{Bmatrix} i \\ m\,j \end{Bmatrix}\mathscr{H}^j - \frac{1}{2}\partial_m\,G\,\mathscr{H}^i\,. \tag{107}$$

This differential equation is resolvable through the ansatz:

$$\mathscr{H}^i = a\,g^{ij^*}\,\partial_{j^*}\,e^{bG}\,, \tag{108}$$

with:

$$g^{ij^*}\,g_{mj^*} = \delta^i{}_m\,. \tag{109}$$

Inserting (108) in (107) and remembering (94), the equation is satisfied for:

$$a = \frac{2e}{p} = 4e;\quad b = p = \frac{1}{2}\,. \tag{110}$$

It has been therefore determined, by the analysis of Bianchi identities, the dependence of the gravitino field and of the auxiliary field of spin 1/2 from the Kähler potential $G(z,\overline{z})$:

$$S = i\,e\exp(\frac{G}{2})\,; \tag{111}$$

$$\mathscr{H}^{i} = 2e(g^{ij^*}\partial_{j^*}G)\exp(\frac{G}{2})\,. \qquad (112)$$

6. CONCLUSIONS

In this paper we have considered in a "geometric approach" the coupling of the scalar multiplets of Wess-Zumino to D=4, N=1 pure supergravity. Supergravity is the effective theory of superstring theory [8-11]. The "geometric approach" considers all fields as superforms in superspace. We have used the concepts of supersymmetry, superspace, rheonomic principle, Bianchi identities. The found results allow the construction of the complete action of pure supergravity coupled to scalar multiplets in a mathematically very elegant and general way.

REFERENCES

[1] P. Di Sia, *Supergravità nel superspazio: panoramica generale e analisi tecnica*, Roma, Italy: Aracne Editrice, 2014.

[2] F. Mandl, G. Show, *Quantum field theory*, UK: John Wiley & Sons, 1984.

[3] P. Di Sia, "Relevant tools in building supergravity theories", *Integrated Journal of British*, vol. 2, issue 3, pp. 1-5, May-June 2015.

[4] P. Di Sia, "About the importance of supersymmetry and superspace for supergravity", *International Journal of Innovative Science, Engineering and Technology* (IJISET), vol. 2, issue 4, pp. 495-500, April 2015.

[5] P. Di Sia, "Kähler manifolds as target space of supergravity theories: characteristics for D=4, N=1 theory", *International Journal of Current Research*, vol. 7, issue 1, pp. 11971-11980, Jan 2015.

[6] P. Di Sia, "Rheonomic principle, Bianchi identities and supergravity", *International Journal of Innovative Science, Engineering and Technology* (IJISET), vol. 2, issue 5, pp. 615-619, May 2015.

[7] P. Di Sia, "Geometric construction of D=4, N=1 pure supergravity", *International Research Journal of Engineering and Technology* (IRJET), vol. 2, issue 4, pp. 16-20, July 2015.

[8] D. Z. Freedman, A. Van Proeyen, *Supergravity*, Cambridge: Cambridge University Press, 2012.

[9] M. B. Green, J. H. Schwarz, E. Witten, *Superstring Theory: 25th Anniversary Edition*, Cambridge: Cambridge Monographs on Mathematical Physics, Cambridge University Press, 2012.

[10] P. Di Sia, *Extreme Physics and Informational / Computational Limits*, Journal of Physics, Conference Series, vol. 306, p. 012067, 2011.

[11] P. Di Sia, *Exciting Peculiarities of the Extreme Physics*, Journal of Physics, Conference Series, vol. 442, issue 1, p. 012068, 2013.

BIOGRAPHY

Paolo Di Sia is currently Adjunct Professor by the University of Verona (Italy), Professor at ISSR, Bolzano (Italy) and member of ISEM, Palermo (Italy). He obtained a 1st level Laurea in Metaphysics, a 2nd level Laurea in Theoretical Physics, a PhD in Mathematical Modelling applied to Nano-Bio-Technology. He interested in Classical Quantum Relativistic Nanophysics, Planck Scale Physics, Supergravity, Quantum Relativistic Information, Mind Philosophy, Quantum Relativistic Econophysics, Philosophy of Science, Science Education. He wrote 187 publications, is reviewer of 2 mathematics books, reviewer of 11 international journals, 8 Awards obtained, included in Who's Who in the World 2015 and 2016. He is member of 6 international scientific societies and member of 22 International Advisory/ Editorial_Boards.

paolo.disia@gmail.com
www.paolodisia.com

